ANALYSIS OF A NON-PREEMPTIVE PRIORITY QUEUEING SYSTEM WITH SETUP TIMES

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ABSTRACT

Two classes of calls arrive at a service station in Poisson process. Service times for calls of each class are independently and arbitrarily distributed. A setup time is required, whenever service is changed from one class to the other. A setup time is also required at the beginning of each busy period. Setup times for each class are independently and arbitrarily distributed. Calls of class 1 have non-preemptive priority over calls of class 2, but the order of service within each class is first-come, first-served. The system is assumed to be under-saturated.

The joint queue-length distribution is studied by the imbedded Markov chain method, and its generating function is obtained. Then, by applying the ordinary method, Laplace-Stieltjes transforms for the distribution function of waiting time are derived. Expressions for the mean queue length and the mean waiting time for calls of each class are given. Finally, some numerical examples are given and the results are compared with those for the alternating priority queueing system with setup times.

It is noted that the non-preemptive priority queueing system with changeover times is investigated by M. Eisenberg. He also studied some variations of the system. An alternating priority queueing system with changeover times and setup times was investigated by Miller. In this paper, we will consider a non-preemptive priority queueing system with changeover times and setup times.

1. INTRODUCTION

Priority queueing systems with Poisson inputs have been investigated by many authors. Suppose that two classes of calls arrive at a service station and are served by a single server. The following three priority disciplines are usually considered.

Preemptive: Calls of class 1 have exogenous priority over calls of class 2. If a call of class 1 arrives when a call of class 2 is being served, the server interrupts the current service and immediately starts serving the call of class 1.

Non-Preemptive: Calls of class 1 have exogenous priority over calls of class 2. If a call of class 1 arrives when a call of class 2 is being served, the server never interrupts the current service. The current service is continued to completion.

Alternating: Calls of each class can have endogenous priority depending upon the current service. If the current service is of class 1, calls of class 1 have endogenous priority over calls of class 2. The server serves all calls of class 1 that are present and also all the new arrivals as long as there are calls of class 1 in the system. If the current service is of class 2, calls of class 2 have endogenous priority.

In two-class queueing systems, the server may require an adjustment or preparation period whenever service is changed from one class to the other. A preparation period may be also required at the beginning of each busy period. Times for these preparations are called as changeover times, orientation times, or setup times by some authors. In this paper, we use the following definitions.

Changeover time: A changeover time is required when service is changed from one class to the other.

Orientation time: When the system becomes empty, the server is oriented towards one of two classes. If a call of the oriented class arrives during the idle period, the server starts serving the call. If a call of the other class arrives, an orientation time is required.

Setup time: A setup time is required whenever a call arrives during an idle period, regardless of class of the arriving call.

Priority queueing systems with changeover times, orientation times, or setup times are investigated by some authors. Gaver investigated a preemptive priority queueing system with changeover times and orientation times, and the results were compared with those of the first-come, first-served queueing system. Jaiswal characterized the transient behavior of this queueing system, and Mevert considered some variations. A non-preemptive priority queueing system with changeover times was investigated by Eisenberg. He also studied some variations of the system. An alternating priority queueing system with changeover times and setup times was investigated by Miller. In this paper, we will consider a non-preemptive priority queueing system with changeover times and setup times.
2. MODEL AND NOTATION

Suppose that two classes of calls arrive independently at a service station. The arrivals are Poisson with arrival rates \( \lambda_1, \lambda_2 \), and the service times are independently and arbitrarily distributed. Denote by \( H_1(t) \) the distribution function of the service time of a call of class 1 and by \( H_2(t) \) that of a call of class 2. Suppose that calls of class 1 are given "high priority" status, and those of class 2 are of "low priority". Suppose further that the interaction between high and low priority classes is non-preemptive (head-of-the-line). It is assumed that the order of service within each class is first-come, first-served.

Suppose that a changeover time is required whenever service is changed from one class of calls to the other. Suppose further that a setup time is required whenever a call arrives during an idle period. It is assumed that the changeover times and the setup times for each class are independently and identically distributed. Thus, it is convenient to use the same terminology for changeover times and setup times, and both are referred to as "setup times" henceforth. Denote by \( U_1(t) \) the distribution function of a setup time for class 1 and by \( U_2(t) \) that of a setup time for class 2.

We will use the following notation:

\[
\begin{align*}
H_i(s) &= \int_0^\infty e^{-st} dH_i(t), \\
U_i(s) &= \int_0^\infty e^{-st} dU_i(t), \\
\lambda_i &= \int_0^\infty dH_i(t), \\
\mu_i &= \int_0^\infty t dH_i(t), \\
\rho_i &= \lambda_i/\mu_i, \\
\Delta_i &= 1 - \rho_i.
\end{align*}
\]

for \( i = 1, 2 \).

3. THE EQUILIBRIUM DISTRIBUTION OF QUEUE LENGTHS

Consider an epoch at which a service for a call of class 1 \((i = 1, 2)\) is commenced, and let \( n_1 \) and \( n_2 \) be the numbers of calls of class 1 and class 2 respectively. The generating function of \( n_1 \) and \( n_2 \) is given by

\[
G_1(x_1, x_2) = Q_1(x_1, x_2) = Q_1(0, x_2) - Q_1(0, 0) + p_2Q_1(0, 0)x_2.
\]

Using \( G_2(x_2) \), Eq. (3.3) becomes

\[
x_2Q_2(x_1, x_2) = [Q_2(0, x_2) - Q_2(0, 0)]H_2^n(x_1 + z_2)
+ Q_2(x_2)\mu_2U_2^n(x_1 + z_2)H_2^n(x_1 + z_2) + Q_2(0, 0)\mu_2U_2^n(x_1 + z_2).
\]

Then, by putting \( x_1 = 0 \) in Eq.(3.6), it is obtained that

\[
Q_2(0, x_2) = \frac{G_2(x_2)U_2^n(\lambda_1 + z_2) - Q_2(0, 0)}{H_2^n(\lambda_1 + z_2)}.
\]

From Eqs.(3.6) and (3.7), it is obtained that

\[
Q_2(x_1, x_2) = \left( \mu_2H_2^n(\lambda_1 + z_2) \right)H_2^n(x_1 + z_2)
- \frac{x_2Q_2(0, 0)}{H_2^n(\lambda_1 + z_2)}H_2^n(x_1 + z_2).
\]

Similarly, consider an epoch at which a setup (or changeover) for a call of class 1 is commenced, and let \( \xi_1 \) and \( \xi_2 \) be the numbers of calls of class 1 and class 2 respectively. The generating function of \( \xi_1 \) and \( \xi_2 \) is given by

\[
G_1(x_1, x_2) = Q_1(x_1, x_2) = Q_1(0, x_2) - Q_1(0, 0) + p_1Q(0, 0)x_1.
\]

Substituting Eqs.(3.7) and (3.8) in Eq.(3.9), it is obtained that

\[
Q_1(x_1, x_2) = \left( \mu_1H_1^n(\lambda_1 + z_2) \right)H_1^n(x_1 + z_2)
- \frac{x_2Q_1(0, 0)}{H_1^n(\lambda_1 + z_2)}H_1^n(x_1 + z_2).
\]

Since Eq.(3.2) is written as

\[
x_1Q_1(x_1, x_2) = \left( Q_1(0, x_2) - Q_1(0, 0) \right)H_1^n(x_1 + z_2)
+ G_1(x_1, x_2)U_1^n(x_1 + z_2)H_1^n(x_1 + z_2),
\]

it is obtained that

\[
Q_1(x_1, x_2) = \left( A(x_1, x_2) - x_2B(x_1, x_2) - x_2 \right)
+ C(x_1, x_2)Q(0, 0)
- D(x_1, x_2)Q_2(0, 0)
- H_1^n(x_1 + z_2)
+ \frac{\Delta_1^nH_1^n(x_1 + z_2)}{\xi_1 - H_1^n(\lambda_1 + z_2)}.
\]

where we put

\[
A(x_1, x_2) = \left( U_1^n(x_1 + z_2) + \frac{\Delta_1^nU_1^n(\lambda_1 + z_2)}{\xi_1 - H_1^n(\lambda_1 + z_2)} \right)
- H_1^n(x_1 + z_2)H_2^n(x_1 + z_2),
\]

\[
B(x_1, x_2) = \frac{\Delta_2^nU_2^n(x_1 + z_2)}{\xi_2 - H_2^n(\lambda_1 + z_2)},
\]

Next, consider an epoch at which a service for a call of class 2 is commenced, and let \( n_2 \) be the number of calls of class 2. The generating function of \( n_2 \) is given by

\[
Q_2(x_2) = Q_2(0, x_2) - Q_1(0, 0) + p_2Q(0, 0)x_2.
\]
\[
C(x_1, x_2) = p_2 x_2 + p_1 x_1 H^*_2(x_1 + x_2), \quad (3.15)
\]
\[
D(x_1, x_2) = H^*_2(x_1 + x_2) - H^*_2(0 + x_2) \quad \text{and} \quad x_2 - H^*_2(x_1 + x_2) - U^*_2(x_1 + x_2). \quad (3.16)
\]

If \( p_2 = \lambda_2, h < 1 \) and \( |x_1| \leq 1 \), then by using Rouché's theorem, we can prove that the denominator of the right-hand side of Eq. (3.12) has exactly one root, say \( y_1 \), in the domain \( |x_1| \leq 1 \). We note that
\[
y_1 = \frac{x_2}{1 - \lambda_2}, \quad (3.17)
\]
where \( \tau^*_1(s) \) is the Laplace-Stieltjes transform of the probability that the length of a busy period is \( \leq t \) in a single server queue with a Poisson input of density \( \lambda_2 \) and with mutually independent service times having a common distribution \( \mu \). Since \( |Q_1(x_1, x_2)| \leq 1 \) if \( |x_1| \leq 1 \) and \( |x_2| \leq 1 \), it follows that \( y_1 \) must also be a root of the numerator of the right-hand side of Eq. (3.12). Thus, it is obtained that
\[
(A(y_1, x_2) - x_2 B(y_1, x_2) - x_2 Q_2(x_2)/x_2 = Q_1(0, 0) - C(y_1, x_2)Q_2(0, 0) + D(y_1, x_2)Q_2(0, 0), \quad (3.18)
\]
which leads to
\[
G_2(x_2) = \frac{(Q_1(0, 0) - C(y_1, x_2)Q_2(0, 0) + D(y_1, x_2)Q_2(0, 0))}{Q_2(0, 0)}/x_2. \quad (3.19)
\]
Substituting Eq. (3.19) in Eqs. (3.12) and (3.8), \( Q_1(x_1, x_2) \) and \( Q_2(x_1, x_2) \) are obtained by using \( Q_1(0, 0), Q_2(0, 0) \) and \( Q(0, 0) \).

4. THE METHOD OF FINDING \( Q_1(0, 0), Q_2(0, 0) \) AND \( Q(0, 0) \)

Let \( x_1, x_2 \to 1 \) in Eq. (3.12), then we have
\[
Q_1(1, 1) = 1 - \left\{ (A(x_1, 1) - B(x_1, 1 - 1)G_2(1, 1)) + C(x_1, 1)Q_2(0, 0) - D(x_1, 1)Q_2(0, 0) \right\} \quad (4.1)
\]
Here, \( G_2(1) \) is obtained by letting \( x_2 \to 1 \) in Eq. (3.19) and by applying L'Hospital's theorem. After some calculations, it is shown that
\[
G_2(1) = \frac{p_2(1 + \lambda_1 u_1)(1 - H^*_2(1))Q(0, 0)}{(1 - \rho_1 - \rho_2)V(\lambda_1) - \lambda_2 u_1 u_2(1 - H^*_2(\lambda_1))} + \frac{\lambda_2(u_1 + u_2)(1 - H^*_2(\lambda_1))}{(1 - \rho_1 - \rho_2)V(\lambda_1) - \lambda_2(u_1 + u_2)(1 - H^*_2(\lambda_1))}. \quad (4.2)
\]
where we put
\[
V(\lambda_1) = 1 - H^*_2(\lambda_1) + U^*_2(\lambda_1)H^*_2(\lambda_1). \quad (4.3)
\]
Then, it is derived that
\[
Q_1(1, 1) = \frac{p_2(1 + \lambda_1 u_1)V(\lambda_1)Q(0, 0)}{(1 - \rho_1 - \rho_2)V(\lambda_1) - \lambda_2 u_1 u_2(1 - H^*_2(\lambda_1))} + \frac{\lambda_2(u_1 + u_2)(1 - H^*_2(\lambda_1))}{(1 - \rho_1 - \rho_2)V(\lambda_1) - \lambda_2(u_1 + u_2)(1 - H^*_2(\lambda_1))}. \quad (4.4)
\]
In a similar way, it is shown that
\[
Q_2(1, 1) = \frac{p_2(1 + \lambda_1 u_1)V(\lambda_1)Q(0, 0)}{(1 - \rho_1 - \rho_2)V(\lambda_1) - \lambda_2 u_1 u_2(1 - H^*_2(\lambda_1))} + \frac{\lambda_2(u_1 + u_2)(1 - H^*_2(\lambda_1))}{(1 - \rho_1 - \rho_2)V(\lambda_1) - \lambda_2(u_1 + u_2)(1 - H^*_2(\lambda_1))}. \quad (4.5)
\]
Since
\[
Q_1(1, 1) + Q_2(1, 1) = 1, \quad (4.6)
\]
it is obtained that
\[
(1 - \rho_1 - \rho_2)V(\lambda_1) - \lambda_2 u_1 u_2(1 - H^*_2(\lambda_1)) + \lambda_2(u_1 + u_2)(1 - H^*_2(\lambda_1)) = 0. \quad (4.7)
\]
Eq. (4.7) gives a relation between \( Q(0, 0) \) and \( Q_2(0, 0) \). Here, we note that
\[
Q_1(1, 1) = p_1, \quad Q_2(1, 1) = p_2 \quad (4.8)
\]
is derived from Eqs. (4.4), (4.5) and (4.7).

Next, we will derive another relation. Applying Rouché's theorem to the denominator of the right-hand side of Eq. (3.19), we can prove that the denominator has exactly one root, say \( y_2 \), in the domain \( |x_2| < 1 \). Since the function \( G_2(x_2) \) is analytic in this domain, it follows that \( y_2 \) must be a root of the numerator of the right-hand side of Eq. (3.19). Thus, it is obtained that
\[
Q_1(0, 0) - C(y_2, v_2)Q_2(0, 0) + D(y_2, v_2)Q_2(0, 0) = 0, \quad (4.9)
\]
where we put
\[
y_2 = \frac{x_2}{1 - \lambda_2}. \quad (4.10)
\]
Eq. (4.7), (4.9) and (4.4) give simultaneous equations for \( Q_2(0, 0), Q_2(0, 0) \) and \( Q(0, 0) \). Put
\[
K_1 = (1 + \lambda_1 u_1)V(\lambda_1), \quad K_2 = (1 + \lambda_2 u_1 v_2 - u_1 V(\lambda_2)), \quad K_3 = (1 - \rho_1 - \rho_2)V(\lambda_1) - \lambda_2(u_1 + u_2)(1 - H^*_2(\lambda_1)), \quad \text{then we have}
\]
\[
Q_1(0, 0) = \frac{K_2(1 - D(y_2, v_2))}{K_1(1 - D(y_2, v_2)) + K_2(1 - C(y_2, v_2))} \quad (4.12)
\]
\[
Q_1(0, 0) = \frac{K_2(C(y_2, v_2) - D(y_2, v_2))}{K_1(1 - D(y_2, v_2)) + K_2(1 - C(y_2, v_2))} \quad (4.13)
\]
\[
Q_2(0, 0) = \frac{K_2(1 - C(y_2, v_2))}{K_1(1 - D(y_2, v_2)) + K_2(1 - C(y_2, v_2))} \quad (4.14)
\]
Thus, the generating functions \( Q_1(x_1, x_2) \) and \( Q_2(x_1, x_2) \) are completely determined.
The mean queue lengths for calls of class 1 and class 2 are calculated by

\[ q_1 = \lim_{x_1 \to 1} \left( \frac{d}{dx_1} \right) Q_1(x_1, 1)/Q_1(1, 1), \]  
\[ q_2 = \lim_{x_2 \to 1} \left( \frac{d}{dx_2} \right) Q_2(1, x_2)/Q_2(1, 1), \]

respectively. From Eqs.(3.12) and (3.8), it is obtained that

\[ Q_1(x_1, 1) = \left\{ A(x_1, 1) - B(x_1, 1) - l \right\} G_2(1) - Q_1(0, 0) + C(x_1, l)Q(0, 0) \times H_1(z_1) - D(x_1, 1)Q_2(0, 0) \]  
\[ Q_2(1, x_2) = \left\{ U_2(x_2) + \frac{U_2^*(x_2) + H_2^*(x_2)}{x_2 - H_2(x_1 + z_2)} \right\} \times \frac{Q_2(1, x_2)}{x_2 - H_2(x_1 + z_2)}. \]

Differentiating Eq.(5.3) two times with respect to \( x_1 \) and then putting \( x_1 = 1 \), it is obtained that

\[ 2(1 - P_1)Q(1, 1) - \]  
\[ 2(1 - P_1)Q(1, 1) = \]  
\[ (A_1 - B_1)G_2(1) + C_2Q(0, 0) - D_2Q(0, 0) \]

where we put

\[ A_1 = \lim_{x_1 \to 1} \left( \frac{d^2}{dx_1^2} \right) A(x_1, 1), \]
\[ B_1 = \lim_{x_1 \to 1} \left( \frac{d^2}{dx_1^2} \right) B(x_1, 1), \]
\[ C_1 = \lim_{x_1 \to 1} \left( \frac{d^2}{dx_1^2} \right) C(x_1, 1), \]
\[ D_1 = \lim_{x_1 \to 1} \left( \frac{d^2}{dx_1^2} \right) D(x_1, 1), \]

for \( i = 1, 2 \).

Thus, after some calculations, it is obtained that

\[ q_1 = \frac{\lambda_1^2 u_1^2}{2(1 - \rho_1)} + \frac{\lambda_1\lambda_2u_2^2}{2(1 - \rho_1)} - \frac{\lambda_1^2 u_1^2}{2(1 - \rho_1)} - \frac{\lambda_2^2 u_2^2}{2(1 - \rho_1)} + \frac{\lambda_1^2 u_1^2}{2(1 - \rho_1)} - \frac{\lambda_2^2 u_2^2}{2(1 - \rho_1)} \]

Next, we will find \( q_2 \). Differentiating Eq.(5.4) with respect to \( x_2 \), and then putting \( x_2 = 1 \), it is obtained that

\[ q_2 = \frac{\lambda_1^2 u_1^2}{1 - H_2^*(1)} - \frac{\lambda_2^2 u_2^2}{1 - H_2^*(1)} \]

where we put

\[ H_2^*(1) = \lim_{x_2 \to 1} \left( \frac{d}{dx_2} \right) H_2(x_2), \]
\[ (U_2(x_2) + H_2^*(x_2)) = \lim_{x_2 \to 1} \left( \frac{d}{dx_2} \right) U_2(x_2)H_2(x_2). \]

Then, it is obtained that

\[ q_2 = \frac{(\lambda_1 + \lambda_2)u_2}{1 - H_2^*(1)}. \]

It is noted that \( G_2(x_2) \) is calculated by differentiating \( G_2(x_2) \) which is given in Eq.(3.19). But the final result is complicated and is abbreviated in this paper.

6. THE EQUILIBRIUM DISTRIBUTIONS OF WAITING TIMES

For a stationary process, denote by \( W_i(t) \) the probability that the waiting time of a call of class \( i \) is \( \leq t \). Let \( W_i(s) \) be the Laplace-Stieljes transform of \( W_i(t) \), i.e.,

\[ W_i(s) = \int_0^\infty e^{-st} dW_i(t) \]

for \( i = 1, 2 \). Since the queue length of class \( i \) immediately after the departure of a call of class \( i \) is equal to the number of calls of class \( i \) arriving during the waiting time and the service time of the departing call, it is obtained that

\[ Q_1(x_1, 1)/Q_1(1, 1) = \frac{W_1^*(1 - x_1)H_2^*(1)}{1 - H_2^*(1)}, \]
\[ Q_2(1, x_2)/Q_2(1, 1) = \frac{W_2^*(1 - x_2)H_2^*(1)}{1 - H_2^*(1)}. \]

Thus, for \( s < \lambda_1 \),

\[ W_1(s) = \frac{Q_1(x_1, 1)}{Q_1(1, 1)} = \frac{W_1^*(1 - x_1)H_2^*(1)}{1 - H_2^*(1)}, \]

and for \( s < \lambda_2 \),

\[ W_2(s) = \frac{Q_2(x_2, 1)}{Q_2(1, 1)} = \frac{W_2^*(1 - x_2)H_2^*(1)}{1 - H_2^*(1)}. \]
The mean waiting times for calls of class 1 and class 2 are given by

\[ W_1 = -1 \lim_{s \to 0} \frac{dW_1(s)}{ds} = \frac{q_1}{\lambda_1 - h_1}, \quad (6.5) \]

\[ W_2 = -1 \lim_{s \to 0} \frac{dW_2(s)}{ds} = \frac{q_2}{\lambda_2 - h_2}, \quad (6.6) \]

respectively.

7. THE MEAN BUSY PERIODS FOR CALLS OF CLASS 1 AND CLASS 2

Consider an epoch at which a setup (or changeover) for a call of class 1 is commenced, and let \( \xi_1 \) be the number of calls of class 1. The probability generating function of \( \xi_1 \) is given by \( G_1(x_1, 1)G_1(1, 1) \). After the lapse of the setup time, service for the call of class 1 is commenced. Let \( \bar{N} \) be the mean number of calls of class 1 at the end of the setup. Then, it is obtained that

\[ \bar{N} = -1 \lim \left( \frac{d}{dx_1} \frac{G_1(x_1, 1)U_1^N(x_1)}{G_1(1, 1)} \right). \quad (7.1) \]

Put \( x_1 = 1 \) in Eq. (3.11), then, by using Eq. (3.5), it is obtained that

\[ Q_1(x_1, 1) = (-G_2(1) - Q_1(0, 0) + p_2Q(0, 0) + G_1(x_1, 1)U_1^N(x_1)) \quad \frac{H_2^N(x_1)}{x_1 - H_1^N(x_1)}. \quad (7.2) \]

Since

\[ Q_1(1, 1) = -1 \lim \frac{d}{dx_1} \frac{G_1(x_1, 1)U_1^N(x_1)}{x_1 - 1} \quad (7.3) \]

is derived from Eq. (7.2), it is obtained that

\[ \bar{N} = (1 - \rho_2)Q_1(1, 1)G_2(1, 1). \quad (7.4) \]

Let \( \bar{T}_1 \) be the mean length of busy period for calls of class 1, then it is evident that

\[ \bar{T}_1 = \bar{N} \lambda_1, \quad (7.5) \]

where we put

\[ Y_1 = -1 \lim \frac{dY_1(s)}{ds} = h_1/(1 - \rho_1). \quad (7.6) \]

\( Y_1 \) is the mean busy period in a single server queue with Poisson input of density \( \lambda_1 \) and with mutually independent service times having a common distribution function \( H_1(t) \). Using Eqs. (7.4), (7.5), (7.6) and (4.8), \( \bar{T}_1 \) is calculated as

\[ \bar{T}_1 = \rho_1h_1/G_1(1, 1) = \rho_1/(1 + \lambda_2)G_1(1, 1). \quad (7.7) \]

Next, consider an epoch at which the system becomes empty. Then, the system remains idle until a new call arrives. Let \( \bar{T}_0 \) be the mean idle time, then, by the assumption of Poisson inputs with densities \( \lambda_1 \) and \( \lambda_2 \), it is obtained that

\[ \bar{T}_0 = 1/(\lambda_1 + \lambda_2). \quad (7.8) \]

Finally, consider an epoch at which a setup (or changeover) for a call of class 2 is commenced. After the lapse of the setup time, service for the call of class 2 is commenced. Let \( \bar{T}_2 \) be the mean length of busy period for calls of class 2, then, in statistical equilibrium, it must be hold that

\[ \bar{T}_1G_1(1, 1)/\rho_1 = \bar{T}_2G_2(1)/\rho_2 = \bar{T}_0Q(0, 0) \]

\[ + \bar{w}_2G_2(1, 1) = \bar{w}_1G_1(1, 1)/(1 - \rho_1 - \rho_2). \quad (7.9) \]

Thus, by using Eq. (7.7), it is obtained that

\[ \bar{T}_2 = \rho_2/(\lambda_1 + \lambda_2)G_2(1). \quad (7.10) \]

8. NUMERICAL EXAMPLES

In this section, some numerical illustrations are given. Fig. 1 and Fig. 2 show the mean waiting times \( \bar{w}_1 \) (broken lines) and \( \bar{w}_2 \) (solid lines) for different values of parameters. It is noted that \( \bar{w}_1 \) converges to some constant value as \( \rho_1 \) increases, though \( \bar{w}_2 \) diverges infinitely. Moreover, it is interesting to note that, for small values of \( \rho_1 \), at first \( \bar{w}_1 \) increases as \( \rho_2 \) increases, then begins to decreases gradually, and finally converges to a constant value. This property may be explained as follows:

Consider an epoch at which a call of class 1 arrives when there are only calls of class 2 in the system. If a setup for calls of class 2 is in progress, the arriving call must wait during the remaining setup time and a service time for a call of class 2. If a service for a call of class 2 is in progress, the arriving call must wait during the remaining time of the current service. Thus, the waiting time in the former case is greater than that in the latter case. The probability that the arriving call of class 1 encounters the former case decreases as \( \rho_2 \) increases and the probability that the arriving call of class 1 encounters the latter case increases as \( \rho_2 \) increases. It is evident that, for a small value of \( \rho_2 \), a large value of \( \rho_1 \), an arriving call of class 1 often encounters one of the above two cases. This is the explanation of the behavior of \( \bar{w}_1 \).

Fig. 3 and Fig. 4 show the probability that the system is empty.

Finally, we shall compare numerically our model with the alternating priority queueing model investigated by Miller. As is mentioned in the introduction, Miller has analyzed the latter model in detail and has given some examples for a symmetric queue. Here, the symmetric queue means that \( \lambda_1 = \lambda_2 = \lambda_1, H_1(t) = H_2(t) = H(t), \) and \( U_1(t) = U_2(t) = U(t) \). Miller's results for the symmetric queue are shown in Table 2, where a blank means saturation state. Our results for the symmetric queue with same values of parameters are shown in Table 2. It is observed that the alternating priority queueing system has more equilibrium states than the non-preemptive priority queueing system has, when changeover times and setup times are required. This fact coincides with our intuition.
Fig. 2 The mean waiting times $\bar{w}_1$ and $\bar{w}_2$. ($h_1 = 1, h_2 = 0.5, u_1 = 1, u_2 = 1$)

Fig. 3 $Q(0, 0)$: The probability that the system is empty ($h_1 = 1, h_2 = 0.5, u_1 = 1, u_2 = 1$)

Fig. 4 $Q(0, 0)$: The probability that the system is empty ($h_1 = 1, h_2 = 0.5, u_1 = 1, u_2 = 1$)

Table 1 $\bar{w}$ and $Q(0, 0)$ for the alternating priority queue with setup times ($h = h_1 = h_2 = 1, \lambda_1 = \lambda_2, u = u_1 = u_2$)
Table 2 \( w \) and \( Q(0, 0) \) for the non-preemptive priority queue with setup times

\[
(h = h_1 = h_2 = 1, \lambda = \lambda_1 = \lambda_2, u = u_1 = u_2)
\]

9. CONCLUSION

Analyses are made for the non-preemptive priority queueing system with setup times. The joint queue-length distribution is studied and its generating function is obtained. Laplace-Stieltjes transforms for the distribution function of waiting time are also obtained by applying the ordinary method. Expressions for the mean queue length and the mean waiting time are given for calls of each class. Finally, some numerical examples are given and the results are compared with those for the alternating priority queueing system with setup times.

REFERENCES