A METHOD FOR THE OPTIMIZATION OF TELEPHONE TRUNKING NETWORKS WITH ALTERNATE ROUTING

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ABSTRACT

A method for the economical optimization of hierarchically organized telephone networks with alternate routing is described. First an exposition of the problem is given. The proposed method is explained showing that all routes obey the same optimization equation. It is important to note that two parameters, the mean and the variance, are used to define the traffic. The iterative algorithm used by the computer program that makes the method operational is also described. Finally the paper concludes with an evaluation of the scope of the method.

1. THE PROBLEM

The aim of the proposed method is the economical optimization of telephone trunk networks with alternate routes and hierarchical structure.

Before presenting the problem in a concrete way, the starting hypothesis and basic concepts referring to it will be defined.

1.1. Network definition

Fig. 1 shows a network with alternate routes and hierarchical structure in a conventional area. The depicted network has three levels. In the picture

○ represents a local exchange
○ " a primary center
△ " a secondary center

It is important to remember that the fundamental reason for the necessity of switching centers of a high order is the high telephone density rather than the geographical extension. As new local exchanges are being opened, it may not be economically interesting to interconnect them totally; instead, it is resorted to centers which will collect the local overflow traffic, or primary centers; also the total interconnection of the primary centers may not be advantageous from an economical point of view, then secondary centers are established to collect the primary overflow traffic, and so on.

The network trunks are all unidirectional.

FIG. 1
1.2. Routing law

Fig. 2 shows a schematic representation of the hierarchical structure of the network in Fig. 1. The dotted vertical lines separate the exchanges which are located in the same primary zone; the full vertical lines separate the exchanges which are located in the same secondary zone. Let us consider any two local exchanges, number 3 and 13 for example. In Fig. 3a these two exchanges have been represented with all their centers of superior order (primary and secondary). The drawing also shows the existing trunks between these two exchanges in the direction from 3 to 13, and their routing law (or overflowing law).

![Fig. 2](image)

![Fig. 3a](image)

![Fig. 3b](image)

Fig. 3b depicts all possible routes and their routing law between the exchanges in Fig. 3a, assuming that there is no restriction in the number of successive overflows and that the switching equipment of an exchange can perform The trunks in heavy lines are those which exist in the network in Fig. 1.

The network structure and its routing law, as seen in the above example, is the one we have considered, because it is the most generally accepted. However, the proposed method is independent from the structure and serves for networks with a different routing law.

1.3. Overflow traffic

Overflow traffic has been defined by two parameters: mean and variance. The procedures used to solve the traffic problems arising from the proposed method are based on Wilkinson’s theory and Lotze’s equations; full and limited availability has been considered. In the Annex the equations used to solve traffic problems are given.

1.4. Network service quality

Two criteria are generally followed to define the service quality of the network, i.e. "point to point" and "per communication". According to the first criterion the lost traffic mean on each final route would be given as a percentage (grade of service) of the mean of offered traffic to the particular route. The second criterion consists of admitting a lost traffic mean on each final route, given as a percentage of the sum of offered traffic among exchanges that use said final route.

Both criteria can be used in the proposed method.

1.5. Problem definition

Given a number of exchanges with a certain offered traffic matrix, and a hierarchical structure, the problem is to determine the most economical network configuration that will satisfy a defined service quality.

The costs taken into account are those of the telephonic elements which come into play: unitary cost of the trunks on each route, and cost of the switching equipment in the exchanges.

2. STUDY OF A SIMPLE CASE

Fig. 4 represents part of a network. Routes 2, 4, and 5 are final routes, while numbers 1 and 3 are high usage routes. Let us try to dimension this part of the network in the most economical way, so that it carries traffic A offered by exchange I to exchange J, under conditions of a given congestion in the final routes.

It will be assumed that the cost of each route, i, is proportional to its number of circuits, Wi, with unitary cost Ci. This cost includes the switching costs in origin and destination as well as the cost of the trunk itself. Therefore the total cost of the network can be expressed as follows:

\[ C = C_1 N_1 + C_2 N_2 + C_3 N_3 + C_4 N_4 + C_5 N_5 \]  

[2.1]

The cost is a function of as many independent variables as high usage routes exist in the network, for once these have been determined, the number of circuits in the final routes is determined by the traffic offered to them and by the congestion imposed on them. Therefore we shall take as independent variables the number of trunks on the high usage routes, \( N_1 \) and \( N_2 \).

Our aim is to minimize function C. For this we shall calculate the derivatives of C with respect to \( N_1 \) and \( N_2 \):

\[ \frac{DC}{DN_1} = C_1 + C_2 \frac{DN_1}{DN_2} + C_4 \frac{DN_2}{DN_3} + C_5 \frac{DN_3}{DN_5} \]  

[2.2]

\[ \frac{DC}{DN_2} = C_2 \frac{DN_1}{DN_2} + C_3 \frac{DN_3}{DN_4} + C_5 \frac{DN_3}{DN_5} \]  

[2.3]
For the calculation of the derivatives that appear in the second member we shall have in mind that variables \( N_2, N_4 \) and \( N_s \) are functions of \( N_1 \) and \( N_3 \), as shown by the following expressions:

\[
\begin{align*}
M_{01} &= M_{01} (M_{11}, V_{11}, N_1) \\
V_{01} &= V_{01} (M_{11}, V_{11}, N_1) \\
N_2 &= N_2 (M_{01}, V_{01}, N_1) \\
M_{03} &= M_{03} (M_{01}, V_{03}, N_3) \\
V_{03} &= V_{03} (M_{01}, V_{03}, N_3) \\
N_4 &= N_4 (M_{01}, V_{03}, M_{03}) \\
N_s &= N_s (M_{01}, V_{03}, M_{03})
\end{align*}
\]

where these variables represent:

- \( M_{01} \): offered traffic mean to route \( i \).
- \( V_{01} \): offered traffic variance to route \( i \).
- \( M_{0i} \): overflow traffic mean from high usage route \( i \).
- \( V_{0i} \): overflow traffic variance from high usage route \( i \).
- \( M_{ai} \): lost traffic mean admitted on the final route \( i \).

The previous expressions imply the hypothesis that the traffic on any route is completely defined by its mean and its variance. The admitted lost traffic mean on the final routes, \( M_{ai} \), can in general be given in one of the following forms: either by a value of the grade of service \( E_i \) in each final route, so that:

\[
M_{ai} = E_i \cdot M_i \quad [2.5]
\]

or by a grade of service relative to the traffic between the exchanges that use route \( i \) as a final route

\[
M_{ai} = \frac{\eta}{3} \quad [2.6]
\]

Bearing this in mind we go on to calculate the derivatives of the second members in (2.2.) and (2.3.)

\[
\begin{align*}
\frac{\partial M_{0i}}{\partial N_{1i}} / M_{0i} &= \frac{\partial M_{0i}}{\partial N_{1i}} / M_{0i} + \frac{\partial M_{0i}}{\partial N_{1i}} / M_{0i} \frac{\partial N_{1i}}{\partial N_{1i}} = \\
&= \frac{-\lambda_{01}}{\eta_{01} + \lambda_{1i}} + \lambda_{1i} \\
\frac{\partial V_{0i}}{\partial N_{1i}} / V_{0i} &= \frac{\partial V_{0i}}{\partial N_{1i}} / V_{0i} + \frac{\partial V_{0i}}{\partial N_{1i}} / M_{0i} \frac{\partial M_{0i}}{\partial N_{1i}} = \\
&= \lambda_{1i} \frac{-\lambda_{01}}{\eta_{01} + \lambda_{1i}} + \lambda_{1i} \frac{-\lambda_{01}}{\eta_{01} + \lambda_{1i}} \frac{\partial N_{1i}}{\partial N_{1i}} = \\
\frac{\partial N_{4i}}{\partial N_{1i}} / N_4 &= \lambda_{1i} \lambda_{01} \frac{-\lambda_{01}}{\eta_{01} + \lambda_{1i}} + \lambda_{1i} \lambda_{01} \frac{-\lambda_{01}}{\eta_{01} + \lambda_{1i}} \\
\frac{\partial N_s}{\partial N_{1i}} / N_s &= 0
\end{align*}
\]

In the previous expressions we have introduced a series of variables which are characteristics of high usage routes and of final routes. Those referring to high usage routes, \( i \), are defined as follows:

\[
H_{0i} = \frac{1}{\partial M_{0i}} / V_{0i} \quad \text{Inverse of the marginal overflow mean with respect to the number of trunks, (with minus sign).}
\]

\[
H_{1i} = \frac{1}{\partial V_{0i}} / V_{0i} \quad \text{Inverse of the marginal overflow variance with respect to the number of trunks, (with minus sign).}
\]

\[
H_{1i} = \frac{\partial M_{0i}}{\partial V_{0i}} / V_{0i} \quad \text{Marginal overflow mean with respect to the offered traffic mean.}
\]

\[
\delta_1 = \frac{\partial M_{0i}}{\partial V_{0i}} / V_{0i} \quad \text{Marginal overflow variance with respect to the offered traffic mean.}
\]

\[
\delta_2 = \frac{\partial M_{0i}}{\partial V_{0i}} / V_{0i} \quad \text{Marginal overflow variance with respect to the offered traffic variance.}
\]

The characteristic variables of final routes, \( i \), are defined as follows:

\[
\lambda_{Mi} = \frac{\partial N_{4i}}{\partial M_{0i}} / N_4 \quad \text{Marginal number of trunks with respect to the offered traffic mean.}
\]

\[
\lambda_{Vi} = \frac{\partial N_{4i}}{\partial V_{0i}} / N_4 \quad \text{Marginal number of trunks with respect to the offered traffic variance.}
\]

Substituting the calculated values of the derivatives, expressions (2.2.) and (2.3.) will become:

\[
\begin{align*}
\frac{\partial C}{\partial N_{1i}} / N_3 &= C_1 - C_2 \left[ \frac{\partial M_{0i}}{\partial V_{0i}} + \frac{\partial N_{4i}}{\partial V_{0i}} \right] - C_3 \left[ \frac{\partial M_{0i}}{\partial V_{0i}} + \frac{\partial N_{4i}}{\partial V_{0i}} \right] + \\
&+ \lambda_{1i} \left[ \frac{\partial M_{0i}}{\partial V_{0i}} + \frac{\partial N_{4i}}{\partial V_{0i}} \right] - C_5 \left[ \frac{\partial M_{0i}}{\partial V_{0i}} + \frac{\partial N_{4i}}{\partial V_{0i}} \right] + \\
&+ \lambda_{1i} \left[ \frac{\partial M_{0i}}{\partial V_{0i}} + \frac{\partial N_{4i}}{\partial V_{0i}} \right] \quad [2.7]
\end{align*}
\]

\[
\frac{\partial C}{\partial N_{1i}} / N_3 = C_1 - C_2 \left[ \frac{\partial M_{0i}}{\partial N_{1i}} + \frac{\partial N_{4i}}{\partial N_{1i}} \right] - C_3 \left[ \frac{\partial M_{0i}}{\partial N_{1i}} + \frac{\partial N_{4i}}{\partial N_{1i}} \right] [2.8]
\]

We shall now define new parameters, \( \phi_{Mi} \) and \( \phi_{Vi} \), characteristics of each route \( i \) in the final routes:

\[
\phi_{Mi} = \frac{\partial M_{0i}}{\partial N_{1i}} \quad \text{is the marginal cost of the route with respect to the traffic mean offered to the route.}
\]

\[
\phi_{Vi} = \frac{\partial V_{0i}}{\partial N_{1i}} \quad \text{is the marginal cost of the route with respect to the traffic variance offered to the route.}
\]

In the high usage routes:

\[
\phi_{Mi} = \frac{\partial M_{0i}}{\partial N_{1i}} \quad \text{is the marginal cost of the network with respect to the traffic mean offered to the route.}
\]
is the marginal cost of the network with respect to the traffic variance offered to the route.

Using the parameters that have just been defined, equations (2.7) and (2.8) can be written as follows:

\[
\frac{\partial C}{\partial N_i} = C_i - \frac{\partial M_1 + \partial M_2}{\partial N_i} - \frac{\partial M_3 + \partial M_4}{\partial N_i} \quad (2.9)
\]

\[
\frac{\partial C}{\partial N_j} = C_j - \frac{\partial M_1 + \partial M_2}{\partial N_j} - \frac{\partial M_3 + \partial M_4}{\partial N_j} \quad (2.10)
\]

Now note that, according to their definition, \( \partial N_i \) and \( \partial N_j \) can be written

\[
\frac{\partial N_i}{\partial N_i} = \text{\[a.e.1\]}
\]

then equations (2.9) and (2.10) become

\[
\frac{\partial C}{\partial N_i} = C_i - \frac{\partial M_1 + \partial M_2}{\partial N_i} - \frac{\partial M_3 + \partial M_4}{\partial N_i} \quad (2.12)
\]

\[
\frac{\partial C}{\partial N_j} = C_j - \frac{\partial M_1 + \partial M_2}{\partial N_j} - \frac{\partial M_3 + \partial M_4}{\partial N_j} \quad (2.13)
\]

The previous formulas are very simple expressions of the derivatives \( \frac{\partial C}{\partial N_i} \)

which where to be calculated. Note that the numerators of the second members depend on the parameters of the marginal costs corresponding to the routes over which, the route whose derivative has been calculated overflows on first choice. The denominators are the parameters \( \lambda_m \) and \( \lambda_v \) corresponding to said route. All marginal cost \( \partial M_i \) and \( \partial N_j \) except \( \partial M_3 \) and \( \partial N_3 \), are calculated directly from parameters \( \lambda_m \) and \( \lambda_v \) defined before. \( \partial M_3 \) and \( \partial N_3 \) are obtained from the other using equation (2.11.).

Equalizing expressions (2.12.) and (2.13.) to zero, the following equations result

\[
1 = \frac{\partial M_1 + \partial M_2}{\partial N_i} + \frac{\partial M_3 + \partial M_4}{\partial N_i} \quad (2.14)
\]

\[
1 = \frac{\partial M_1 + \partial M_2}{\partial N_j} + \frac{\partial M_3 + \partial M_4}{\partial N_j} \quad (2.15)
\]

and they define the values of \( N_i \) and \( N_j \) which minimize the economic function.

3. GENERAL EQUATIONS

In the following we intend, in the first place, to determine the general form of the optimization equation. This equation will have to be fulfilled by each independent variable of the problem, that is, by the number of circuits on each of the high usage routes.

Let us consider any route in Fig. 5, for example route 1.

The traffic offered to this route has as destination those exchanges whose higher order exchange is B, (exchanges D and D'in Fig. 5.)

The minimum condition requires that

\[
\frac{\partial C}{\partial N_i} = 0 \quad (3.1)
\]

where \( C \) is the network cost and \( N_i \) the number of circuits on route 1. The cost \( C \) can be expressed as follows:

\[
C = C_1 N_i + C' \quad (3.2)
\]

\( C_1 N_i \) being the cost of route 1, and \( C' \) the cost of the remaining network.

When the number of circuits in all the high usage routes, except in route 1, is constant, the cost \( C' \) depends on \( N_i \) through \( M_{D1} \) and \( V_{D1} \), mean and variance of the overflow traffic of route 1.

Therefore we have

\[
C' = C'(M_{D1}, V_{D1}) \quad (3.3)
\]

Equation (3.1.) can be written in the following way:

\[
C_1 + \frac{\partial C'}{\partial M_{D1} N_i} \cdot M_{D1} N_i + \frac{\partial C'}{\partial V_{D1} N_i} \cdot V_{D1} N_i = 0 \quad (3.4)
\]

Assuming we know the parameters of marginal cost \( \partial M_i \) and \( \partial N_j \) of each route on which the traffic of 1 overflows as a first choice (routes 2, 3, and 3'), and of the routes to which the traffic carried by 1 (routes 4 and 4') is offered as a first choice, the derivatives of \( C' \) in respect of \( M_{D1} \) and \( V_{D1} \) can be written as follows:

\[
\frac{\partial C'}{\partial M_{D1}} = \partial M_{D1} M_{D1} - \partial M_{D1} M_{D1} M_{D1} - \partial M_{D1} M_{D1} \quad (3.5)
\]

\[
\frac{\partial C'}{\partial V_{D1}} = \partial V_{D1} V_{D1} - \partial V_{D1} V_{D1} V_{D1} - \partial V_{D1} V_{D1} \quad (3.6)
\]
Where $M_{D}$ and $M_{D'}$ represent the addends of the traffic mean, $M_{j}$, offered to route $1$ and whose destinations are, respectively, $D$ and $D'$. The variables $V_{D}$ and $V_{D'}$ have an analogous significance in respect of the traffic variance, $V_{j}$, offered to route $1$.

The preceding equations (3.5.) and (3.6.) implicitly assume that

$$M_{OD} = M_{OD} M_{D}$$  \hspace{1cm} $$V_{OD} = V_{OD} V_{D}$$ \hspace{1cm} (3.7)$$

that is, that the mean and variances of the overflow traffic are the sums of the addends proportional to the means and variances of traffic offered to the route.

Going back to equation (3.4.) the terms that appear in it can be written, using the definition of the parameters $H_{M}$ and $H_{V}$ in paragraph (2), in the following way

$$\frac{\partial M_{D}}{\partial N_{1}} = -\frac{1}{H_{M}} \hspace{1cm} \frac{\partial V_{D}}{\partial N_{1}} = -\frac{1}{H_{V}} \hspace{1cm} (3.8)$$

Keeping in mind these expressions, and expressions (3.5.) and (3.6.), equation (3.4.) can be written as follows:

$$\phi_{M} + \phi_{M} = \frac{M_{D}}{M_{D}} + \frac{M_{D}}{M_{D}} - \frac{M_{D}}{M_{D}} - \frac{M_{D}}{M_{D}} + \frac{M_{D}}{M_{D}} = c_{i} H_{M}$$

$$+ \phi_{V} + \phi_{V} = \frac{V_{D}}{V_{D}} + \frac{V_{D}}{V_{D}} - \frac{V_{D}}{V_{D}} - \frac{V_{D}}{V_{D}} = c_{i} H_{V} \hspace{1cm} (3.9)$$

This equation expresses the condition that must be fulfilled by the number of circuits in route $1$. Note that the parameters that intervene in it are, on the one hand, the marginal costs of the routes to which it is offered as a first choice the overflow traffic, or the traffic carried by the route under consideration; on the other hand the parameters $H_{M}$ and $H_{V}$ of that route. The influence of $N_{i}$ is obtained through these parameters. The equation of optimization corresponding to any of the high usage routes of the network takes the form of equation (3.9.); its general expression can be written

$$z_{j} \frac{\partial (\phi_{M})}{\partial N_{1}} \frac{M_{j}}{M_{j}} + z_{j} \frac{\partial (\phi_{V})}{\partial N_{1}} \frac{M_{j}}{M_{j}} = 1 \hspace{1cm} (3.10)$$

Where the index $i$ extends to all destination exchanges that receive traffic offered to the route being considered, and the index $j$ extends for each destination, $i$, to all the routes to which it is offered as a first choice the overflow traffic of the route under consideration, or the traffic carried by it.

Equation (3.10) establishes, therefore, the minimum condition of the economic function, as a function of the marginal costs $\phi_{M}$ and $\phi_{V}$. The calculation of these parameters can be made in a simple way as described below. There are two quite different cases according to whether the marginal costs on final routes or on high usage routes are to be calculated. In the first case the calculation is direct, as it was already seen in the previous paragraph; in fact

$$\phi_{M} = \frac{\partial C_{j}}{\partial M_{j}} V_{i} = \frac{\partial (C_{j} - N_{j})}{\partial M_{j}} V_{i} = C_{j} + \lambda_{M} \hspace{1cm} (3.11)$$

$$\phi_{V} = \frac{\partial C_{j}}{\partial V_{j}} M_{j} = \frac{\partial (C_{j} - N_{j})}{\partial V_{j}} M_{j} = C_{j} + \lambda_{V} \hspace{1cm} (3.12)$$

The calculation of marginal costs on high usage routes can be made by taking into account that, by definition of them,

$$\phi_{M} = \frac{\partial C_{j}}{\partial M_{j}} V_{i} + \frac{\partial C_{j}}{\partial M_{j}} V_{i} + \frac{\partial C_{j}}{\partial M_{j}} V_{i} = \frac{\partial C_{j}}{\partial V_{j}} M_{j} + \frac{\partial C_{j}}{\partial V_{j}} M_{j} + \frac{\partial C_{j}}{\partial V_{j}} M_{j} \hspace{1cm} (3.13)$$

$$\phi_{V} = \frac{\partial C_{j}}{\partial V_{j}} M_{j} + \frac{\partial C_{j}}{\partial V_{j}} M_{j} + \frac{\partial C_{j}}{\partial V_{j}} M_{j} \hspace{1cm} (3.14)$$

The terms $\left( \frac{\partial C_{j}}{\partial M_{j}} \right)$ and $\left( \frac{\partial C_{j}}{\partial V_{j}} \right)$ are given by expressions (3.5.) and (3.6.). Note that they are the numerators of each of the addends of the left hand side of the optimization equation; therefore in a general way they can be expressed according to what was done in the case of equation (3.10). The factors that multiply these terms in the previous equations are the characteristic parameters of the high usage routes as defined in paragraph (2), that is, the marginal overflow means and variances, with respect to the mean and variance of the offered traffic. Therefore equations (3.13) and (3.14) can be written as follows:

$$\phi_{M} = \left( \frac{\partial C_{j}}{\partial M_{j}} \right) + \left( \frac{\partial C_{j}}{\partial M_{j}} \right) + \left( \frac{\partial C_{j}}{\partial M_{j}} \right) \hspace{1cm} (3.15)$$

$$\phi_{V} = \left( \frac{\partial C_{j}}{\partial V_{j}} \right) + \left( \frac{\partial C_{j}}{\partial V_{j}} \right) + \left( \frac{\partial C_{j}}{\partial V_{j}} \right)$$
which is the general form of equation (2.11) obtained in the particular case studied in the previous paragraph.

Summarizing: Equations (3.10) establishes the condition of optimum economic for each of the high usage routes; two types of parameters come into play here: those designated \( \lambda_H \) and \( \lambda_V \), which are characteristic of each route, functions of traffic offered, and number of trunks in the route (they are, therefore calculated directly), and the marginal costs parameters of the routes to which it is offered as a first choice, the traffic carried or the overflow traffic of the high usage route under consideration.

Equations (3.11.) and (3.12) give the procedure for the calculation of the marginal costs of the final routes. Equations (3.15), applied in a successive way beginning with the route next to last in the hierarchy, till the direct routes are reached, give the calculation procedure for the marginal costs on the high usage routes.

4. RESOLUTION ALGORITHM

For the solution of preceding equation system the following iterative produce has been followed:

1. Estimation of parameters \( \lambda_H \) and \( \lambda_V \) of the final routes, and parameters \( b_H, b_V, c_H \) and \( c_V \) of the high usage routes.

2. Calculation of parameters \( \phi_H \) and \( \phi_V \) on all final routes by means of equations (3.11.) and (3.12).

3. Making a "sweep" of the network from the final routes to the direct routes, parameters \( \phi_H \) and \( \phi_V \) are evaluated by means of the reiterative application of equations (3.15): for fractions \( M_i/\Sigma M_i \) and \( V_i/\Sigma V_i \) it has been assumed that all \( M_i \) and \( V_i \) are equal for any \( i \).

4. Using the optimization equation (3.10), calculation on each direct route of the number of trunks that satisfy that route, according to the traffic offered. With this number, evaluation of the overflow traffic of these routes that will be offered to the overflow routes corresponding to each of them in accordance with the network structure.

5. Making a "sweep" of the network from the direct routes to the final routes, calculation of the number of circuits by means of the optimization equation on each of the high usage routes and of the overflow traffic on each of them; the overflow traffic will be offered to those routes defined by the network structure.

6. Calculation of the number of circuits on the final routes, the traffic offered to each of them and the lost traffic mean defined by the grade of service being known.

7. Calculation of parameters \( \lambda_H \) and \( \lambda_V \) on the final routes, and parameters \( \phi_H \) and \( \phi_V \) on the same routes.

8. Making a "sweep" of the network from the final routes to the direct routes, valuation of parameters \( b_H, b_V, c_H \) and \( c_V \) on each of them, and of parameters \( \phi_H \) and \( \phi_V \). The used traffic values are those obtained in the last "sweep" carried out from the direct routes to the final routes.

9. Repeat the described process from point 4. The optimum will be reached when the result of two consecutive iterations is identical.

5. COMPUTER PROGRAM

A computer program has been prepared using the methods we have just described. The most important characteristics of this program are:

- FORTRAN IV(?) programming language. Therefore the program can be executed in the majority of available computers.
- It has a modular structure; this means that changes can be made with a minimum reprogramming effort, being necessary to substitute only the module or modules which are affected by the desired change.
- All input variables as well as results are stored in a work disk so that only the program logic is stored in the internal memory. This makes the size of the networks that can be studied with the program practically unlimited.
- The internal memory required is approx. 70 kilobytes.
- The program has been conceived in such a way that it may deal with most real cases; therefore it can accept restrictive conditions of such type as maximum and minimum number of trunks on a particular route, non-existence of some routes, election a priori of particular routes as final routes, etc.

6. CONCLUSIONS

The method described is specially suitable for its treatment in a computer insofar as the form of the optimization equation is the same for all routes. Then the computer program is relatively simple.

The method is independent from the used traffic models; a change in these models would affect only the modules of the program which operatively perform the model. Special mention must be made of the hypothesis expressed by equations (3.7.); this hypothesis is very questionable, above all in the case of networks with "crossed" routes, and is presently under study. The method is also independent from the routing law; a change in this law would imply a change in the modules that perform the routing "logic" in the program.

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8. ANNEX

Here, the procedure used to solve the traffic problems arising from the proposed method is given. According to whether the availability is full or limited we have grouped the formulas in two parts:

Full availability
- Overflow traffic from groups of N trunks and pure chance offered traffic A:

Mean \( M = A \cdot E(A,N) \)

Variance \( \frac{V}{M} = 1 - M + \frac{A}{N+1+M-A} \)

where \( E(A,N) \) is the Erlang function

\[ E(A,N) = \frac{A^N}{N!} \cdot \frac{e^{-A}}{\epsilon^N} \cdot \frac{1}{t^n} \cdot dt \]

- Wilkinson's equations. Equivalent group, pure chance traffic \( A^* \) and number of trunks \( N^* \), being of offered traffic defined by its mean, \( M \), and its variance, \( V \),

\[ M = A^* \cdot E(A^*,N^*) \]

\[ \frac{V}{M} = 1 - M + \frac{A^*}{N^*+1+M-A^*} \]

Limited availability Lotze's equations.
- Overflow traffic from groups of N trunks, availability \( K \), and pure chance offered traffic A:

Mean \( M = \frac{E(A_0,N)}{E(A_0,N-K)} \cdot A \)

\[ M = A_0 [1 - E(A_0,N)] \]

Variance

\[ \frac{V}{M} = 1 + \frac{K}{N} \left[ \frac{A}{1+K-A(1-M/A)} \cdot M \right] \left[ 1 + 0.25 \left( 1 - \frac{K}{N} \right) \right] \]

\[ \frac{M}{A} = E(A_1,K) \]

- Generalized Wilkinson's equations.

Equivalent group, pure chance traffic \( A^* \) number of trunks, \( N^* \), availability \( K^* \) being offered traffic defined by its mean, \( M^* \), and its variance, \( V^* \),

\[ M = \frac{E(A^*,N^*)}{E(A^*,N^* - K^*)} \cdot A^* \]

\[ M = A^* - A^* [1 - E(A^*,N^*)] \]

\[ \frac{V}{M} = 1 + \frac{K^*}{N^*} \left[ \frac{A^*}{1+K^* - A^*[1-M^*/A^*]} \cdot M \right] \left[ 1 + 0.25 \left( 1 - \frac{K^*}{N^*} \right) \right] \]

\[ \frac{M}{A^*} = E(A^*,K^*) \]

\[ N^*/K^* = 2 \cdot N/K \]

9. REFERENCES

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