TRAFFIC MODEL FOR THE SIMULATION OF ENTIRE EXCHANGES

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ABSTRACT

For simulation of entire exchanges, an agreement must be made concerning the treatment of calls (internal and from other exchanges) terminating on subscribers found busy.

For simulation according to the Kosten model, three different simplifying assumptions are investigated which are relatively easy to implement in a simulation program:

A. If the called subscriber is found busy, another subscriber is selected at random; this process is repeated until a free subscriber is found.

B. Calls meeting with subscriber-busy condition are treated as ordinary lost calls, i.e. they contribute with the mean holding time to the traffic offered.

C. Calls meeting with subscriber-busy condition are treated as special lost calls, the holding time of which is considered zero in the model, i.e. they do not contribute to the traffic offered.

Following a discussion of these three assumptions, preference is given to Model C for practical reasons.

When defining the model, it becomes apparent that the theoretical definition of traffic offered is in contrast to the interpretation normally used in practice.

Model C is fully described by a set of formulas which are used for the preparation and evaluation of the simulations. Any traffic distribution among subscribers can be chosen. This makes it necessary to approximate the probability of subscriber-busy condition prior to simulation.

1. PROBLEM AND ASSUMPTIONS

For economic reasons, modern centrally-controlled switching systems use switching networks comprising multi-stage link systems. All types of calls are usually set up in one step on the conditional path-finding principle. The determination of traffic handling capacity and the dimensioning of such networks represent special problems of traffic theory. It is nowadays customary to apply, for the determination of capacity or at least for verification of calculation methods, traffic simulation procedures.

![Switching network and connections](image)

Fig. 1 Switching network and connections

For the simulation of less-centralized exchanges, the switching network can be broken down into parts which can be investigated separately, normally considering only one type of traffic. In contrast, modern switching networks (Fig. 1) must be treated as an entity, and several types of traffic as well as connection possibilities (Fig. 2) must be applied simultaneously for their investigation.

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![Connections in an exchange](image)

Fig. 2 Connections in an exchange

1. Internal call
2,3 Outgoing calls
4,7 Incoming calls from other exchanges
5,6,8,9 Transit calls
In order to outline a specific problem, a simple case should be discussed here, where register traffic and special lines (coin telephones, speech services etc.) will not be considered. Any existing subscriber busy calls connections shown in Fig. 2 and the resulting traffics. Hence, the traffic sources are subscribers, incoming and both-way junctors. It is assumed that the subscriber traffic having Engset distribution. Special subscriber behavior (repeated call attempts) will not be considered, because it is difficult to generalize and would complicate the simulation. The incoming and both-way junctors pass to the switching network traffic of Erlang distribution. The traffic sinks are subscribers as well as outgoing and both-way junctors which handle the traffic originated by the sources. When a call is switched to a free junctor of the desired direction, it exists for a random duration. The further call establishing process cannot be observed, since only one exchange is being simulated. Calls terminating in the exchange (internal calls as well as incoming calls from other exchanges) are routed to the subscribers. They may be lost because they cannot reach the wanted subscriber due to blocking in the switching network, or because the wanted subscriber is found busy.

In the following, it will be shown that the treatment of calls meeting with subscriber-busy condition constitutes a major problem. There are several obvious approaches, three of which will be discussed here; however, they considerably affect the simulation results, so that a critical choice must be made in order to obtain results which reflect actual conditions. It will be seen that the customary definition of traffic offered will meet with difficulties and that a definition, anyhow, is very problematic.

2. MODEL VARIANTS

Traffic simulation is performed according to the model of Kosten [1]. The various types of calls (internal, outgoing, incoming and transit) have negative exponentially distributed holding times of equal mean value. The entire system is assumed to operate on the loss principle for all types of calls.

Simulations according to the Kosten model are well-known [2,3]. This paper deals with the simulation of entire exchanges and, in particular, with the effects of the treatment of subscriber-busy calls. The final objective is to establish rules for the allocation of random numbers for the various random events.

Below, three models are discussed to which the above-mentioned assumptions are applied; calls meeting with subscriber-busy condition, however, are treated in different ways. Underlying these models are simplifying assumptions which only approximate real conditions. However, they are relatively easy to include in a simulation program.

2.1 Model A

When a call finds the wanted subscriber busy because he either generated or received a call, a different subscriber is selected for this call on a random basis. This process is repeated until the subscriber found is free. In practice, it will never occur that all subscribers are busy, since the switching network does not offer that many connection paths.

In model A there are, consequently, no subscriber-busy calls, although their percentage is relatively high in reality (20 % or even more). This leads to pessimistic simulation results, i.e. excessive blocking probabilities, because at times of heavy traffic and resultant limited connection possibilities, it is attempted to set up all connections although many calls find the wanted subscribers busy and actually should no longer load the switching network.

The distribution function of total traffic in the switching network is substantially falsified in this model and a significant simulation result - the probability of subscriber busy condition at unequal subscriber traffic values - is in this case unobtainable. These are the principal drawbacks of model A.

2.2 Model B

Subscriber-busy calls are treated as ordinary lost calls as they occur due to blocking in the switching network. They are considered to have the same mean holding time as the other calls, and they contribute with this holding time to the traffic offered.

Underlying this model is the well-known definition of traffic offered [4], i.e. all call attempts (including subscriber-busy calls) are multiplied with the mean holding time which is known from measurements of real traffic. Since subscriber-busy probability is normally quite high, there is a considerable difference between the traffic offered to the switching network and the carried and measurable traffic load.

The traffic values specified for dimensioning of exchanges are normally based on measurements of traffic loads. In order to derive from the traffic load the traffic offered, the subscriber-busy probability must be known which, however, has so far been difficult to measure. It is therefore assumed that in practice only the loss resulting from blocking in the switching network is being considered in the calculations. This would be contradictory to model B, and the simulation results would be too optimistic since the load on the switching network during the simulation would be smaller, by the number of subscriber-busy calls, than expected.

It is obvious that there is a discrepancy between theoretical definition and practical interpretation of the traffic offered. It should therefore be clarified in each instance how the traffic offered, as specified for dimensioning, should be interpreted.

2.3 Model C

Subscriber-busy calls are treated as special lost calls which in practice have very short holding times and in the model, according to the assumption, holding times of zero length. In this model, subscriber-busy calls do not contribute to the traffic offered.

The assumption with respect to subscriber-busy calls is in contradiction to the known definition of traffic offered [4]. On the other hand, it comes very close to the interpretation of traffic offered as it is presumably generally applied. In a simulation according to model C, the load on the switching network will be smaller, by the amount of loss occurring in the switching network, than the traffic offered. This clearly shows how problematic it is to give a generally applicable definition of the traffic offered.
During the simulation, subscriber-busy calls are treated in the same way in models B and C; they have no effect on the traffic flow in the switching network because they are rejected as lost calls before a new call arrives. The only difference is in the length of the random number intervals and, therefore, in the generated and carried traffic.

2.4 Choice of model

For practical reasons, preference is given to model C over model B, although no proof can be presented that would justify this choice and although this model is in contradiction to the definition of traffic offered. Model A is excluded because of the unrealistic assumption with respect to subscriber-busy calls. Further fundamental work will have to be performed to clarify the situation.

Two examples will illustrate why model C has been given preference:

From the exchange B, a trunk group arrives at the investigated exchange A. The traffic load of this trunk group can be measured to derive from it the traffic offered. It can be assumed that this calculation does not take into consideration the subscriber-busy probability of exchange A, and this probability is usually unknown and would be difficult to determine as it depends on subscriber traffic distribution. Subscriber-busy calls are included in the measured traffic and they affect the actual mean holding time, but the calculated traffic offered exceeds the measured traffic load only by the amount corresponding to the loss of the trunk group. Model C takes this into account in a considerably better manner than model B.

The second example illustrates the following problem: Assume that the dimensioning specifications indicate 0.10 Erlang traffic per subscriber. What is the traffic load to be expected per subscriber in a switching network properly dimensioned for approx. 1% loss? Considering internal traffic only, model B has only 0.089 Erlang traffic at equal load on all subscribers, or 0.079 or even 0.069 Erlang which is closer to real load distribution. The traffic load is further dependent on the share of external traffic, since subscriber-busy condition is not considered for outgoing traffic. Independent from the load distribution and the percentage of outgoing traffic, model C simulates 0.099 Erlang traffic carried per subscriber.

The second example clearly shows the effect of model choice on the blocking values obtained by simulation or, in case of predetermined blocking values, on the traffic capacity which is normally a statement of the permissible total traffic offered.

3. DESCRIPTION OF MODEL C

A traffic model for the simulation of complete exchanges will be fully defined, i.e. formulas will be given which make it possible to derive, from the predetermined traffic data for the simulation, the necessary input data and, after the simulation, the output data.

Each individual subscriber of the exchange is observed and, with respect to this observation, represented in the computer. The total traffic (outgoing and incoming) must be freely selectable in order to be able to simulate any distribution of traffic. Internal, outgoing, and incoming calls must be considered, and a distinction must be made between successful and rejected calls, noting whether calls have been rejected due to the lack of free connection paths or because of subscriber-busy condition. In order to be able to use the Kostens simulation method in its simple form, it is assumed for this traffic model that all types of calls have a negative exponential distribution of holding times with the same mean value. In view of the assumption that subscriber-busy calls have a holding time equal to zero, this assumption is not matter-of-fact.

3.1 Notations and definitions

In view of the variety of call, traffic and loss types, it appears advisable to use some predetermined notations.

The following indices will be used:

\[ \lambda \] - any subscriber of the exchange

\[ \sigma \] - originating

\[ \gamma \] - terminating

\[ \eta \] - blocked call due to lack of free paths

\[ \xi \] - lost call due to subscriber-busy condition

\[ \iota \] - internal

\[ \epsilon \] - external

The number of calls or call attempts per hour is designated \( c \), for example

\[ C_{\lambda} \] - Number of originating call attempts of subscriber \( \lambda \)

\[ C_{\lambda} = (C_{\lambda}^\sigma + C_{\lambda}^\epsilon + C_{\lambda}^\xi + C_{\lambda}^\iota + C_{\lambda}^\sigma + C_{\lambda}^\epsilon + C_{\lambda}^\xi + C_{\lambda}^\iota) \]

This equation shows the necessary breakdown of call attempts of the subscriber \( \lambda \). The quadruple indices are self-explanatory. In model C it has been assumed that internal calls can be rejected because of subscriber-busy condition \(( C_{\lambda}^\iota \) ), while there is no such loss for outgoing calls.

The traffic load per subscriber is designated \( \gamma \), e.g.

\[ \gamma_{\lambda} = \text{load of subscriber } \lambda \]

\[ \gamma_{\lambda} = \gamma_{\lambda}^\sigma + \gamma_{\lambda}^\epsilon + (\gamma_{\lambda}^\iota + \gamma_{\lambda}^\epsilon + \gamma_{\lambda}^\iota + \gamma_{\lambda}^\epsilon + \gamma_{\lambda}^\iota) \]

In model C, the traffic offered is defined by

\[ \alpha_{\lambda} = (C_{\lambda}^\gamma + C_{\lambda}^\epsilon) \]  

where \( \epsilon \) represents the mean holding time. Lost calls due to subscriber-busy condition are not considered in this formula (1), according to the assumption. The traffic offered per subscriber is, for example,

\[ \alpha_{\lambda} = \text{traffic offered of subscriber } \lambda \]

\[ \alpha_{\lambda} = \alpha_{\lambda}^\sigma + \alpha_{\lambda}^\epsilon = (\alpha_{\lambda}^\sigma + \alpha_{\lambda}^\epsilon) + (\alpha_{\lambda}^\iota + \alpha_{\lambda}^\epsilon) \]

The following loss probabilities are used in the relationships between traffic offered and traffic carried:

\[ B_{\lambda} \] - loss probability for internal calls due to blocking

\[ B_{\text{IC}} \] - Loss probability for internal calls (switching network plus trunk group)

\[ B_{\text{EC}} \] - Loss probability for outgoing traffic

\[ B_{\text{EC}} \] - Loss probability for incoming external calls due to blocking in the switching network.
Examples:
\[ \eta_{\lambda \text{oi}} = \lambda_{\lambda \text{oi}} (1 - B_i) \]
\[ \eta_{\lambda \text{oe}} = \lambda_{\lambda \text{oe}} (1 - B_{\text{oe}}) \]
\[ \eta_{\lambda \text{te}} = \lambda_{\lambda \text{te}} (1 - B_{\text{te}}) \]

The probability of subscriber-busy condition is equal for terminating internal and external calls, since their occurrence is random.

Therefore,
\[ B_i = B_{\text{si}} = B_{\text{se}} \quad \text{subscribers busy probability.} \]

3.2 Call and release probability

For negative-exponentially distributed holding time, the probability that an existing call will terminate within the short time interval \( \Delta t \) is
\[ P_r(\Delta t) = \frac{\Delta t}{\lambda}. \]

The conditional probability for a call attempt by subscriber \( \lambda \) during the time interval \( \Delta t \), the condition that the subscriber is free, is
\[ P_{c,\lambda}(\Delta t) = \frac{C_{\lambda \text{oe}} \Delta t}{1 - \gamma_{\lambda}} = \frac{C_{\lambda \text{oe}} \Delta t}{1 - \gamma_{\lambda}}. \]

During the non-active intervals \( 1 - \gamma_{\lambda} \), subscriber \( \lambda \) must initiate \( C_{\lambda \text{oe}} \) call attempts on the average. As mentioned earlier,
\[ C_{\lambda \text{oe}} = (C_{\lambda \text{oe}} + C_{\lambda \text{oi}} + C_{\lambda \text{os}}) \]

applies, and with the definition of traffic offered as applied to model C,
\[ \lambda_{\lambda \text{oe}} = \lambda \left( C_{\lambda \text{oe}} + C_{\lambda \text{oe}} \right) \]

\[ \lambda_{\lambda \text{oi}} = \lambda \left( C_{\lambda \text{oi}} + C_{\lambda \text{oi}} + C_{\lambda \text{os}} \right) \]

\[ \lambda_{\lambda \text{os}} = \lambda \left( C_{\lambda \text{os}} + C_{\lambda \text{os}} + C_{\lambda \text{os}} \right) \]

Since the subscriber-busy probability is
\[ B_{\text{si}} = B_{\text{se}} = \frac{C_{\lambda \text{si}}}{C_{\lambda \text{oi}} + C_{\lambda \text{oi}} + C_{\lambda \text{si}}} \]

we can conclude for the call probability (3)
\[ P_{c,\lambda}(\Delta t) = \left( \frac{\lambda_{\lambda \text{oi}}}{(1 - \gamma_{\lambda})(1 - B_i)} + \frac{\lambda_{\lambda \text{oe}}}{1 - \gamma_{\lambda}} \right) \Delta t. \]

3.3 Generation of artificial telephone traffic

For traffic simulation according to the method of Kosten [1], it is necessary to determine the intervals for calling and release numbers.

Assume that the release intervals for junctors are unity [3]; according to equation (5), the calling number interval of subscriber \( \lambda \) then is
\[ r_{\lambda} = r_{\lambda \text{oi}} + r_{\lambda \text{oe}} = \frac{\lambda_{\lambda \text{oi}}}{(1 - \gamma_{\lambda})(1 - B_i)} + \frac{\lambda_{\lambda \text{oe}}}{1 - \gamma_{\lambda}}. \]

The first term of the sum represents the internal calls, the second one the external calls. This subdivision is necessary in view of the fact that internal calls can meet with subscriber-busy condition.

The length of the calling number intervals for terminating incoming traffic (Erlang distribution) of subscriber \( \lambda \) is obvious
\[ r_{\lambda \text{te}} = \frac{\lambda_{\lambda \text{te}}}{1 - B_{\text{te}}}. \]

The random number generator is defined by joining together all intervals of calling and release numbers, i.e. each possible random number is classified and its occurrence initiates a well-defined event.

In the case of transit traffic, the interval length corresponds to the traffic offered. For incoming and transit traffic, the interval length must be divided by \( (1 - B) \) if the loss on the trunk group is not negligibly small.

When preparing the simulation (input data for the computer), only the different values of traffic offered are known. The traffic loads \( \eta_{\mu \lambda} \) can be substituted in the above formulas by the traffic offered \( \lambda_{\mu \lambda} \) in the case of low loss, or they must be computed for estimated loss figures. The subscriber-busy probability can be estimated as shown in the following section.

3.4 Subscriber busy probability

If personal and business relations are excluded and if there are \( n \) subscribers, the probability of subscriber \( \lambda \) calling subscriber \( \mu \) is
\[ \lambda_{\mu \lambda} = \frac{1}{\sum_{k=1}^{N} \lambda_{k \lambda}} \]

Personal and business relations can also be considered, but for a general investigation of switching networks this seems to be superfluous.

The probability of finding subscriber \( \mu \) busy is obviously \( \eta_{\mu \lambda} \). Therefore, the probability of any originated internal call of subscriber \( \lambda \) meeting with subscriber-busy condition is
\[ B_{\text{si} \lambda} = \frac{\sum_{k=1}^{N} \lambda_{k \lambda} \eta_{\mu \lambda}}{\sum_{k=1}^{N} \lambda_{k \lambda}}. \]
If the number of subscribers is large, the influence of subscriber $\lambda$ can be neglected and we can write more general

$$B_s^* - B_s^i - B_{se} = \sum_{\mu=1}^{\infty} \alpha_{\mu} \frac{\eta_{\mu}}{\sum_{\mu=1}^{\infty} \alpha_{\mu}}$$

(8)

In most cases, $\eta_{\mu}$ can be substituted by $\alpha_{\mu}$, and very often $\alpha_{\mu+1} = \alpha_{\mu}$, i.e. $\alpha_{\mu+1} = \alpha_{\mu}/2$

Hence, we obtain as an approximation for the subscriber-busy probability

$$B_s^* = B_{s1}^* - B_{se} = \sum_{\mu=1}^{\infty} \alpha_{\mu}$$

(9)

For simulations, it is often assumed that subscriber traffic is equally distributed, i.e. $\alpha_{\mu} = \alpha$; according to equation (9), the result then is simply

$$B_s^* - B_{s1}^* - B_{se} = \alpha .$$

3.5 Calculation of output data

In order to perform the simulation, it has been necessary to introduce many approximations because the loss probability and the subscriber-busy probability were not known. After the simulation, we know these values as they have been derived by sampling. Using the same formulas, we can now derive from the random number intervals the actual traffic offered, which in most cases will slightly deviate from the original values because the simulation was prepared on the basis of approximations.

4. CONCLUSIONS

During the development of a traffic model for the simulation of entire exchanges, it became apparent that the treatment of subscriber-busy calls constitutes a problem. The discussion of several solutions (models A, B, and C) leads to a fundamental problem of traffic theory - the definition of traffic offered. A discrepancy between the theoretical definition and the practical interpretation of this term became obvious.

All developments of electronically controlled switching systems are supported by traffic simulations. Usually, entire exchanges are simulated in order to determine the traffic handling capacity of the switching networks. The discussion of the models A, B, and C revealed that the choice of the model has a major impact on the simulation results. For this reason, everybody using simulation results should know the model employed, and he should further know whether the results are optimistic, pessimistic, or realistic.

In order to simplify the situation, one particular model should be standardized and introduced internationally.

REFERENCES


