ABSTRACT

The numerical treatment of more or less complicated mathematical problems often is quite easy to be done by means of a time-sharing computer. For this purpose a particularly useful type of program is that one, which provides the solution to a whole class of problems, where any special case can be handled by merely describing an appropriate set of parameters. An example for such a general program is the numerical treatment of a system of difference equations for the instationary behavior of a m-server queueing system with constant service times. The variety of its applications covers telephone traffic as well as road traffic, and all kinds of service stations in post-offices, mountain-railways, custom-houses and so on. The dialogue with the time-sharing computer is demonstrated.

A problem frequently encountered in traffic theory is the computation and numerical evaluation of complicated mathematical expressions. In general these evaluations can be accomplished only by means of an electronic computer. Obviously great advantages are offered by using a time-sharing computer: by the terminal of a time-sharing system the computer is always directly accessible. So in programming a problem and also in debugging this program the programmer saves a considerable amount of waiting time for the computer and all the ways to the computing center. Furthermore any newly developed computing program can immediately made available to all outstations of the time-sharing system. From this viewpoint, a particularly useful type of program is that one, which provides the solution to a whole class of problems, where any special case can be handled by merely describing an appropriate set of parameters. Consider for instance the probabilities of the number of units waiting in the queue of a delay system. For a large class of delay systems the vector of these probabilities can be determined from a set of linear difference equations. The special properties of the considered queueing system, as there are arrival rates, number of service points and so on, are merely expressed by the coefficients of these equations. So we have developed a simple, but relatively general computer program for this problem and we shall briefly illustrate how it can solve a number of different problems with the aid of a time-sharing data processing system.

We consider a m-server queueing system with constant service times $T_0$, and examine it at isolated time points $t_n$, where the difference between $t_{n+1}$ and $t_n$ equals the constant time $T_0$: $t_{n+1} - t_n = T_0$ for all $n$. 

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Suppose then that at time $t_n$ there are exactly $k = k_n$ units present in the waiting field of our queueing system. Then the numbers $k_n$ form a Markov process. If $P_n(k)$ is the probability of exactly $k$ units in the waiting field at time $t_n$, then it is easy to derive the following set of difference equations for the unknown $P_n(k)$:

$$P_{n+1}(0) = \sum_{k=0}^{m} P_n(k) \sum_{i=0}^{m-k} W(i)$$

for all $n$ and $i = 1, 2, \ldots$

$$P_{n+1}(i) = \sum_{k=0}^{i+m} P_n(k) W(i+m-k)$$

where $W(i)$ is the probability of exactly $i$ units arriving at the system during $T_o$. If we assume, that the coefficients $W(i)$ are known, then the $P_n(k)$ can easily be computed. In teletraffic theory these equations describe a delay system with $m$ serving lines and constant holding times $T_o$. The distribution of the intervals between the arriving calls enters merely into the coefficients $W(i)$, which can be specified manually or by means of a suitable subroutine. Due to this fact it is possible to change the value of the offered traffic at the different time points $t_n$ and to study the effects of these changes. In the same easy manner also the rate of calls served at $t_n$ can be altered.

The very same set of equations is obtained in the theory of road traffic, if $P_n(i)$ is interpreted as the probability of $i$ vehicles waiting in front of a traffic signal at the end of the green phase, assuming that $m$ vehicles can pass on green and that the traffic flow is examined at definite time points differing by one cycle length $T_o$ of the signal.

Quite generally, this set of equations is qualified for the treatment of queueing problems at any kind of service stations, for instance ticket counters, post office windows, machines in an assembly-line production chain, escalators, cable cars, toll booths on bridges and highways, queues before narrow street or highway sections and so on. As the theory applies also to the instationary states of the system, it is particularly useful for short range predictions of queue lengths and delays.

Traffic in a whole network can be managed on the basis of the predictions from a single traffic control center, which communicates with a time-sharing computer via a modem. Should it be necessary, this method could be generalized to the extent of including a treatment of processes with an optional distribution of the rate of the served units, where service times are not constant.

As an example an instationary queueing process was treated with the system of difference equations described above. The results are shown in Fig.1. On the left-hand side we have a constant service rate of 13 units per time interval. The mean of the offered traffic (step-line) increases rapidly and climbs considerably over the service rate. Therefore we get an overload in the range of the time intervals 8 to 19. On the right, Fig.1 shows an adaption of the service rate from 13 to 16 units per time interval between the intervals 6 and 23. The arriving traffic has Poisson distribution with mean values, which are also shown by a step-line. The graphs for the mean of the queue-length and also for the probabilities, that the queue-length is less or equal this mean value, are drawn as functions of the time.

Fig.2 shows the results of a similar queueing process and is printed directly by the terminal (teletype-writer) of a time-sharing computer. The mean queue-length is indicated by "\(\bar{q}\)" and the lower 2σ-bound by "-" and the upper 2σ-bound by "+". Both the distribution of the arriving traffic and the service rate have been changed several times by means of the terminal. Because of the great size of the input for a Poisson-distribution the corresponding dialogue with the computer isn't copied in the figure. Therefore the mean of the offered traffic and the service rates are shown by a solid and a dashed line.

In Fig.3 a little part of the dialogue is shown for another, more simple example.

Fig.4 shows another part of this dialogue; here especially the effect of changing the service rate can be seen. In both figures the user's answers to the computer's questions are underlined.
WARTESYSTEM M·KONST·ABFERTIGUNGSRATE

ANZAHL D·WARTEPLÄTZE? 10

GEBEN SIE JETZT DIE EINFAHRLWÄHRSLKTN·AN, WENN
P(Y)=.XXXX, GEBEN SIE SIE IN DER FORM: Y=..XXXX AN

WIE LAUTET DAS 1·TE WERTEPAAR? 2·16
WIE LAUTET DAS 2·TE WERTEPAAR? 3·24
WIE LAUTET DAS 3·TE WERTEPAAR? 4·24
WIE LAUTET DAS 4·TE WERTEPAAR? 5·20
WIE LAUTET DAS 5·TE WERTEPAAR? 6·14

ABFERTIGUNGSRATE? 4

WIEVIELE ZYKLEN WOLLEN SIE JETZT RECHNEN? 10

Ø = MITTL·ANZAHL M·D·WARTENDEN
- = M·2·SIGMA D·WARTEVERTEILUNG
+ = M·2·SIGMA DESGL.

0-----------------------------+---+++---+++---+++---+++--- 10 ZYKL.

0

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WOLLEN SIE MIT GLEICHEN EINFAHRLWÄHRSCHEINLICHKEITEN WEITERRECHNEN? JA

ABFERTIGUNGSRATE? 4

WIEVIELE ZYKLEN WOLLEN SIE JETZT RECHNEN? 10

Ø = MITTL·ANZAHL M·D·WARTENDEN
- = M·2·SIGMA D·WARTEVERTEILUNG
+ = M·2·SIGMA DESGL.

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WOLLEN SIE MIT GLEICHEN EINFAHRLWÄHRSCHEINLICHKEITEN WEITERRECHNEN? JA

ABFERTIGUNGSRATE? 4

WIEVIELE ZYKLEN WOLLEN SIE JETZT RECHNEN? 10

Fig. 3
WÖLLEN SIE MIT GLEICHEN EINFALLWAHRSCHENLICHKEITEN WEITERRECHNEN? JA
ABFERTIGUNGSRATE? 5
WIEVIELE ZYKLEN WÖLLEN SIE JETZT RECHNEN? 10

\[ \begin{align*}
\theta &= \text{MITTL. ANZAHL M D. WARTENDEN} \\
- &= M - 2 \times \text{SIGMA D. WARTEVERTEILUNG} \\
+ &= M + 2 \times \text{SIGMA DESGL.}
\end{align*} \]

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\hline \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
\end{array} \]

WÖLLEN SIE MIT GLEICHEN EINFALLWAHRSCHENLICHKEITEN WEITERRECHNEN? JA
ABFERTIGUNGSRATE? 3
WIEVIELE ZYKLEN WÖLLEN SIE JETZT RECHNEN? 10

\[ \begin{align*}
\theta &= \text{MITTL. ANZAHL M D. WARTENDEN} \\
- &= M - 2 \times \text{SIGMA D. WARTEVERTEILUNG} \\
+ &= M + 2 \times \text{SIGMA DESGL.}
\end{align*} \]

\[ \begin{array}{cccccccccccc}
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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\end{array} \]

WÖLLEN SIE MIT GLEICHEN EINFALLWAHRSCHENLICHKEITEN WEITERRECHNEN? JA
ABFERTIGUNGSRATE? 4
WIEVIELE ZYKLEN WÖLLEN SIE JETZT RECHNEN? 15

\[ \begin{align*}
\theta &= \text{MITTL. ANZAHL M D. WARTENDEN} \\
- &= M - 2 \times \text{SIGMA D. WARTEVERTEILUNG} \\
+ &= M + 2 \times \text{SIGMA DESGL.}
\end{align*} \]

\[ \begin{array}{cccccccccccc}
\hline \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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WÖLLEN SIE MIT GLEICHEN EINFALLWAHRSCHENLICHKEITEN WEITERRECHNEN? NEIN
WÖLLEN SIE M. NEUEN EINF. WAHRSCHENL. KTN. IM ERREICHT. ZUSTAND WEITERRECHNEN? NEIN

Fig. 4