ABSTRACT

This paper presents the theoretical development of a minimum-cost algorithm applicable to satellite networks wherein preassigned and demand assigned facilities are utilized in an alternate routing arrangement. The cost factors considered include the respective variable cost components for terrestrial plant as well as the satellite circuit lease charges pertinent to each operational mode. Of particular focus is a proposal for generalizing previously-advanced "time homogeneous" design procedures through the formal analytical treatment of non-coincident traffic peaks among the various routes comprising the network.

1. BACKGROUND

The forthcoming introduction of Demand Assignment Multiple Access (DAMA) systems will precipitate notable challenges in the area of network design as related to the routing of international telephony via satellite. Accordingly, certain economic ground rules, which have prevailed during the present era of exclusive Preassigned Multiple Access (PAMA) operation, must give way to those appropriate for the "mixed" PAMA/DAMA configurations of the future. Already, widespread attention is being directed towards coping with these challenges, including the establishment of recommended practices in the application of computers for network planning and dimensioning [ref. 1]. Complex design procedures are very much in use by some administrations [refs. 2 & 3], and the general availability of high speed electronic computers has helped foster the development of highly sophisticated algorithms for this purpose.

Two recent papers [refs. 4 & 5] both featured the concept of employing the DAMA portion of a given satellite network to route overflow traffic, with the coexisting PAMA connections serving as high-usage trunks on the respective links. This routing discipline provides the basis for the present paper inasmuch as the fundamental objective is the same: to find the minimum-cost size for all PAMA satellite circuit groups. The most significant departure from these previous papers is a proposal for utilizing available data on each link's 24-hour traffic profile to take advantage of time-separation of congestion periods among various links within the network. In addition, marginal capacity factors are herein defined in a slightly altered fashion.

2. BASIC MODEL AND COST PARAMETERS

At present, prior to the inception of DAMA service, the satellite network is made up entirely of permanently assigned circuit groups providing direct connections among a large number of international transit-centres (CT's). The simple four-node routing pattern of Figure 1(a) is amply illustrative of this situation. The width variations among paths are intended to denote the non-homogeneity of busy-hour traffic offerings typically associated with different CT-pairs. The

FIGURE 1

344/1
effect of replacing this PAMA system by an equivalent alternate routing network is shown in Figure 1(b). Overflow loads in this latter case are assumed handled at the satellite in a channels-pair: fully variable mode as, for instance, is characteristic of the SPADE system [ref. 6] which will soon be deployed. The broken lines represent DAMA access-circuit groups, one being associated with each participating earth station. Upon implementation, the DAMA system itself will assume a role similar to that of CT. While it is conceivable that some DAMA access-circuit groups will serve as high-usage groups, such considerations are not included in this analysis.

An isolated overflow path between any two earth stations consists of three distinct sections in tandem. Inclusion of the high-usage path gives rise to the trapezoidal alternate routing pattern of Figure 2. The annual cost, \( C_f \), therein includes both the per-circuit satellite lease charge, plus the cost of equipment and transmission facilities, linking earth station with its CT on both ends of each PAMA connection. \( C_f \) is the equivalent annual lease charge allocable to each DAMA satellite circuit. The parameter \( L \) represents the annualized capital outlay per access-circuit, a figure which includes the price for each full duplex channel unit and a prorated share of the terrestrial interfacing costs.

3. BUILDING THE ALGORITHM

The proposed algorithm deals individually with each route within the given DAMA "community." It is therefore sufficient to narrow the initial scope of this analysis to consideration of a single link \((i,j)\) as portrayed in Figure 2. The minimum-cost apportionment of the traffic offered, \( A_{ij} \), between the high-usage and overflow paths in this instance corresponds to the condition where the costs associated with the marginal traffic load carried by the high-usage group's

\[
\frac{\partial C_{ij}}{\partial n_{ij}} = C_p + C_f \sum_{N_n} n_{ij} B_{s} B_{a} A_{ij} - \sum_{N_n} n_{ij} B_{s} A_{ij}
\]

(1)

Given some sequential ordering of the CT's, the \( \sum \) operator is interpreted as a summation taken over all links \((i,j)\) with \( i < j \). The \( n_{ij} \) term corresponds to the quantity of PAMA circuits associated with the particular link being considered, and \( N \) represents the necessary number of full duplex circuits per satellite to accommodate the aggregate overflow from all links while maintaining a mean loss probability for this segment of the overflow path equal to \( B_s \). \( N_1 \) and \( N_2 \) are the respective sizes of the access-circuit groups at the ith and jth earth stations, each group having \( B_s \) as the specified mean loss probability. [A full list of parameter definitions is provided for the reader in the Glossary.]

By way of facilitating algorithm development, it will be initially assumed that the busy-hour or maximum congestion intervals for all links occur in-phase relative to one another; from this, it follows that the maximum congestion interval for each section of a given link's overflow path is time-coincident with the peak load carried by the link's high-usage group. This situation is hereafter referred to as the "time-homogeneous" case. With respect to link \((i,j)\) alone, the minimum-cost traffic apportionment corresponds to the condition:

\[
\begin{align*}
&\frac{\partial C_{ij}}{\partial n_{ij}} = 0 = C_p + C_f \sum_{N_n} n_{ij} B_{s} B_{a} A_{ij} - \sum_{N_n} n_{ij} B_{s} A_{ij} \\
&+ L \left( \sum_{N_n} n_{ij} B_{s} B_{a} A_{ij} + \sum_{N_n} n_{ij} B_{s} A_{ij} \right)
\end{align*}
\]

(2)

By factoring out the marginal occupancy term that pertains to the high-usage group, equation (2) can be rearranged so that it takes the form:

\[
\frac{f(n_{ij}, A_{ij})}{A_{ij}} = A_{ij} \left( E_{n_{ij}} - E_{n_{ij}} \right)
\]

(3)

\[
\begin{align*}
&f(n_{ij}, A_{ij}) = A_{ij} \left( E_{n_{ij}} - E_{n_{ij}} \right)
\end{align*}
\]

wherein \( f(n_{ij}, A_{ij}) \) represents the marginal occupancy function while \( E_{n_{ij}}(A_{ij}) \) denotes the Erlang loss function. Since the parameter \( n_{ij} \) is restricted to positive integer values, an exact equality will generally not be feasible for expression (3). To allow for this in an unambiguous manner, the convention selected for application of this crucial expression is the "less than or equal to" inequality condition as specified.

Each link has associated with it three marginal capacity factors, one for each section of the overflow path. With respect to the DAMA "pool" at the satellite, the marginal capacity is given by:

\[
(\beta)_{ij} = \frac{3a_{ij} A_{ij}}{A_{ij}}
\]

Furthermore, the terms:

\[
(\beta)_{ij} = \frac{3a_{ij} A_{ij}}{A_{ij}}
\]

define marginal capacity for the two access-circuit groups common to link \((i,j)\). It should be pointed out that the foregoing does reflect a slight departure from the marginal capacity concept expressed in a predecessor paper [ref. 5]. This present form, being more direct, is better suited for rigorous handling of \( a_{ij} \), the mean overflow from the high-usage group on link \((i,j)\) whose nonrandomness can be analytically described [ref. 7]. This is not a small advantage; the marginal capacity terms collectively exert a strong influence upon the numerical accuracy of expression (3), and, as will be seen, the degree of convergence of these terms wholly determines the point of termination for the algorithm itself. [The Appendix section outlines a proposed method for evaluating \( (\beta)_{ij} \).]
The right-hand side of expression (3) is simply
the product of two constant terms which, by way
of convenience, will be designated $K_1$ and $K_2$.
The mean loss quantities, $B_a$ and $B_s$, are essen-
tial dimensioning determinants. However, dis-
pensing with the "time homogeneous" assumptions
of the previous section raises the question: with
which interval or intervals should these input
parameters be identified? Notwithstanding this
dilemma, even if suitable reference intervals
could be specified, there would remain the problem
of making sure that the point-to-point mean loss
probability $(B_s+2B_a)$ for all overflow paths is
not exceeded during any other period of a normal
day. The purpose of this section is to suggest
how these difficulties might be overcome through
the introduction of some new concepts, while re-
taining the basic rational developed for the
"time homogeneous" algorithm.

Given the number of time-zones traversed by a par-
ticular link, in addition to its maximum offered
traffic intensity, a 24-hour traffic profile may be
reliably deduced [refs. 8 & 9]. Relative to each
overflow path, respective maximum congestion
intervals, the DAMA satellite circuit "pool" and
the ith and jth station access-circuit groups, can be
found; correspondingly, these may be designated by $\tau_1$, $\tau_2$, and $\tau_3$. Next, it is necessary to in-
troduce the following partial derivative terms:

$$\begin{align*}
R_1 &= \frac{3a_{ij}}{3n} |A_{ij}(a_{ij})| A_{ij'}(a_{ij'}) \\
R_2 &= \frac{3a_{ij}}{3n} |A_{ij}(a_{ij})| A_{ij'}(a_{ij'}) \\
R_3 &= \frac{3a_{ij}}{3n} |A_{ij}(a_{ij})| A_{ij'}(a_{ij'})
\end{align*}$$

It is likewise important to redefine the three
marginal capacity terms per overflow path:

$$\begin{align*}
{\beta}_{ij} &= \frac{3a_{ij}}{3n} |B_aA_{ij}(a_{ij})| \\
{\beta}_{ij} &= \frac{3a_{ij}}{3n} |B_aA_{ij}(a_{ij})| \\
{\beta}_{ij} &= \frac{3a_{ij}}{3n} |B_aA_{ij}(a_{ij})|
\end{align*}$$

[The evaluation method of the Appendix becomes
wholly applicable to the foregoing $(\beta)_{ij}$ term by
simply replacing $a_{ij}$ by $a_{ij}(a_{ij})$ and $A_{ij}$ by $A_{ij}(a_{ij})$.]

Finally, expression (3) can now be rewritten:

$$\begin{align*}
\frac{f(n_{ij}', a_{ij})}{f(n_{ij}, a_{ij})} &= \frac{C_f}{C_p} \left( \frac{\beta}{\beta_{ij}} \right) \left( \frac{R_1}{R_{ij}} + \frac{R_2}{R_{ij}} + \frac{R_3}{R_{ij}} \right) \\
&\leq \left( \frac{\beta_{ij}}{\beta} \right) \left( \frac{R_1}{R_{ij}} + \frac{R_2}{R_{ij}} + \frac{R_3}{R_{ij}} \right)
\end{align*}$$

The similarities between expressions (5) and (3)
are quite obvious. The marginal occupancy func-
tion and the $K_1$ factor both remain unchanged.
It is the $K_2$ factor which assumes a somewhat mod-
ified form in expression (5). The previous con-
dition of $K_2$ less than unity is no longer assured.
Accordingly, with reference to Figure 3, $\phi_1$ can
no longer be greater than $\phi_2$ for certain links. The logic
of this revised algorithm is laid out in Figure
4. The practice of utilizing two optimizing
criteria -- one approximate, the other precise --
is clearly shown. It is presumed that the re-
quisite time-zone information is fed into the pro-
gram from the PAMA Data Base File. Otherwise,
relative to the "time homogeneous" case, the in-
put parameters are identical.
FOR EACH LINK FIND \((p_{ij})\) THE NEW MARGINAL CAPACITY OF THE DAMA SATELLITE POOL.

SOLVE FOR \(s\) AND \(a\) FROM:

\[ A^*e^*(A^*) = A \]

\[ H(s^*, a^*) = \bar{v} \]

COMPUTE:

\[ a = \sum_{i \in I} a(r_{ij}) \]

\[ \bar{v} = \sum_{i \in I} v(r_{ij}) \]

FIND \(a(r_{ij})\) AND \(v(r_{ij})\) FROM:

\[ a(r_{ij}) = A_{r1} \]

\[ v(r_{ij}) = V_{r1} \]

ESTABLISH \(n^*\) USING "FIRST APPROXIMATION" OPTIMAL EQUATION.

ESTABLISHED \(n^*\) FROM "REFINED" VERSION OF OPTIMAL EQUATION.

HAS KEYING INDICATOR BEEN SET?

NO

EVALUATE AND ASSIGN \(r_{1}, r_{2}, r_{3}\) FACTORS TO ALL LINKS WHERE \(n_{ij} < n_{ij}\)

COMPUTE RESPECTIVE \(A_{r1}, A_{r2}, A_{r3}\) AND \(\bar{v}, \bar{v}\) QUANTITIES FOR ALL STATIONS TENTATIVELY PARTICIPATING IN DAMA OPERATION.

SOLVE FOR \(s^*, s^*\) AND \(a^*, a^*\) FROM:

\[ A^*e^*(A^*) = A_{r1} \]

\[ H(s^*, a^*) = \bar{v} \]

FOR EACH LINK, FIND \((p^*_{ij})\) AND \((p_{ij})\) WHERE APPLICABLE. THESE ARE THE NEW MARGINAL CAPACITIES OF THE RESPECTIVE TERMINALS' DAMA ACCESS-CIRCUIT POOL.

YES

PRINT LINK-BY-LINK AND COUNTRY-LEVEL COST INFORMATION AS WELL AS FINAL HIGH-USAGE AND ALTERNATE ROUTING TRUNKING REQUIREMENTS.

CALL EXIT

NOTE: THIS INDICATOR ACTS AS A FLAG TO THE PROGRAM BY DESIGNATING WHICH OPTIMIZATION PROCEDURE IT SHOULD FOLLOW: THE "FIRST APPROXIMATION" MODE OR THE "REFINED" MODE.
The problem of dimensioning preassigned and demand assigned circuit groups for telephony service through a satellite network has been examined using alternate routing concepts. The proposed computer program tends to converge towards a traffic apportionment between pre-assigned and demand assigned facilities on each route such that the overall annualized expenditures borne by system participants is minimized. Input cost parameters consist merely of two ratios which relate to respective space segment lease charges and to expenditures for earth station/terrestrial equipment. Desired point-to-point loss probabilities can be specified individually for each route. The basic "time homogeneous" algorithm need undergo only a very modest refinement to satisfactorily account for the non-coincidence of busy hours among the various traffic streams within the network.

ACKNOWLEDGEMENTS

The authors are indebted to Messrs. William Kaht and Robert Swensen of the Systems Engineering Division, and to Mr. John Puente of the Comsat Laboratories, for their consistent and generous support of our endeavors in this particular area of investigation. We also wish to express our appreciation to Miss Alice Kim for her superlative typing of the final manuscript, despite its rather arduous content.

REFERENCES


APPENDIX

The flow diagram of Figure 4 reveals the central importance of the marginal capacity terms throughout the execution of the algorithm. Generally, they must be recomputed with each iteration. In addition, their convergence properties over a long series of iterations determine the exact point at which the algorithm is terminated. Needless to say, computational accuracy is an imperative. The method outlined below, while strictly for the quantification of $(S)_{ij}$, can readily be adapted for evaluating the $(S)_{ij}$ and $(S)_{ij}$ terms.

As presented in Section 3, the marginal capacity associated with the DAMA satellite "pool" is defined as a partial derivative. This term can be further reduced to:

$$(S)_{ij} = \frac{3a_{ij}}{3N}
\begin{pmatrix}
\frac{3a_{ij}}{3N} & B_{S_{ij}}A_{ij}
\end{pmatrix}
\begin{pmatrix}
\frac{3a_{ij}}{3N} A_{ij}
\end{pmatrix}$$

The remainder of this Appendix will be devoted to an analytical discussion of how the two component partial derivatives on the right-hand side of the foregoing equation may be evaluated numerically.

As is well known, the total mean overflow from all links $(i,j)$ is related to the equivalent random offered load, $A S$, according to:

$$A = A S + A$$

(hence,$$A S = A - A$$)

It follows that $B_S$, the mean loss probability assigned to the satellite DAMA circuit group, is given by:

$$B_S = \frac{A S + N(A S)}{A S + A S} = \frac{E_{S+N}(A S)}{E_{S}(A S)}$$

(A-2)

Differentiating both sides of equation (A-1) with respect to $A S$ yields:

$$3A S | B_S = 3A S | E_{S}(A S) + A S \frac{3E_{S}(A S)}{3A S}$$

(A-3)

Equation (A-3) is easily quantified, all of its right-hand terms being known before this point of each iterative cycle is reached. With this result, the derivative of $\bar{V}$ with respect to $A S$, $\bar{V}$ fixed, becomes immediately solvable. That is, because the variance of the combined overflow from all links $(i,j)$ is:

$$\bar{V} = H(S, A S) = A | 1 - A + A(S + 1 + A - A S)^{-1}$$

differentiation leads to:

$$3\bar{V} | S = 3A S | E_{S}(A S) + A S \frac{3E_{S}(A S)}{3A S}$$

(A-4)

Thus, with the aid of (A-3), the right-hand side of equation (A-4) is devoid of unknowns. Similarly, separate differentiations of $A$ and $\bar{V}$ with respect to $S$, holding $A S$ constant, produce the known quantities:
The relationship between equations (A-6) and (A-4) and between (A-5) and (A-3) may be more fully appreciated by reference to the upper plot of Figure A-1. The $\delta \ell$ vector therein represents an infinitesimal shift occasioned by an equally minute decrease in the mean overflow contributed by a single link $(i,j)$, with $A_{ij}$ assumed constant. The slope of the constant-$V$ curve is simply equation (A-4) divided by (A-6).

$$\tan \phi = \frac{\partial V}{\partial A_{ij}} = \frac{2S^*}{A_{ij}}$$  \hspace{1cm} (A-7)

By analogy, the slope of the constant-$A$ curve may be expressed as (A-3) divided by (A-5):

$$\tan \theta = \frac{\partial V}{\partial S^*} = \frac{2A_{ij}}{3S^*}$$  \hspace{1cm} (A-8)

The total overflow contribution from link $(i,j)$ has a variance which is described by:

$$v_{ij} = H(n_{ij}, A_{ij}) = a_{ij} \left\{ 1 - a_{ij} + a_{ij}(n_{ij} + 1 + a_{ij} - A_{ij})^{-1} \right\}$$

The mean and variance of the combined overflow of all links in the network are simply the respective linear summations:

$$A = \sum a_{ij} \quad \text{and} \quad V = \sum v_{ij}$$

where $\sum$ denotes a summation taken over all $(i,j)$ under the condition, $i<j$. Thus, for a single link, taking the partial derivative of $v_{ij}$ with respect to $a_{ij}$, all other parameters held constant, gives:

$$a(n_{ij}, A_{ij}) = \frac{\partial v_{ij}}{\partial A_{ij}} = \frac{\partial V}{\partial S^*}$$ \hspace{1cm} (A-9)

$$= 1 - 2a_{ij} + 2a_{ij}(n_{ij} + 1 + a_{ij} - A_{ij})^{-1}$$

$$- a_{ij} 2 \left( n_{ij} + 1 + a_{ij} - A_{ij} \right)^{-2} \left( 1 + \frac{2n_{ij}}{2S^*} \frac{1}{A_{ij}} \right)$$

The final term of the foregoing equation is the reciprocal of the marginal occupancy function, $f(n_{ij}, A_{ij})$. All other terms having been quantified prior to this juncture in the iterative cycle, moreover, equation (A-9) can be computed.

Next, $\tan \rho$ is derivable by employing certain trigonometric relationships intrinsic to the microscopic view which is depicted in the upper plot; it can be shown that:

$$\tan \rho = \frac{1}{1 + \frac{\tan \phi}{\tan \theta}} = a(n_{ij}, A_{ij}) \hspace{1cm} (A-10)$$

Solving explicitly for $\tan \rho$, equation (A-9) may be rewritten as:

$$\tan \rho = - \frac{3S^*}{A_{ij}} \frac{1}{A_{ij}} \frac{3A_{ij}}{3S^*} \frac{1}{A_{ij}}$$

$$= \frac{\partial V}{\partial S^*} = \frac{2S^*}{A_{ij}}$$

$\frac{\partial V}{\partial S^*} - a(n_{ij}, A_{ij}) \frac{3A_{ij}}{3S^*} S^*$

The final term of the foregoing equation is the reciprocal of the marginal occupancy function, $f(n_{ij}, A_{ij})$. All other terms having been quantified prior to this juncture in the iterative cycle, moreover, equation (A-9) can be computed.
The parameters of the lower plot in Figure A-I are functionally dependent upon those discussed with respect to the upper plot; it is likewise helpful to relate certain partial derivatives to their geometric equivalences. For instance, the slope of the constant-$E_\ast(A\ast)$ curve can be expressed as the ratio of two partial derivatives terms. The first one is:

$$\frac{\partial E_\ast(A\ast)}{\partial A\ast} \bigg| A\ast = E_\ast(A\ast) \left( E_\ast(A\ast) + \frac{S\ast - A\ast}{A\ast} \right)$$

The denominator term, $\left(\frac{\partial E_\ast(A\ast)}{\partial A\ast}\right)_{A\ast}$, was evaluated in equation (A-5). Therefore:

$$\tan \gamma = \frac{3S\ast}{3A\ast} \bigg| E_\ast(A\ast) = \left( E_\ast(A\ast) + \frac{S\ast - A\ast}{A\ast} \right) \log S\ast - \psi$$

Guided by the form of the above equation, $\tan \gamma$ can be written:

$$\tan \gamma = \frac{3(S\ast + N)}{3A\ast} \bigg| E_\ast(N) = \left( E_\ast(N) + \frac{S\ast + N - A\ast}{A\ast} \right) \log S\ast + N - \psi$$

where $\psi = \frac{3(\log A\ast(S\ast + N))}{3(A\ast + N)}$. $E_\ast(N) = B_\ast E_\ast(A\ast)$ is given by equation (A-1) so that the right-hand side of (A-11) is completely defined. From equation (A-1) it also follows that:

$$\frac{\partial E_\ast(N)}{\partial A\ast} \bigg| A\ast = B_\ast \frac{\partial E_\ast(A\ast)}{\partial A\ast} \bigg| A\ast = \frac{B_\ast}{\partial A\ast} A\ast$$

(A-12)

Once again, invoking certain trigonometric relationships for the purpose of making appropriate substitutions permits equation (A-12) to become:

$$\left( \frac{\partial E_\ast(N)}{\partial A\ast} \bigg| \frac{S\ast + N}{A\ast} \right) \left( 1 + \frac{\tan \rho}{\tan \gamma} \right) = B_\ast \left( \frac{\partial E_\ast(A\ast)}{\partial A\ast} \bigg| \frac{S\ast}{A\ast} \right) \left( 1 + \frac{\tan \rho}{\tan \gamma} \right)$$

It is now a straightforward matter to solve for $\tan \rho$. Alternatively, $\tan \rho$ may be expressed in its more fundamental form:

$$\tan \rho = -\frac{\partial E_\ast(N)}{\partial A\ast} \bigg| A\ast = B_\ast \frac{\partial E_\ast(A\ast)}{\partial A\ast} \bigg| A\ast$$

Hence:

$$\frac{\partial A\ast}{\partial N} \bigg| B_\ast, A\ast = \tan \rho - \tan \rho' = 1$$

Finally, the other decisive partial derivative term is defined by:

$$\frac{\partial A\ast}{\partial \lambda} \bigg| A\ast = \lambda \frac{\partial A\ast}{\partial A\ast} \bigg| A\ast = E_\ast(A\ast) + A\ast \frac{\partial E_\ast(A\ast)}{\partial A\ast} \bigg| A\ast$$

which, by utilizing the result on the left-hand side of equation (A-12), may be expanded to:

$$\frac{\partial A\ast}{\partial \lambda} \bigg| A\ast = E_\ast(A\ast) + A\ast \left( E_\ast(A\ast) + \frac{S\ast - A\ast}{A\ast} \right) \left( 1 + \frac{\tan \rho}{\tan \gamma} \right)$$

GLOSSARY

A $$\hat{\lambda}$$ Mean value of the combined overflow traffic from all routes within the particular satellite system (or "pool") under study.

A* $$\hat{\lambda}$$ Equivalent random offered traffic as related to the $A$, $V$ quantities.

A ij $$\hat{\lambda}$$ Total offered traffic flow of route $(i,j)$ as calculated via Erlang loss formula from known values of $A_{ij}$ and $B_p$.

A i, A j $$\hat{\lambda}$$ Mean value of the combined overflow traffic from all routes terminated at the $i$th and $j$th earth stations, respectively.

A i* A j * $$\hat{\lambda}$$ Equivalent random offered traffic as related to the $A_i$, $V_i$ and the $A_j$, $V_j$ quantities, respectively.

a ij $$\hat{\lambda}$$ Mean value of traffic lost (or overflowing) from the remaining preassigned satellite circuits, $A_{ij}'$, on route $(i,j)$.

A(τ) ij $$\hat{\lambda}$$ Total offered traffic flow of route $(i,j)$ during the hourly interval, $\tau$. Note that maximum $A(\tau)_{ij}$ is synonymous with $A_{ij}$. $A(\tau)_{ij}$ $$\hat{\lambda}$$ Mean traffic overflowing from $A_{ij}'$ during given hourly interval, $\tau$. Note that maximum $a(\tau)_{ij}$ is synonymous with $A_{ij}'$.

B_p $$\hat{\lambda}$$ Assumed loss probability for each link operating in the original PAMA mode.

B_s $$\hat{\lambda}$$ Mean loss probability assigned to DAMA circuit group at particular satellite in question.

B_a $$\hat{\lambda}$$ Mean loss probability assigned to each earth station's DAMA access-circuit group.

C_{ij} $$\hat{\lambda}$$ Total annualized cost applicable to route $(i,j)$.

C_P $$\hat{\lambda}$$ Annualized cost per PAMA high-usage connection.

C_F $$\hat{\lambda}$$ Annualized cost per DAMA satellite circuit.

L $$\hat{\lambda}$$ Annualized cost representing the added investment required per access-ckt. trunk termination.

N $$\hat{\lambda}$$ Calculated number of DAMA satellite circuits.
\( N_i, N_j \) calculated number of DAMA access-circuit units for \( i \)th and \( j \)th earth stations, respectively.

\( n_{ij} \) number of original PAMA satellite circuits associated with route \((i,j)\).

\( n^*_{ij} \) resultant number of high-usage PAMA circuits pertaining to route \((i,j)\).

\( s^* \) size of the equivalent high-usage circuit group related to the \( A, V \) quantities.

\( S^i, S^j \) size of the equivalent high-usage circuit groups related to the \( A^i, V^i \) and \( A^j, V^j \) quantities, respectively.

\( v \) variance of combined overflow traffic from all routes within particular satellite system (or "pool") under study.

\( v^i, v^j \) variance of the combined overflow traffic from all routes terminated at the \( i \)th and \( j \)th earth stations, respectively.

\( v_{ij} \) variance of the traffic lost (or overflowing) from the remaining preassigned satellite circuits, \( n_{ij} \), on route \((i,j)\).

\( v(t)_{ij} \) variance of the traffic overflowing from \( n_{ij} \) during the given hourly interval, \( t \).

\( (s)_{ij} \) marginal capacity in the final DAMA satellite circuit group relative to route \((i,j)\) under conditions of constant congestion, \( B_s \).

\( (s^*)_{ij} \) intermediate or trial value for the marginal capacity quantity, \((s)_{ij}\).

\( (s^i)_{ij}, (s^j)_{ij} \) marginal capacity for \( i \)th and \( j \)th earth stations respectively in the final DAMA access-circuit groups, relative to route \((i,j)\) under conditions of constant \( B_a \).

\( (s^*_i)_{ij}, (s^*_j)_{ij} \) intermediate or trial values for the marginal capacity quantities, \((s^i)_{ij}\) and \((s^j)_{ij}\).

\( \epsilon \) chosen tolerance value for ensuring sufficient closeness between \((s)_{ij}\) and \((s^*)_{ij}\) in magnitude.

\( \epsilon^i, \epsilon^j \) chosen tolerance value for insuring sufficient closeness respectively between \((s^i)_{ij}\) and \((s^*_i)_{ij}\) in magnitude and between \((s^j)_{ij}\) and \((s^*_j)_{ij}\) in magnitude.