AN APPROACH OF REAL/FEED-BACK TYPE/TELEPHONE TRAFFIC AND ITS EFFECT ON THE CHARACTERISTICS OF QUEUING SYSTEMS

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ABSTRACT
Owing to the characteristics of the telephone traffic usually a feed-back type traffic composed by pairs of demands /trebles of them, etc/ arrives to the input of the common control systems of telephone exchanges. This could produce significant differences in the quality of service as compared with that of a Poisson /Engset/ input process in case of heavy traffic and priority queue discipline. The service order giving priority to the second member of a pair of demands /selection type connections/ is favourable in all cases. As a consequence in examining the effects of overload and in dimensioning high speed common control systems it seems advisable to take into account the properties of real telephone traffic.

INTRODUCTION
In telephone exchanges the common control system performs several operations while handling a particular call and for doing this it connects itself several times to the same cell. This means that the above mentioned operations are generated by separate demands belonging to the same call, therefore time-interrelations exist between them. This is the case e.g. when the control system first connects a register and subsequently the called line to a calling subscriber's line so that /in case of no waiting/ actually the dialling time elapses between the appearances of the two demands for connection.

If an input process combined of such demands appears at the input of the control system /as a server/ theoretically it can not be regarded as a Poisson-process /or as an Engset-process in case of a finite number of subscribers/ because the arrival times of the individual demands are not completely independent of each other. The complete input process may generally be regarded as consisting of a Poisson-process /or Engset process/ - this corresponds to the first arrivals of the calls /primary process/ - and one or more /secondary, etc./ processes are added to it in which the appearances of demands are influenced by the primary process. The secondary /tertiary, etc./ processes return to the same server, therefore the complete input process may be regarded as a feed-back type process. The fact that time-interrelations exist between the partial processes affects the characteristics of the waiting system and for the above mentioned reason such a complex input process consists of pairs of demands /trebles of them, etc./.

Our tests have been aimed at finding the effect of the before mentioned input process type on the service characteristics.

The examination of the question is justified by the fact that in dimensioning common control systems the above feed-back type characteristic of the telephone traffic is usually not taken into account although we have no knowledge of results supporting this simplification. The theoretical examination of the feed-back type waiting systems have been conducted by several authors /1,2/. The discussed cases may however be regarded as the marginal case of the system presently examined as the time between the completion of serving the primary demand and the repeated appearance of the call was supposed as being zero.

The examination is also justified by the fact that the opinions about the optimum service order of the demands appearing in a telephone exchange are different from each other. According to certain opinions /e.g./3/ priority should be given to the secondary demands /selection type connections/, while others - primarily for psychological reasons - deem it advisable to serve the primary demands /register type connections/ as quickly as possible. It seems obvious that the question could only be satisfactorily answered if the difference between the grade of service given
by the different methods was determined.

1. TEST PRINCIPLE AND METHOD

The tests were carried out by simulation. A time-true simulation method was employed the program of which produced the mean waiting times, the distribution of the waiting times, the average lengths of the waiting lines, the density function of the line lengths, etc.

Minio experiments were carried out in every case and the accuracy of the obtained results is related to a confidence interval of 95 per cent. For the sake of the single demonstration of the characteristics properties the input process consisted of two parts in the examined case. The primary process was supposed to be a Poisson-process and the system consisted of a single server only. The \( T_1 \) and \( T_2 \) service times were constants, and the \( T_2 \) section time between the departures of the first demand and the arrival of the second demand of a pair of demands was a normally distributed random variable.

![Fig. 1 A pair of demands as an example of the feed-back type input process](image)

Each cell spends \( T_1 + T_2 \) time in the system after the first arrival and then leaves it at the first departure. After the time \( T_1 \) the cell appears with a new demand at the second arrival and having spent \( T_1 + T_2 \) time in the system leaves it at the last departure. The \( T_1 \) and \( T_2 \) random variables designate the waiting times of the primary and secondary demands. The queue discipline was FQFS (first come first served and non preemptive priority type, respectively).

The results obtained by the simulation were compared to the corresponding calculated values of the M/G/1 system Poisson input process, arbitrary service time distribution function, single server. By this comparison we would have liked to determine whether any change of the usual calculation method is justified because of the degree of the deviations from the theoretical results of the M/G/1 system or not.

2. REFERENCE FORMULAE

2.1. FCFS queue discipline

For defining the average waiting time the Pollaczek-Khintchine formula was used

\[ W = \frac{\lambda}{2} \int_0^\infty t^2 dF(t) \]

where \( \lambda \) is the intensity of the input process, \( \phi \) is the traffic of the serving organ, \( F(t) \) is the distribution function of the service time,
The mean waiting time in our case is given by

\[
W = \frac{1}{k} (W_1 + W_2)
\]

(2a)

The distribution function of the waiting time for nonpreemptive priority in a general case was given by Kersten and Runnenburg (6). The general form of the Laplace-Stieltjes transform of the waiting time distribution function for calls with a priority of \( k = 1 \) is:

\[
\Psi_k(p) = \left(1 - \frac{\lambda_1}{\lambda_k}\right)p + \sum_{i=2}^{r} \lambda_i \left(1 - \Phi_i(p)\right)
\]

where \( \lambda_i \) is the mean value of the service time in the \( i \)-th priority class

\( \Phi(p) \) is the Laplace-Stieltjes transform of the distribution function of the service time in the \( i \)-th priority class

\( r \) is the number of priority classes

For the tested case \( \tau_1(t) = 1 (t - T_1) \)

\( \Phi_1(p) = e^{-pT_1} \)

and if the primary process has priority

\[
\Psi_2(p) = \left(1 - \frac{\lambda_1}{\lambda_2}\right)p + \lambda_2 \left(1 - e^{-pT_2}\right)
\]

The equation being valid for the priority of the secondary process is obtained by changing the values \( \lambda_1 \), to \( \lambda_2 \), and \( T_1 \) to \( T_2 \), respectively.

The form of the waiting time distribution function can be derived by inverse transform:

\[
W_k(t) = \begin{cases} W_{1k}(t) & \text{if } t < T_2 \\ W_{1k}(t) + W_{2k}(t) & \text{if } t \geq T_2 \end{cases}
\]

(3)

where for the case of the priority of the primary process again:

\[
W_{1k}(t) = e^{-\lambda_1T_1} \sum_{n=0}^{k} \frac{e^{-\lambda_1T_1} \left(1 - \frac{\lambda_1}{\lambda_k}\right)^n}{n!} + \frac{\lambda_2}{\lambda_1} \tau_1(t) - \lambda_2 \tau_2(t) - \lambda_2 T_1,
\]

\[
W_{2k}(t) = e^{-\lambda_1T_1} \sum_{n=0}^{m} \frac{e^{-\lambda_1T_1} \left(1 - \frac{\lambda_1}{\lambda_k}\right)^n}{n!} + \frac{\lambda_2}{\lambda_1} \tau_1(t) + \frac{\lambda_2}{\lambda_1} \tau_2(t) + \frac{\lambda_2}{\lambda_1} T_2
\]

and furthermore,

\[
kT_1 \leq t \leq (k+1)T_1
\]

\[
mT_1 + T_2 \leq t \leq (m+1)T_1 + T_2
\]

3. TEST RESULTS

3.1 General

In the case of the FCFS queue discipline there is only one queue waiting for the server both the primary and secondary demands connect to this. Two cases were tested for service with priority, in one case the primary demands /primary first = PF/ and in the other the secondary demands /secondary first = SF/ had priority. In these cases the primary and secondary demands were arranged into separate queues waiting for the server, but within the queues the FCFS principle was used.

The preliminary tests showed that significant deviations from the theoretical reference values can be experienced only when \( \varphi > 0.9 \) erl. As a consequence of this the tests were generally carried out for \( \varphi = 0.9 \) erl, but to present a survey of the situation the variation of a few characteristic values was given as the function of the traffic of the server.

\[ M \gamma / \gamma \] the mean value of the action time distribution was sixteen time units, and \( B^2 / \gamma \)

the standard deviation was 2.87 time units during the tests. The value \( T_1 + T_2 \) was chosen as time unit. The effects of the parameters \( M / \gamma \) and \( B^2 / \gamma \) were also examined.

In the figures the reference values are connected with a thin line and the measured values are connected with a thick one. The 95 per cent confidence limits are also given together with the measured values.

3.2 Average waiting times

In Figure 2, the average waiting times can be seen for FCFS and for the two priority disciplines.

![Fig. 2 Comparison of calculated average waiting times with those obtained with simulation](image-url)
In the FCFS case the measured average waiting time curve has no substantial deviation from the calculated one, the PP curve has about 30 per cent higher as an average, and the SF curve is practically identical to the point T2/T1 = 1 and then gradually travels over the calculated value.

3.3 Average waiting times in the priority classes

Figure 3 shows the average waiting times for the individual priority classes in the PP and SF cases. The reference values are the W1 and W2 expressions in formula /2/, respectively, and ψ = 0.9 erl.

In the PP case the measured waiting times of the priority demands are in complete agreement with the calculated values - which is reasonable because then the demands of the Poisson input process are served first. This means that the deviation of the average waiting time in Figure 2 is caused solely by the longer waiting times of the secondary demands served in the second class.

In the SF case up to the value T2/T1 = 1.5 there is no deviation from the calculated curve, however, on the right from that point an interesting phenomenon can be experienced. The average waiting time of the secondary demands having priority rapidly increases, but the average waiting time of the secondly served primary demands is more favourable as expected. After all with a minor increase of the average waiting time the advantage of the priority almost disappears because the primarily served secondary demands are also waiting for strikingly long times.

3 " Effect of the server traffic

In Figure 4 the average waiting times of the cases PP, T1 = 1.6, T2 = 0.4 and SF, T1 = 0.4, T2 = 1.6 are plotted as the function of the traffic. For comparison purposes the W1 and W2 expressions of formula /2/ and the averages of them were used.

It can be seen that the deviations from the calculated values become more and more striking with the increase of the traffic.

3.5 Effects of M / ψ / and D2 / ψ /

In Figure 5 the effect of the change in the mean value of the action time can be seen in case of SF service /T1 = 0.4, T2 = 1.6, ψ = 0.9/, D2 /ψ/ = 1.53/. The waiting times increase linearly with the increase of the average action time. The horizontal lines plotted for comparison show the calculated reference values. The importance of the phenomenon is increasing because of the acceleration of the operation speed of the common control systems. Namely, the action time is dependent on some human action /e.g. selection, pressing the button/ which can not be significantly accelerated and so the action time will be longer and longer related to the average holding time unit of the common control system.

Figure 6 illustrates the effect of the change in the standard deviation of the action time in case of SF service /T1 = 0.4, T2 = 1.6, ψ = 0.9/, M /ψ/ = 15/. The waiting time values decrease with the increase of the standard deviation, however, the character of the deviation from the calculated values is not changed. To get an average waiting time level less than the
calculated value is an unreal possibility because the action time generally has a well-defined lower limit value — with which the greatest value of the standard deviation is also defined.

\[ P \geq 0 = \phi \] was valid. The waiting probability of the secondary process is illustrated in Figure 7 for the PF and in Figure 8 for the SF case, respectively. It can be seen that in the SF case the anomaly experienced in the character of the average waiting time manifests itself in the value of the waiting probability too.

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4. CONCLUSIONS

4.1 From service quality point of view significant difference between the feed-back type input process which takes into account the action time, and the Poisson input process can be observed when \( q > 0.6 \text{ erl} \) and priority discipline is used.

4.2 The queue discipline which gives priority to the secondery demands is always favourable, however, it should be taken into account that the differences between the waiting times of the priority classes almost disappear from \( T_2/T_1 > 2 \).

4.3 In the case of the input process used for tests the average waiting times are increased as compared with those of a Poisson input process of the same intensity. The magnitude of the deviation increases with the increase of the traffic, and so, the effects of the overload appear earlier both in the case of a long-lasting overload and in the cases of short duration traffic peaks.

4.4 The increase of the average waiting times is dependent on the relative length of the action time, therefore the phenomenon appears to a greater extent in high speed control systems.

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414/6