PROGRAM FOR OPTIMIZING NETWORKS WITH ALTERNATIVE ROUTING

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ABSTRACT.
The planning problems in a fast growing telephone system are difficult to solve when there are no methods to evaluate projects that include large number of interdependent variables. In order to alleviate this situation the administration has decided to mechanize the basic planning functions by means of a chain of programs that will permit the study of well defined parts of the system. One of these parts is the interconnection network between exchanges. The study of this problem is the object of this report which concerns only with the program for minimizing the cost of networks with the alternative routing facility.

The problem has been treated to a great extend in the literature, therefore we will describe only the considerations made in our application.

The program has the possibility of optimizing networks with full availability or gradings. The optimizing method is based on the derivatives of the congestion formula and the optimum is approach iteratively. The congestion formula employed in the full availability case is the Erlang B formula. For gradings we utilize the Erlang's Interconnection Formula because it gives approximate values in a great number of practical gradings as has been shown by Bretschneider and Elldin and because is easy to use in the computer. The optimization model takes into consideration only the mean value of the overflow traffic. The partial derivatives that are required in the process were obtained numerically by recursive algorithms. The program has been written in FORTRAN IV for the IBM 360-40 computer with the disc operating facility.

1. THE METHOD.

There are several methods for optimizing networks with overflow facilities but among the most simple and known there are two: the one parameter 3) 4) method which considers the mean value of the overflow traffic into the optimum relations and the two parameter method which takes into account not only the mean but also the variance of the overflow traffic for the optimum relations. The two parameter method 4) 5) is more accurate but the expressions are more complicated and time consuming when calculated numerically, therefore it is necessary to find approximations when the method is going to be included in an iterative process. On the other side, the improvement in accuracy for the values found in applications is small compared with the one parameter method. For these reasons we have decided to develop the program for the one parameter method. In a few words this method consist basically in defining a cost function which includes all the routes and switching utilized for carrying the traffic between two points. The cost function gives by derivation the condition for the minimum. This condition is obtained in an implicit form, therefore the minimum has to be found with an iterative procedure, which starts from a given arbitrary value and ends until the optimum reached. In Fig. 1 it is shown an example of the simplest case of optimization. The route between the exchanges A and B is high usage and the routes AC and CB are final. The cost function of the arrangement is

\[ C = C_1 N_1 + C_2 N_2 + C_3 N_3 \]  \hspace{1cm} (1)

where \( C_1, C_2, C_3 \) are the marginal costs of the trunks in each route and \( N_1, N_2, N_3 \), their number.

The minimum is obtained with the derivative of equation (1) respect to \( N_1 \).

\[ \frac{\partial C}{\partial N_1} = C_1 C_2 \frac{\partial N_2}{\partial N_1} + C_2 C_3 \frac{\partial N_3}{\partial N_1} = 0 \]  \hspace{1cm} (2)

When equation (2) is satisfied we have obtained the minimum. The partial derivatives are related by the loss functions of the routes.

When we applied this method to a network that has overflow routes common to several traffics, the optimum obtained in a point to point bases have to be recalculated because the original conditions in the overflow routes change for every new point to point calculation. For this reason we have to continue the process until...
the point to point calculations do not change any more the state of the overflow system. Expressed in other words we can say that the process has two iterative levels, one for the point to point routes and other for the whole overflow system.

The overflow cases considered in the process are shown in Fig. 2. This scheme permits the optimization of most of the routes of our system and gives also good approximations for -- more complicated routing cases because the optimum depends strongly on the first alternative route and gradually less on the second third etc. In these cases we make a separate check manually and usually we do not find any difference.

The equation (2) is solved in the process in terms of the following derivatives.

\[
\frac{\partial E(N,A)}{\partial N} = (3)
\]

\[
\frac{\partial A(N,E)}{\partial N} = (4)
\]

\[
\frac{\partial A^1}{\partial A} = (5)
\]

where:

- \( A \) = The offered traffic to the route.
- \( E \) = The congestion in the route.
- \( A^1 \) = The overflow traffic from the route.
- \( N \) = The number of trunks in the route.
- \( E(N,A) \) = The congestion as function of the traffic and the number of trunks.

\[ A(E,N) = \text{The offered traffic as a function of the congestion and the number of trunks.} \]

In the case of gradings we have considered that the availability \( K \) is given in the input data.

The expression employed for calculating the numerical values of the derivatives are:

**Full Availability**

\[
\frac{\partial E_1(A,N)}{\partial N} = \psi_{N+1} \cdot E_1(A,N)^1
\]

where:

\[
E_1(A,N) = \frac{\frac{A}{N}}{\sum_{j=0}^{N} \frac{A}{j!}}
\]

the numerical values of the equation (7) are obtained recursively with

\[
E_1(A,N) = \frac{A}{N} \cdot E_1(A,N-1)
\]

and the starting value is

\[
E_1(A,1) = \frac{A}{1!}
\]

The numerical values of the \( \psi_{N+1} \) function are obtained by means of the recursion

\[
\psi_{N+1} = \left\{ 1 - E_1(A,N) \right\} \cdot \left\{ \psi_{N+1} \right\}^1
\]

with the starting value

\[
\psi_1 = \int_0^A e^{-t} dt
\]

\[
\frac{\partial A(N,E)}{\partial N} = \frac{\psi_{N+1}}{\psi_{N+1} + \psi_{N+1} \cdot E_1(A,N)}
\]

1) formula given by Akimaru in reference 1

\[
\frac{\partial A^1}{\partial A} = (1+N)E_1(A,N) \cdot E_1(A,N) \cdot \left\{ E_1(A,N)-1 \right\}
\]

**Limited Availability;** In this case we have obtained the derivatives by numerical approximations and we have used the following expressions:

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\[ \frac{\partial E_2(A,N,K)}{\partial N} = \frac{1}{2} \left\{ E_2(A,N+1,K) - E_2(A,N-1,K) \right\} \]

where:

\[ E_2(A,N,K) = \frac{Z_{AK}^K \cdot T_K + \ldots + Z_{AN}^N \cdot T_N}{\sum_{i=0}^{r-1} A_i^i \cdot T_i + \sum_{r=r+1}^r A_r^r \cdot T_r} \]

and:

\[ T_r = \prod_{s=K}^{r-1} \left( 1 - \frac{S}{K} \right) \cdot \frac{Z_{s}^s}{\left( \frac{K}{N} \right)} \]

the equation (15) is solved recursively when \( A,N,K \) are given.

\[ \frac{\partial A(E,N,K)}{\partial N} = \frac{1}{2} \left\{ A(E,N+1,K) - A(E,N-1,K) \right\} \]

where \( A \) is obtained iteratively from the equation (15).

\[ \frac{\partial A^1}{\partial A} = \frac{1}{2} (A + \Delta A) \cdot E(A + \Delta A, N, K) - (A - \Delta A) \cdot E(A - \Delta A, N, K) \]

2. THE PROGRAM

The program has been designed in a form that permits to introduce new mathematical formulae for the calculation of the partial derivatives and new traffic cases. The traffics are given in a point to point form. The first part of the program collects the point traffic which have no direct route and forms a new matrix of fresh traffics. Then it is calculated the initial number of circuits. In order to save computer time we have considered that in the high usage routes the initial improvement factor - (the additional carried traffic per trunk) is in average 0.3. With this factor we start the process of point to point optimization of the high usage routes. When we come to the last high usage route we compare one by one the number of trunks with the initial value, if there are differences we start the point to point process again and we continue until the number of conductors is equal in the last two cycles. At this moment we stop the process and we make a print of the results.

2.1 THE INPUT DATA.

Because the size of the networks with alternative routing is large, we have placed the input information for the program in a disc file, this makes the whole process slower but is the only way to store the information in a commercial computer.

The disc files can contain the information of 35,000 routes, each route requires the following data:

Number of the route.
Name of originating exchange.
Name of the terminating exchange.
Type of switching (full availability or grading)
Availability of the route.
Loss value if the route is final.
Marginal cost per trunk (This cost might include the associated switching equipment)
Overflow facilities for the first alternative route.
Overflow facilities for the second alternative route.
Overflow facilities for the third alternative route.

2.2 THE OUTPUT DATA.

When the process stops and we order the print-out, we obtain the following information per route.
Bottom traffic (first choice traffic)
Congestion
Number of trunks
Cost of the route

In this print we also include all the input data of the route and the great totals of cost, the number of trunks, the traffic, the number of routes and the last values of the derivatives obtained in the process.

REFERENCES.