ABSTRACT

This paper deals with some problems appearing on dimensioning the outgoing trunks from an exchange in a multi-exchange area in a city.

A model for iterative calculation on a computer is presented, and some exchanges in Copenhagen are examined by the aid of this model in order to study the problems. The model takes into consideration that the automatical exchanges in Copenhagen are of the L.M.Ericsson types ARF 10 and AKF 10. The model therefore is arranged for calculation of the congestion on the supposition that the trunks are connected to a link system, where the internal congestion and possibly the existence of gradings influence the final congestion. "The geometric group concept", presented by Smith and Bridgford, is used as a convenient method of calculation in connection with a computer.

The calculations show how the use of different dimensioning scales for the primary routes affects the economy, the congestion and the ability of overloading. The results show among other things that the running cost has a very flat minimum, when the distribution of trunks on direct routes and the tandem route is altered. The results also show that it is very important to control the congestion values by high traffic, since the overload capacity of a network with alternative routing and with an economical dimensioning often is very low.

INTRODUCTION

It is wellknown that a network with alternate routing contains a smaller number of trunks and therefore gives a cheaper value for the running cost than a network without alternate routing. It is also wellknown that the system with alternate routing can be optimized to give a minimum for the running cost. This occurs when you take the price for routing directly and for routing via a tandem exchange into consideration and determine the number of trunks in the primary routes so that the routes are exactly carrying an amount of traffic for which there is an economical argument, while the rest of the traffic is routed as overflow traffic to the tandem exchange.

In the following is regarded a network in a city area with many local exchanges having alternate routing via one tandem exchange. The complete annual price for carrying the traffic \( A_p \) from one of the existent exchanges A to a new exchange B will be

\[
U = N_p \cdot u_0 + A_p \cdot B_p \cdot \frac{u_1}{d_1} + \frac{u_2}{d_2},
\]

where

- \( N_p \) is the number of trunks on the direct route,
- \( B_p \) - congestion
- \( u_0 \) - annual price for one trunk on the direct route,
- \( u_1 \) - to the tandem exchange LTB
- \( u_2 \) - from the tandem exchange LTB
- \( d_1 \) - marginal utilization of a trunk to LTB
- \( d_2 \) - from LTB.

\( U \) has a minimum, when

\[
\frac{d(A_p, B_p)}{dN_p} = \frac{u_1}{u_2},
\]

i.e. when

\[
N_p = \frac{1}{u_1} + \frac{1}{u_2}
\]

the direct route is dimensioned from an improvement value

\[
F_{1, N_p} (A_p) = F_B = \frac{1}{u_1} + \frac{1}{u_2}
\]

(1)

According to this it should be easy to determine the number of trunks on the direct routes. In practice, however, a lot of circumstances must be taken into consideration. Some of them are mentioned in the following.

1. The use of formula (1) demands knowledge of the marginal utilization values \( d_1 \) and \( d_2 \). In a big network the appearance of one new exchange A will not affect the \( d \)-values much, since the route from LTB is carrying traffic from many exchanges. It is therefore defensible to use the existing values for \( d_2 \). The value of \( d_1 \) only depends on the dimensioning of the outgoing routes from the exchange A; therefore it is only possible to determine the \( d_1 \)-value by aid of iterative calculation of all routes from the exchange, where the start value of \( d_1 \) is arbitrarily fixed and where the calculations are repeated until \( d_1 \) at the beginning and the end of a calculation series has almost the same value.

2. The method of economical optimizing only ensures that the trunks are used in the most economical manner. The amount of traffic lost on account of congestion is however not treated by the method. The value of the final...
INPUT OF SERVICE CRITERION "c", AND "d" VALUES, TRAFFIC VALUES, DISTANCES ECT.

CALCULATION WITH NEW "d"-VALUE

n = 0

CALCULATION OF "d"-VALUE

SELECTION OF SCALE

DETERMINATION OF THE NUMBER OF TRUNKS Np

Np > 0

YES

NO

CALCULATION OF THE CONGESTION

SUMMING UP MEAN AND VARIANCE OF THE OVERFLOW TRAFFIC

n = 0

YES

NO

DETERMINATION OF "d"-VALUE CORRESPONDING TO THE PEAKEDNESS VALUE

CONGESTION > B2

YES

NO

DETERMINATION OF "d"-VALUE FOR THE PEAKEDNESS VALUE FOR THE OVERFLOW TRAFFIC

FIXING OF B2

NEW RUN

n = n + 1

CALCULATION OF THE CONGESTION

CONGESTION > B1

YES

NO

N1 = N1 - 25

Calculation of B1

NO

N2 = N2 + 1

Calculation of B2

NO

U IS INCREASED BY COSTS FOR N1 AND FOR THE TRUNKS CARRYING TRAFFIC FROM LT8 TO THE ARRIVAL EXCHANGES

SUMMING UP MEAN AND VARIANCE OF THE OVERFLOW TRAFFIC

n = n + 1

NEW RUN

NO

YES

SUMMING UP THE RUNNING COST U

NO

YES

OUTPUT OF THE CALCULATIONS

NEW RUN

END

FIG.1 FLOWDIAGRAM OF THE CALCULATION MODEL.

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congestion for the offered traffic $A_p$ is

$$B_t = B_p \cdot ((B_s+B_t+B_u)-(B_s\cdot B_t+Bu\cdot B_s+Bu\cdot B_t+Bs\cdot B_t \cdot Bu)),$$

where $B_t$ is the congestion on the links in LTB, $B_s$ - route to LTB and $B_u$ - from LTB.

When $B_s$ and $B_u$ are essentially smaller than $B_t$, the expression is simplified to

$$B_t = B_p \cdot B_s.$$

The supposition corresponds with the case where the traffic via the tandem exchange is only meeting a negligible congestion in the tandem exchange itself and on the route from the exchange.

The final congestion to different B-exchanges will not have the same value due to the use of different improvement values for the direct routes. It is a problem, why the traffic to one or more B-exchanges is meeting a final congestion bigger than allowed, although the average congestion has a satisfactory value.

3. By fixation of the maximum value of the congestion it is expedient to look at the things by normal traffic as well as by overload. Overload is here regarded as a condition that occurs on a few days during the year, where the traffic is especially high, but not catastrophe affected.

By fixation of the congestion by overload the network is guaranteed a good overload capacity.

Networks with alternate routing will often have a very low overload capacity. The grade of service by normal traffic must therefore be set to a very low value to ensure a good service by overload condition.

4. By dimensioning the number of trunks by use of $B_p$-scales and by calculation of the corresponding congestion values it must be taken into consideration that the routes are often connected to selector systems with grading and internal congestion. It is therefore not correct to use the Erlang formula, which must be replaced by one or several approximation formulas. By the selection of an approximation formula significant value shall be taken to the accuracy and to the suitability of the formula for calculation on an electronic computer.

5. By the reflections concerning the dimensioning scales it must be considered whether there is a reasonable relation between the economical profit attained by using more complicated dimensioning scales and the additional work connected to this.

In the following a description is given of a model for iterative calculation of the outgoing network from an exchange by use of a computer. A number of calculation results are presented and are last discussed.

A MODEL FOR ITERATIVE CALCULATION OF THE OUTGOING NETWORK FROM AN EXCHANGE BY AID OF AN ELECTRONICAL COMPUTER

The programme for the model has been written in algol and the calculations have been carried out on the Danish third generation computer RC 4000 manufactured by A/S Regnecentralen, Copenhagen. Fig.1 shows a flow diagram of the model.

The calculations start with calculation of $F_B$ for the different direct routes on the basis of input values for the distances between the exchanges, including the tandem exchange, the cable price of one km trunk connection, and the price of the exchange equipment for one connection by direct routing and by routing to and from the tandem exchange.

In the expression for $F_B$ $d_0$ is regarded to be without influence from the appearance of the exchange itself in the common network. When the purpose is to find the effect of different dimensioning scales on one exchange it can further be regarded as defensible to use the same $d_0$-value for all routes from the tandem exchange. The model therefore is arranged for input of one $d_0$-value.

In a later section the relations are discussed, when the assumption of a fixed $d_0$-value does not hold.

As mentioned in the previous section, $d_1$ is not known at the beginning of the calculation, and the model is therefore using the described iterative calculation method, where the calculations are repeated until $d_1$ has almost the same value at the beginning and the end of a calculation series.

When $F_B$ for a given traffic has been calculated, the next step is to determine the number of trunks $N_p$ in the direct route. The number of trunks $N_p$ is determined so that $A_p (N_p(A_p)-B_p(A))$ is equal to $F_B$ or as near to $F_B$ as possible. Instead of calculating the number of trunks corresponding to all occurring $F_B$-values, it is suitable to prepare scales for $N_p$ as a function of $A_p$ by a limited number of fixed $F_B$-values, covering all in practice occurring values. In the model there are as a maximum used scales corresponding to 6 $F_B$-values. The scale which $F_B$-value is nearest to the calculated value for the traffic $A_p$ is then used.

By the determination of the congestion $B_p$ regard must be taken to the fact that the automatic $p$-exchanges in Copenhagen are of the L.M.Ericsson types ARF 10 and ARF 10, which are working with gradings and with internal congestion in the group selector stages. The calculation method "The geometric group concept", by Smith and Bridgford, is convenient to use on a computer.

By aid of this method and the expression

$$F_B = A_p \cdot (B_p(A_p)-B_p(A))$$

scales have been calculated for the $F_B$-values $1/2$, $1/3$, $1/4$, $1/5$, $1/6$, and $1/10$, and for the cases with selector availabilities of $K=10$, 20 and 40. There have also been calculated scales, where Erlang's B-formula is used for determining the congestion. Fig.2 gives an extract from the scales.

The table shows that for small routes "The geometric group concept" is giving nearly the same number of trunks calculated from "The geometric group concept" and Erlang's B-formula.
When the number of trunks \( N_p \) has been determined for the traffic \( A_p \), the congestion \( B_p \) is calculated by aid of "the geometric group concept" and consequently the overflow traffic from the route is known.

By going over all the primary routes the complete overflow traffic \( A_o \) to the tandem exchange results. This traffic is not quite random. The variance is equal to the sum of the variance values of overflow traffic from all the direct routes. These variance values are in the model determined from 3 curves for variance/mean as a function of the number of trunks by \( k=10, 20 \) and \( 40 \). The curves have been constructed on the basis of earlier presented tables and curves by Wilkinson, Lotze and Bridgford.

A primary route to the tandem exchange with overflow to its normal overflow route is established to carry traffic to exchanges where the traffic values are too small to give own primary routes. To this route also long distance traffic and traffic to special services are offered. The traffic to the route is called \( A_{op} \). According to this all traffic values have possibility of testing 2 routes, and the different congestion values will not disperse so much as by direct testing on the secondary route.

By dimensioning the secondary route, the final congestion \( B_s \) is at first fixed. Then \( B_s \) is determined as

\[
B_s = \frac{A_{op}}{\delta_{B_p}}
\]

It must be taken into consideration that the network shall have a reasonable overload capacity. That means that the bigger exchange the smaller \( B_s \)-value can be accepted. It is not possible to give an exact formula for \( B_s \), as the overload capacity is not only a function of \( \delta_{A_p} \), but also of the number of routes, of the offered traffic to the individual routes and of the availabilities in the different gradings. A \( B_s \)-value, giving a reasonable overload capacity, is therefore best determined individually for the different exchanges.

For determining the number of trunks \( N_s \), the model uses a method where the number of trunks is successively increased, and the process is stopped when the congestion is smaller than or equal to the earlier fixed value for \( B_s \).

The number of trunks fixed by the above-mentioned method is not correct because the traffic is not quite random, and the calculation method for the congestion assumes this. To the value for \( N_s \) there is therefore given an addition \( \delta_N \), the effect of which is to give the desired congestion by the existing peakedness value \( p \). The final value of \( N_s \) is known, the marginal value can be determined as

\[
\delta_N = \frac{A_{op}}{B_s \cdot k}
\]

The expressions for \( \delta_N \) and \( k \) are determined empirically by using curves from the work of Bridgford: "The geometric group concept and its application to the dimensioning of link access systems".

After determining the real value for \( \delta_N \), the calculations are, as previously mentioned, repeated with the new \( \delta_N \)-value in the expression for \( B_s \), and the calculations continue in this manner until \( \delta_N \) approaches a constant value.

When \( N_s \) has been determined, a calculation of the running cost by carrying the traffic from the exchange is made. The cost comprises both the cost for trunks and for exchange equipment for carrying the traffic on to the arrival exchanges. By the calculation of the trunk cost from the tandem exchange to the arrival exchange, the marginal values of traffic on a trunk are used, as the assumption is that the network has been established and the ground expenses have been paid.

For investigation of the state by high traffic the traffic values are at last increased by 20% and the corresponding congestion values are calculated on the supposition that the earlier determined values for \( N_p \) and \( N_s \) are unchanged.

Results

Two exchanges have been examined: Tastrup, a smaller exchange situated far from city in a suburb area, and Valby, a bigger exchange close to city in an area with a good deal of industry.

For both exchanges calculations with dimensioning have been made by using the following scale sets for determining the number of trunks in the direct routes:

a) 18 scales corresponding to \( B_p = \frac{1}{1.2} \cdot 1.5 \cdot 2 \cdot 3 \cdot 5 \) and \( \frac{1}{10} \) calculated for the selector availabilities \( k=10, 20 \) and \( 40 \) by aid of "The geometric group concept" method.

b) 6 scales with the same \( B_p \)-values as above, but calculated by aid of Erlang's \( B \)-formula.

c) As a), but with the number of trunks 1 and 2 changed to 0.

d) As b), but with the number of trunks 1 and 2 changed to 0.

e) Only 1 scale corresponding to \( B_p = \frac{1}{2} \) calculated by aid of Erlang's \( B \)-formula, however with the number of trunks 1 and 2 changed to 0.

f) As e), but for \( B_p = \frac{1}{4} \).

g) As e), but for \( B_p = \frac{1}{1.5} \).

h) As e), but for \( B_p = \frac{1}{2} \).

When an offered traffic value has no coincidence with a scale value, the number of trunks can be determined in two ways: By choosing the number corresponding to the nearest higher or lower traffic value. Calculations for both methods are made for Tastrup, but not for Valby, where coincidence exists.

Extracts of the results are given in fig.3 and 4. As an example the output for a single calculation is shown in fig.5. The computer time for the example was about 7 seconds.

### Table

<table>
<thead>
<tr>
<th>Calcul. number</th>
<th>Scale set</th>
<th>Annual cost in Dkr.</th>
<th>Number of direct routes</th>
<th>Normal traffic Mean</th>
<th>Normal traffic Max. Value</th>
<th>Normal traffic Mean</th>
<th>Normal traffic Max. Value</th>
<th>Normal traffic Mean</th>
<th>Normal traffic Max. Value</th>
</tr>
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<td>105 a</td>
<td>226700</td>
<td>40</td>
<td>0.4 ( \cdot 10^{-7} )</td>
<td>1.6 ( \cdot 10^{-7} )</td>
<td>0.1344</td>
<td>3.2 ( \cdot 10^{-5} )</td>
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<td>0.0668</td>
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<td>0.0376</td>
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<td>0.1200</td>
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<td>0.9 -</td>
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<td>1.6 -</td>
<td>0.0091</td>
<td>0.0146</td>
<td>0.3665</td>
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</table>

Fig. 3 Calculation results for Valby

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\( d_2 = 0.65 \) \( A_p = 442 \) erlang
### Calculations for Traffic Flow at Tøstrup

<table>
<thead>
<tr>
<th>Calcul. number</th>
<th>Scale set/ Routed up or down</th>
<th>Annual cost in D.Kr.</th>
<th>Number of direct routes</th>
<th>Normal traffic</th>
<th>Normal traffic + 20%</th>
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</thead>
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<td>$B_T$ Max. value</td>
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</table>

**Fig. 4** Calculation results for Tøstrup

### Calculations for Traffic Flow at Valby

<table>
<thead>
<tr>
<th>Calcul. number</th>
<th>Scale set/ Routed up or down</th>
<th>Annual cost in D.Kr.</th>
<th>Number of direct routes</th>
<th>Normal traffic</th>
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**Fig. 5** Example of output from a calculation for Valby

### Cost Calculation Number

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**Table 1** Traffic Flow Calculations

*p: p = cm, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50*

### Traffic Flow Calculations

- **Traffic is transferred to the primary route to the tandem exchange LT**

### Example

- **Cost Calculation Number:** 102

### Notes

- For more details, refer to the sources mentioned in the text.
Among other things the following can be drawn from the calculations:

The use of FP-values calculated from Erlang's B-formula gives, roughly speaking, the same economy as the use of FP-values calculated by aid of "The geometric group concept".

The changing of the number of trunks 1 and 2 to 0 has a very little effect on the economy.

The congestion B2 has — especially regarding the big exchange Valby — a very low value. On account of the internal congestion there will be a lower limit for the congestion, and the number of necessary trunks B2 will therefore be great, when B2 is approaching 0. As a result of this the overload capacity will be big. The results from calculation number 112 demonstrate clearly the matter.

When the direct routes are dimensioned for different congestion values — as is the accident when scales computed for FP-values are used — the final congestion values will be different too. The variation will be greatest on exchanges where the primary mean congestion is smallest, and where there are routes with 1 and 2 trunks. For Valby the mean primary congestion by dimensioning after scale set a) is 15%, while for Tastrup it is 9%. This table shows, in accordance with this, that the variation is greater for Valby than for Tastrup. By abolishing the routes with 1 and 2 trunks, the big "outsider" congestion values disappear, and the variation is therefore smaller.

An investigation for the FP-values for the different routes from Tastrup and Valby shows that most of the values are situated in the interval 1/2-1/2.5. The use of only one scale with a FP-value about 1 therefore gives a very good economy. The additional cost compared with the cost by a more differentiated dimensioning is for Tastrup 3-4% and for Valby about 2%.

Rounding up or down the number of trunks does not much influence the economy and the congestion variation by overload in the cases, where there are used an economical dimensioning or simplified scales with a quite good economy.

This is the case by the scale sets a)-d) and f)-g). By scale sets a) and b) it seems to be the best thing to round down, while scale set g) gives a better economy by rounding up.

CRITICAL ESTIMATE OF THE MODEL

1. Influence of the tandem exchange and its outgoing trunks

In the preceding facts it has been provided that the overflow traffic meets no congestion in the tandem exchange and on its outgoing routes. In reality there will often be congestion of not negligible value. The expression for Br will then be altered from

\[ B_r = B_p - B_2 \]  

to

\[ B_r = B_p (B_4 + B_6 + B_9). \]  

The growth factor is \( f = (B_4 + B_6 + B_9).B_p \), and \( f \) will have the greatest value by the calculations where \( B_2 \) is small.

A necessary assumption for a reasonable dimensioning is to fix an upper limit for the final congestion values. It is possible to reduce \( B_2 \) by a more abundant dimensioning of \( B_4 \), but \( B_4 \) can never be less than \( B_p (B_4 + B_6) \) and in practice it will be too expensive to go to the limit, because in principle this demands a secondary route with an infinite number of trunks. The tandem exchange and its outgoing routes therefore have to be dimensioned for a very small congestion, and the overload capacity must be good. The latter thing is especially important, because the alternate routing system can involve a very big increase of the traffic from the tandem exchange by heavy traffic.

2. Variation of \( B_2 \) for overflow traffic from different direct routes

Above it is provided that the secondary route to the tandem exchange gives the same congestion for all over-

flow traffic quantities. It is wellknown that this supposition is only correct when all overflow quantities have the same peakedness value \( p \). By the here treated exchanges nevertheless \( p \) will not variate so much as by full availability conditions. \( p \) will be smallest for routes with few trunks. According to that, the congestion on the small routes will normally be high, but the just described phenomenon will consequently reduce it again. The combined effect will be that big congestion values are reduces, while small values are increased, that is a smoothing of the different values.

3. Variation of \( d_2 \)

In the described model it is provided that \( d_2 \) is constant and has the same value for all routes from the tandem exchange. This supposition is reasonable, when the purpose is to study the effect of different dimensioning scales on a destined exchange. It is only possible to determine the absolute values for the running cost and the congestion values with knowledge of the \( d_2 \)-values for all routes from the tandem exchange. This leads to setting up a bigger model, including the whole network in the area. Propositions for models of this type have been given by several authors. B.Wallström has thus in Ericsson Technics presented a model, which, like this paper, applies "The geometric group concept" as a convenient tool for the calculation of the congestion.

However, it is without a more complicated model possible to draw the following simple conclusion: If a dimensioning method involves an especially high overflow traffic and this method is used on all the exchanges, the effect will be that a very big part of the traffic on the routes from the tandem exchange will be overflow traffic. It is then necessary to dimension these routes especially plentiful, as the overflow traffic is increasing very much by increasing the primary traffic. As an example it can is fig.3 and 4 be seen that dimensioning after scale set e) gives a smaller overflow traffic than after scale set c). For the calculations corresponding to the first case it should therefore be reasonable to use a bigger value for \( d_2 \). The calculations are according to this repeated with the bigger value \( d_2 = 0.70 \) and the running cost is then for Valby reduced from D.Kr.342,900 to 339,000 and for Tastrup from 306,800 to 303,400. The examples show that the expensive dimensioning after scale set e) will be a little cheaper.

4. Effect of the gradings

Calculation of the congestion is made by us of "The geometric group concept"-method. In practice the gradings will often be simplified and will therefore by normal traffic give a bigger congestion than calculated. By heavy traffic, however, the congestion will approach that of the ideal grading and as the critical situations especially occur by heavy traffic, the problem has scarcely any practical effect.

CONCLUSION

The model presented in this paper has been a good tool on the investigation of some dimensioning problems. The calculations carried out by use of the model show that the running cost has a very flat minimum, when the distribution of trunks on direct routes and the tandem route is altered. The results also show that it is difficult to give a simple criterion for the grade of service, but it is important to look at the congestion values by high traffic, because the state here will in reality often be determining for the number of trunks.

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REFERENCES


Bridgford, J.N.: The geometric group concept and its application to the dimensioning of link access systems. ITC London 1964.


