SOME COMPARISONS OF LOAD AND LOSS DATA WITH CURRENT TRAFFIC THEORY

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ABSTRACT

The ultimate test of theory is comparison with reality. The procurement of reliable and reproducible traffic data is relatively difficult since little control can be exercised over the character and level of the input to the load carrying system without invalidating the genuineness of the input. Consequently there is a tendency toward a proliferation of theories with neglect of examining their suitability to describe systems situations of great interest. A modest contribution is offered toward relieving some of the latter shortcomings.

Data are presented and compared with theory in 3 trunking areas: High Usage Groups; Full (nonalternate routed) Groups; and Final Groups.

High Usage Groups

Offered loads versus losses for single hours for several high usage group sizes are compared with Erlang's Loss Theory. Both interlocal and intertoll groups are examined. The data are generally found to agree well with theory.

Use of carried loads to estimate hourly offered loads via Erlang's Loss Theory, however, introduces a bias, and compensating corrections are required. Their magnitudes are indicated.

When busy season carried loads are averaged to reduce the processing effort and to overcome the hourly estimated load bias in Erlang's Loss Theory, the day-to-day load variations introduce other errors of estimation. Their extent and correction (in American practice) are illustrated. The corresponding correction required in Erlang's Loss Formula when estimating mean busy season busy hour overflow load is also determined. Comparison with observed values are made in each instance.

Full Groups

Data are presented showing single hour losses versus offered loads. Erlang's Loss Theory is found to agree well with single hour observations. However when day-to-day effects are included, the average busy season busy hour losses at usual engineering levels agree more satisfactorily with Poisson Theory.

Final Groups

The observed effects of traffic peakedness and day-to-day busy hour variations on final groups are compared with corresponding theory.

1. HIGH USAGE GROUP STUDIES

1.1 Loads and Losses on High Usage Groups

It is commonly assumed that the requirements of a "random" or Poisson input are met when originating traffic is first offered to a restrictive group of paths or switches. It would be difficult and certainly impractical in most operational situations to check such an assumption by examining the call arrival instants or to analyze their interarrival times, as well as make a corresponding study of their service times. Since in any event exact conformity with the theoretical assumptions could not be found, the question would remain as to the relative adequacy with which they were met.

The traffic engineer's usual wish is to describe the blocking which will occur in real life situations when a given average load is offered to a group of paths or switches. It will generally suffice then to compare observed load vs loss relationships with those theoretically derived, rather than attempt an assessment of the agreement of more basic requirements.

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We are directed to the conclusion that although Erlang's loss theory is properly applied only to a system operating in statical equilibrium over a long period of time, for engineering purposes it is suitable for describing conversation type traffic load vs loss results summarized by periods as short as one hour (summary period > 20 average holding time).

Figure 5a shows the load offered, as estimated by equation (1), to three 1-trunk groups during the 3 busiest hours of the day for 20 business days. The load carried (= occupancy here), as determined by switch counting, is plotted as the ordinate. The Erlang relation is drawn as the solid line.

1.2 Estimation of Hourly Offered Loads from Switch Counts

It is customary to furnish only a switch counting device on high usage trunk groups, and to depend on the accuracy of Erlang's loss relation, $E_x(a)$, to estimate the corresponding load, that is solving for $a$ in

$$\delta = a[1 - E_x(a)]$$

where $x$ is the number of trunks in the group. Descloux [1] has shown that a certain amount of bias occurs by this method. The reason for this is made clear by examining Figure 5.

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The points comprise a correlation scatter diagram. The classical regression line of "y on x", that is the expected value of carried load for a selected average offer, would be expected to agree with the Erlang loss relation, since the latter is derived on the basis of offered load as the "independent variable." The "y on x" regression line generated from the data is indeed seen on Figure 5b (dashed broken line) to conform nicely with the Erlang theory.

Unless there were perfect correlation, the "x on y" regression line could not be expected to agree with Erlang's theory, and this is corroborated by the observed regression (solid broken) line on Figure 5b. At lower than average occupancies (and offered loads) this regression line lies slightly below the Erlang theory, and for larger than average occupancies, it lies consistently above. Thus when estimating offered loads from carried loads by Erlang's theory (equation 2) we should expect the values to be smaller than true for lower than average occupancies, and greater than true for larger than average occupancies. For example for one trunk at an observed load carried (occupancy) of 0.75 erlang, Erlang theory would predict an offered load of 3.6 erlangs, while the observed regression line would more nearly suggest an offer of 2.64 erlangs, a difference of 14 percent.

An appropriate theoretical expression for the "x on y" regression in this circumstance is difficult to generate, depending partially as it does on the day-to-day distribution of the offered loads, and this last becomes a matter of observation of customer characteristics. Since the regression lines will be decidedly nonlinear, the theoretical approaches of simple correlation theory are inadequate. In the face of such difficulties, one reverts to the reduction of data taken in the field for practical estimating results.

Figure 6 shows the ratio of two estimates of the offered load $a_2/a_1$, versus observed occupancy of several l-trunk groups. Here $a_1$ is the unbiased estimate from equation (1), and $a_2$ is from equation (2), the latter making use only of the carried load measurement. In spite of the rather wide differences in loadings among the 5 groups shown, the $a_2/a_1$ ratios are quite similar and one would not have too much difficulty in drawing a central ratio line through the field for general correction of $a_2$ values.

![Fig. 6 - Ratio of estimates of single hour loads offered to 1 trunk. Estimate $a_1$ is from switch and call counts; estimate $a_2$ is from switch count and Erlang's loss formula.](image)

Figure 7 shows a similar chart for 3 trunks, indicating the rapidly decreasing ratios with increasing trunk group size, and the tendency for the ratios to approach 1.0 at the lower occupancies. Figure 8 shows a rough summary of the occupancies at which the ratios would exceed 1.05 with various sized trunk groups. Thus one might conclude, for example, that estimates by Erlang's loss formula of hourly offered loads on 12-trunk groups would not be acceptable without correction (that is they would overestimate by more than 5%) for occupancies greater than 0.75.

1.3 Estimation of Average Overflow Load from Average Offered Load During a Busy Season

When an offered load $a$ varies from hour to hour in excess of the amount expected in 1-hour segments of a longtime load in statistical equilibrium [2], the average overflow $o$ over a period of time will exceed that estimated by entering the Erlang loss relation with $a$, that is $o > a^2 E_{a}(x)$. This result is caused by the concave upwards shape of the offered load, overflow load curves. The amount of such excess will depend strongly on the magnitude and character of the offered loads.

Numerous studies have shown that day-to-day busy hour variations in the busy season tend to follow a Type III Pearson distribution,

$$\theta(a) = \frac{a^b}{\Gamma(a)}$$

Typical is the example shown in Figure 9 for a 9-trunk interlocal group.

Again there is a considerable correlation between the day-to-day variance and the mean of such a distribution. Figure 10 shows the field of variances vs means of loads offered during 3 hours each day for 20 days on 72 high usage interlocal groups at Kildare office. Summaries at other exchanges in the United States confirm the general association of variances and means. The general line of regression of variance on mean for the corresponding scatter diagrams is shown by the solid line, whose equation is approximately,

$$\text{Var}(a) \approx 0.31 \, a$$

(A dashed line has also been drawn freely thru the major axis of the elliptical pattern of points. Its equation is

$$\text{Var}(a) \approx 0.13 \, a^{1.56}$$

It will be referred to in a later section.)

If the Erlang loss formula can be used to estimate the proportion of calls which overflow a high usage group,
1.09 1.08

the regression relation of Figure Curves is determined from during a single hour, the average load overflowing, $\bar{a}$, over a series of hours is then calculable from

$$\bar{a} = \frac{\sum a_i}{n}$$

Similarly the day-to-day variance of the overflow loads is determined from

$$\text{Var}(\bar{a}) + \sigma_a^2 = \int_{0}^{\infty} [E_{1+x}(a)]^2 \sigma(a) \, da$$

Curves have been calculated by numerical integration using the regression relation of Figure 10, which give the ratio of $\bar{a}$ to $\sigma_a$, the latter being the overflow corresponding to an offer of $a$, found according to

$$a = E_{1+x}(a)$$

Values of $\bar{a}/\sigma_a$ are given on Figure 11. It is found that at constant loads on the last trunk (shown in 100 call seconds, or CCS) the ratios $\bar{a}/\sigma_a$ are relatively constant; hence a simple table, Table I, using the last trunk load as the index is adequate for many working purposes.

Fig. 9 - Variations in day-to-day busy hour loads, Kildare group No. 67 - 9 trunks, 20 days.

Fig. 10 - Day-to-day variance vs average load in clock hours, 72 interlocal high usage groups, 3 hours, Kildare 1998.

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An example of the need for correcting by the factors of Table I is shown in Figure 12. In the left diagram uncorrected $a_i$ values are plotted against average overflow loads, $a_i$, calculated by the unbiased procedure

$$a_i = \frac{1}{r} \sum_{i=1}^{r} a_i$$

where $r$ is the number of hours summarised for each group.

After correction the overflow estimates are shown in the right diagram to be in much better agreement with the $\bar{a}$ values.

Fig. 11 - Correction required in $a_i$ to estimate $\bar{a}$

Table I - Corrections in $a_i$ to estimate $\bar{a}$

<table>
<thead>
<tr>
<th>Load on the Last Trunk</th>
<th>Range of Corrections</th>
<th>Corrections to $a_i$ for Practical Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCS Erlangs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.22</td>
<td>1.32 - 1.37</td>
</tr>
<tr>
<td>10</td>
<td>0.28</td>
<td>1.22 - 1.28</td>
</tr>
<tr>
<td>12</td>
<td>0.33</td>
<td>1.16 - 1.21</td>
</tr>
<tr>
<td>14</td>
<td>0.39</td>
<td>1.11 - 1.16</td>
</tr>
<tr>
<td>18</td>
<td>0.50</td>
<td>1.05 - 1.09</td>
</tr>
</tbody>
</table>

1.4 Estimation of Average Offered Loads from Average Loads Carried

To obviate the labor of calculating individual hourly estimates of the offered loads from observed hourly carried loads on high usage groups (which would in turn require a correction as discussed in Section 1.2), the carried loads $\bar{f}$ are commonly averaged first; and this average $\bar{f}$ is then entered in Erlang Loss Theory curves or tables to obtain $\bar{a}$ from

$$\bar{a} = \frac{1}{\bar{f}} \int E_{1+x}(\bar{f}) \, d\bar{f}$$

This is done as follows. Choosing a value of $\bar{a}$ as the offer to $x$ trunks, $\bar{a}$ is estimated from the corresponding overflow value $a_i$ corrected by the appropriate factor from Table I. One then obtains $\bar{f}$ from Figure 11. One then obtains $\bar{f}$ from $\bar{a}$, using appropriate factors.
\[ I = \alpha - \bar{a} \] (11)

since that load which does not overflow must naturally be carried. Relation (10) then gives \( \alpha \). A field of values of \( \alpha/\bar{a} \) have been calculated and form the curves of \( I \) Figure 13. We see that the ratio is sensitive both to proportion of overflow and to the number of trunks.

Corresponding data for Kildare groups are shown on Figure 14. Similar forms of probability density distributions are observed.

It will be noted that there is a generally maximum ratio on Figure 13 which tends to occur roughly at \( E_1(a) = 0.2 \), that is at an expected overflow of 20 percent. It is interesting that this maximum correction required on \( I \) occurs squarely in the middle of the most common economic high usage group operating levels.

For practical use we have constructed the traces of the 3-dimensional surface of Figure 13, which correspond to several values of "economic CCS on the last trunk." The theoretical \( \alpha/\bar{a} \) ratios for 8, 14, 20 and 25 CCS on the last trunk are shown on Figure 15. Comparison with data taken on a number of intertoll high usage groups at Memphis, Tennessee is shown in Figure 16. Although the dispersion among individual groups is considerable, the grouped average values show reasonable agreement with theory. Table II shows the values of corrections to \( I \) required to best approximate the true offer \( a \) for the four selected last trunk CCS.

Table II - Average Corrections to be Applied to \( \alpha \) to Obtain Improved Estimates of \( I \)

<table>
<thead>
<tr>
<th>No. Trunks</th>
<th>Correction Factors for y CCS on the Last Trunk</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = 8</td>
<td>1.01 1.02 1.02 1.02</td>
</tr>
<tr>
<td>y = 14</td>
<td>1.02 1.03 1.03 1.03</td>
</tr>
<tr>
<td>y = 20</td>
<td>1.03 1.04 1.04 1.04</td>
</tr>
<tr>
<td>y = 25</td>
<td>1.04 1.05 1.05 1.05</td>
</tr>
<tr>
<td>25 and up</td>
<td>1.05 1.06 1.06 1.06</td>
</tr>
</tbody>
</table>

2. GROUPS WITHOUT ALTERNATE ROUTES

There has long been interest in the different theoretical formulas used by American and European administrations for engineering interlocal trunk groups. The former rely on the cumulative Poisson while the latter have favored Erlang's loss formula. What do data taken on such groups indicate?

In 1959 data were taken on some 30 direct trunk groups terminating in the Arlington, Massachusetts No. 5 crossbar office. Hourly observations comprised numbers of calls offered to each group, the number blocked, and by switch counting, the average load carried. Illustrative of these is group No. 26 with 32 trunks observed for 4 hours a day for 29 business days. Offered loads were estimated for each hour by equation (1). The load versus blocking relationship observed for 116 hours is shown on Figure 17. Superimposed is the Erlang loss relation. The agreement is seen to be very good.

On Figure 18 are shown only the 29 10-11 AM busy hour loads of Figure 17. The agreement with Erlang's theory is again understandably excellent. Other group data comparisons were nearly as satisfactory. We are led to conclude that Erlang's loss formula describes quite well the hourly blocking for conversation traffic on direct groups which do not have alternate routes. Apparently
the return of blocked calls here was of a nature that caused little disturbance of the Poisson character of the offer - or perhaps they contributed to it.

In the lower section of Figure 18 is shown a distribution of the busy hour loads offered to group No. 26 as they varied from day to day over the 6 weeks busy season. For the average offer of $a = 21.37$ erlangs, a variance $\text{Var}(a) = 0.197$ was observed. Since the probability of loss curve in this range is concave upwards, the average load for all the hours tends to exceed the loss at the average load. Thus the theoretical single hour loss at 21.37 erlangs offered is 0.0074. However the observed average loss was found to be 0.0197 when hourly losses were given equal weight (as in American practice). (If the losses had been weighted by the corresponding offered loads, the average loss would be still greater at 0.0267.) Clearly if Erlang's formula describes well the losses for individual hours, it will not usually give an adequate estimate of the average loss over a series of busy hours. However when the more conservative Poisson summation is laid on Figure 18, it is seen to pass directly through the observed points, indicating that at least for the amount of day-to-day variation present in this sample of hours, it provides just the right amount of "improvement" to Erlang's loss formula.

There is then a particular variance of day-to-day busy hour loads which when applied to Erlang's load formula will just produce the average loss of the Poisson formula. Figure 19 shows a field of curves indicating the $\text{Var}(a)$ required for average loads, $a$, such that at the loss levels given, the Poisson summation will closely relate the average busy season busy hour loss to the average offer. On the same field are shown the actual variances versus average offered for the trunk groups in the Arlington study mentioned above. It is seen that the variances found in practice are such that at commonly used interlocal average busy season busy hour grades of service, 0.005 to 0.03, the Poisson summation provides an excellent means of specifying group average capacity. It may be noted that in American practice, the objective grades of service are set to be met by the average (unweighted) blocking in the busy season busy hours.

The dashed line on Figure 19 is a centrally located curve which describes the general relation seen between day-to-day variance and average load offered. It is interesting - and reasonable - that its equation is $\text{Var}(a) = 0.13$ erlangs where $n = 1.58$, identical with that of the central axis line drawn through the high usage group observations on Figure 10. It may be noted that the dotted line on Figure 19 results from choosing $n = 1.50$; it corresponds nicely with Poisson blocking of 0.01 for loads of 10 to 100 erlangs.

3. FINAL GROUPS

Much has been written in the past on estimation of the effect of the nonrandomness of overflow traffic from high usage groups upon the trunking requirements in final groups. Studies have also been made (as in the previous section) showing the added trunks required to accommodate the increased demand on groups where offered loads show significant day-to-day variations. The two effects will usually appear simultaneously in the engineering of final groups. Moreover, loads offered to final groups may be expected to show generally larger day-to-day busy hour variations than do loads to high usage and direct groups not having alternate routes. Figure 20 shows a field of variance versus average loads offered to 28 interlocal final groups in which negligible first-routed traffic was present. A central fitting line, having the equation

$$\log_{10} \text{Var}(a) = 1.8687 \log_{10} a - 0.8861,$$  \hspace{1cm} (12)

has been drawn through the points, or equivalently,

$$\text{Var}(a) = 0.13 a^{-1.84}$$.  \hspace{1cm} (13)
When this variation is assumed, the carrying capacities of 6 to 15 trunks at an 0.02 average blocking are shown illustratively on Figures 21. For 10 trunks, for example, the Erlang capacity for random traffic is reduced from 5.08 to 4.47 erlangs, a drop of 12% resulting from the day-to-day variations.

Thus the Erlang capacity for random traffic is reduced from 5.08 to 4.47 erlangs, a drop of 12% resulting from the day-to-day variations.

Similarly, if day-to-day variations are not introduced, the capacity of 10 trunks is reduced from 5.08 to 3.96 erlangs, a reduction of 28%.

When day-to-day variations and nonrandomness are jointly introduced, the capacity of a group is further reduced; thus the 10 trunks will now accommodate an offer of only 3.57 erlangs, a reduction of 30% from the basic Erlang loss formula value. It is clear that each cause can produce substantial reductions in trunking capacity.

A dramatic illustration of the importance of using theory which considers both day-to-day variations and the peakedness of the traffic is given in Figure 22. The results are shown of observations on a Kildare final group of 59 trunks, with 62 subtending high usage groups plus 4 first-routed traffic items. The estimated peakedness factor is 1.93, and the load-loss characteristics observed for the 20 days are:

<table>
<thead>
<tr>
<th>Hour of Day</th>
<th>Average Offer (erlangs)</th>
<th>Day-to-Day Variance of Offer</th>
<th>Average Blocking</th>
</tr>
</thead>
<tbody>
<tr>
<td>9-10 AM</td>
<td>26.5</td>
<td>26.5</td>
<td>0.001197</td>
</tr>
<tr>
<td>10-11 AM</td>
<td>180.3</td>
<td>180.3</td>
<td>0.04570</td>
</tr>
</tbody>
</table>

As seen on the figure, the simple Erlang loss values are an order of magnitude below the observed losses. Although allowance for nonrandomness makes a marked improvement, it is still far from describing the actual losses. Nor is the Poisson theory which might be expected to compensate partially for day-to-day variations in the 0.01 to 0.03 loss range, a nearly sufficient improvement. But employing the typical day-to-day variation magnitudes of Figure 20 in conjunction with the estimated peakedness of the offer, the solid line load-loss curves for 58 and 60 trunks exactly bracket the average loss values seen for 20 days for the 9-10 AM and the 10-11 AM hours on the 59 trunk final group.

SUMMARY

Examples have been given comparing observation, simulation and theory in various areas of traffic flow on trunks comprising direct and alternate routed plans. Particular attention has been drawn to the need for developing adequate relationships between offered, carried and overflow loads, both single hour and average busy season, suitable to each operating condition. Where possible, a physical understanding of the principal factors is followed by mathematical theory which may require approximate numerical calculations. Further insight may be gained from controlled simulations. All derived relationships to be most useful, must finally be found to agree with the flow of traffic in real life situations.

REFERENCES
