CONSIDERATIONS ON POSSIBLE SAMPLES TO BE OBTAINED FROM AUTOMATIC TRAFFIC FLOW MEASURING EQUIPMENT

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ABSTRACT
A device is considered which makes continuous traffic flow measurements, i.e. it records the arrival of calls (seizures) and the end of successful calls. The processes of the seizures and the ends of successful calls are assumed to have a Poisson distribution and to be independent of each other. A quantity is determined which indicates the percentage of the seizures which can, on the average, be recorded by the device during the busy hour, if the device requires a constant time for recording a seizure. Furthermore, the mean probability is calculated for the case that in the service time required for one seizure the number of successful calls ending is greater than the number of stores in which ends of successful calls can wait to be served. Both quantities are discussed as a function of their parameters.

1. INTRODUCTION
The telephone traffic is subject to strong statistical fluctuations. For this reason, the telephone installations have to be monitored in order to find out in how far the traffic data used at the planning stage are still applicable and the required grade of service can still be observed when complying with requests for the setting up of calls. In the case of some problems (e.g. a determination of the mean holding time or the mean duration of a successful call) it is useful to record the traffic flow and to have it analyzed by computers. However,

a) in general it is not possible to connect all circuits of a group to such a device, and
b) such a device cannot record each change of traffic on the group - i.e. seizure or end of successful call,

because
a) the device has only a limited number of inputs, and
b) a change in traffic cannot be recorded in an infinitely short period of time.

Now we shall consider the problem which percentage of the change in the occupation of a group of circuits can be recorded by a device.

2. THEORETICAL MODEL AND ANALYTICAL SOLUTION
2.1 Let us assume that the device has $N$ inputs, i.e. a maximum number of $N$ circuits of a group can be connected to the device.

To begin with, we shall consider a device which records only seizures. When any of the circuits is seized, this seizure is recorded by the device, if it is free. That means, during a certain period of time the device is occupied. If another circuit is seized during this time, this seizure is lost as far as the recording is concerned. Hence, the device is unable to record several seizures at the same time, i.e. it is a single server system. On the assumption that the seizures on each circuit of a group form a Poisson stream, the summation stream originating on all circuits of the group is a Poisson stream. The input applied to the device, i.e. the summation stream obtained from the $N$ circuits connected, is a Poisson stream. Hence, the preference the incoming traffic has for certain circuits (always starting from a fixed point) and the distribution of the traffic to $N$ circuits are only a special sorting of the individual seizures, the total of which yields the summation stream. As far as the device is concerned, it does not matter whether the input stream originates from a large number of circuits with a low density of seizures or a small number of circuits with a high density of seizures. Consequently, the input stream is
independent of the number $N$ of the circuits connected.

The service time of the device be $\alpha$ and, for simplicity, $\alpha$ be constant. However, $\alpha$ may also be interpreted as the mean time required to set up a call (i.e., time required to dial the digits + time until the wanted subscriber answers).

The device filters the input stream so that there are two partial streams:

![Diagram](image)

a) output 1, the stream of non-recorded seizures
b) output 2, a modified Poisson stream, where the intervals between the seizures are longer than or equal to $\alpha$.

Now we are going to determine the distribution of output 2. Let $X_n$ be the input Poisson stream with the parameter $\lambda$ and $\{\tau_n\}$ the sequence of instants where a call arrives and the device is free.

The differences in time $\tau_n - \tau_{n-1}$ ($n = 1, 2, 3, \ldots$; $\tau_0 = 0$) are independent random variables. The distribution of $\tau_n$ is given by

$$P(\tau_n = x) = \begin{cases} 1 - e^{-\lambda x} & \text{for } x \geq 0, \\ 0 & \text{for } x < 0. \end{cases}$$

The differences in time $\tau_n - \tau_{n-1}$ ($n = 2, 3, \ldots$) are identically distributed random variables with the distribution function $F(x) = 1 - e^{-\lambda x}$ for $x = \alpha$.

For Re(s) > 0, where Re(s) is the real part of s, the Laplace-Stieltjes transform $\Phi(s)$ of $P(x)$ is

$$\Phi(s) = \int e^{-sx} dF(x) = \lambda e^{\alpha} \int e^{-x(s+1)} dx = \frac{\lambda e^{-\alpha}}{s + \alpha}.$$

When $t_n$ stands for the stream of output 2, then

$$P\left(t_n = n\right) = P(t < \tau_{n+1}) = 1 - P(\tau_{n+1} \leq t) = 1 - F(t_{n+1}(t)), $$

where $F_{n+1}(t)$, $n = 0, 1, 2, \ldots$, indicates the $(n+1)$th iterated convolution of the distribution function $F$ with itself.

Then the Laplace-Stieltjes transform of $P\left(t_n = n\right)$ for Re(s) > 0 is

$$e^{-st} dF\left(t_n = n\right) = 1 - (\Phi(s))^{n+1}.$$ 

The following is finally obtained by inverse transformation

$$P\left(t_n = n\right) = \begin{cases} \sum_{j=0}^{n-1} \frac{\lambda}{j!} e^{-\lambda(t-\alpha)} & \text{for } n\alpha = t, \\ 1 & \text{for } n\alpha > t. \end{cases}$$

Furthermore, let $m(t) = E\left[\frac{t}{t}\right]$ be the expectation of the number of events of the stream of output 2, which take place during the interval $(0, t]$.

We obtain

$$m(t) = \sum_{k=0}^{t-1} \left(1 - \sum_{j=0}^{k-1} \frac{\lambda^j}{j!} e^{-\lambda(t-k\alpha)} \right),$$

where $[r]$ is an integer so that $[r] = r - [r] + 1$.

$M(t) = E[X_n] = \lambda t$ be the expectation of the number of the events of the input stream, which occur during the interval $(0, t]$. The quantity

$$p = 100 \cdot \frac{m(t)}{M(t)} = \begin{cases} \frac{t/\alpha}{\alpha} \sum_{k=0}^{t-1} \left(1 - \sum_{j=0}^{k-1} \frac{\lambda^j}{j!} e^{-\lambda(t-k\alpha)} \right) & \text{indicates the percentage of seizures of the input stream, which is, on the average, recorded by a device with the service time during an interval } (0, t]. \end{cases}$$

2.2 Now let us assume that the device can also record the ends of these successful calls, the seizures of which were already recorded by it, i.e., those which originate from output 2. The service time for one end of a successful call shall be very short with respect to the service time $\alpha$ for a seizure. The ends of successful calls may form a Poisson stream with the parameter $\alpha$. The processes of the seizures and those of the ends of successful calls are thus assumed to be independent of each other. Moreover, $z$ stores be available in which the ends of successful calls can wait which arrived while the device was recording. Then

$$P = \frac{m(t)}{M(t)} \sum_{k=0}^{t-1} \left(1 - \sum_{j=0}^{k-1} \frac{\lambda^j}{j!} e^{-\lambda(t-k\alpha)} \right)$$

is the mean probability that more than $z$ successful calls end in the service time $\alpha$ required for one seizure and that the device is thus blocked.

3. EVALUATION

Let us first consider the quantity $p = p(\lambda, \alpha, t)$ according to (1). A busy hour is assumed to be the interval $(0, t]$, i.e., $t = 3600$ s. All other times in formulae (1) and (2) are also expressed in seconds (s).

There applies $\lambda = A/t_{mb}$, where $A$ is the total offer on the N circuits connected to the device. As mentioned above, we assume that the measurements is performed during the busy hour, i.e., that there is a statistical equilibrium, $t_{mb}$ is the mean holding time. Thus we obtain the functional dependence $p = p(A, t_{mb}, \alpha)$.

As stated in sub-section 2.2, the device records the ends of successful calls in a period of time which is very short with respect to $\alpha$. Thus, it is possible to ignore the percentage of seizures not recorded by the device, because it is just recording ends of calls.

Hence, the difference 100-p indicates the percentage of seizures of the input process which is, on the average, not recorded by a device with the service time during the busy hour, i.e., the average percentage of calls which bypasses the device.
Figs. 1, 2, 3 show the quantity $100-p$ as a function of $\alpha$, $t_{mb}$ and $A$, respectively.

Here the higher $\alpha$-values can occur when the device precedes the register, and the lower $\alpha$-values are obtained when the device follows the register. Some values were taken for the the smaller of which correspond to the average holding times in the local telephone service, while the higher ones correspond to the average holding times in the subscriber trunk dialling service. As for $A$, some traffic data were selected which are offered to all the circuits connected to the device. (Hence, $A$ is not the traffic offered to the complete group, but only that offered to the circuits connected.)

Now, we turn to formula (2). Let us assume that the number of stores ranges from 1 to 7. As mentioned above, the following applies to the quantity $\mu = A/t_{mg}$.

Here, $A$ means again the offer to the $N$ circuits connected to the device and $t_{mg}$ is the average duration of a successful call expressed in seconds. It should be noted that in the case of a successful call the holding time is longer than the duration of the call, but with $n$ seizures there are generally less than $n$ successful calls. The mean holding time is therefore shorter than the mean duration of the call. In our model $\mu$ is thus smaller than 1, i.e. the number of seizures offered to the device for recording exceeds the number of the ends of successful calls. From that it results that there are calls which do not mature, a fact which is consistent with practice.
Fig. 5 Blocking probability $P$ for a service time $\alpha = 20$ s, $t_{mb} = 104$ s, $t_{mg} = 143$ s (local telephone service) and an offer $A = 9$ erl, 12 erl, 15 erl, 18 erl, 21 erl as a function of the number of stores $z$.

Fig. 6 Blocking probability $P$ for an offer $A = 9$ erl, a number of stores $z = 4, 5$ and $t_{mb} = 104$ s, $t_{mg} = 143$ s (local telephone service) $t_{mb} = 113$ s, $t_{mg} = 174$ s (S.T.D.) as a function of a service time $\alpha [s]$.

The values taken for the average duration of the call $t_{mg}$ correspond with the chosen average holding times $t_{mb}$. The interrelations between $t_{mb}$ and $t_{mg}$ cannot be expressed as a general function. Measurements showed that for the same $t_{mb}$ the relevant $t_{mg}$ varies considerably from exchange to exchange. Therefore the quantity $P$, the blocking probability of the device according to (2), will not be considered as a function of $t_{mb}$ and $t_{mg}$. Instead of it we consider $P$ - shown in Figures 4, 5 and 6, respectively - as a function of the number of stores $z$ in which ends of successful calls can wait to be served, of the offer $A$ and of the service time $\alpha$, respectively.

4. CONCLUSIONS

4.1 Formulae (1) and (2) can be used for the efficient dimensioning of a device as described above which has one recording instrument (server), $N$ inputs and $z$ stores in which ends of successful calls can wait to be served.

As outlined at the beginning our model device is independent of the number $z$ of circuits connected. However, the number $N$ has to be borne in mind when varying the offer $A$. Our examples apply to a device with 24 inputs, and the offer $A$ ranges therefore from 9 erl and 21 erl. In this case an adequate grade of service (which regard to blocking) can be expected with 4 or 5 stores for waiting ends of successful calls.
4.2 The percentage of seizures recorded by the device depends upon

a) the hunting process by means of which the circuits of a group are sought,

b) the choice of the circuits connected to the device,

c) the quantities \( A \), \( t_{mb} \) and \( \alpha \) as included in formula (1).

The effect of a) and b) was not investigated here. From the discussion of (1) it has become evident that the data obtained by means of the device described above do not allow a conclusion to be drawn on the flow of the daily traffic, since there is no constant relation between the seizures recorded by the device and those occurring in the group of circuits.

REFERENCE [1]