DISCUSSION RECORD
Session No. 32 - QUEUEING SYSTEMS I

PAPER No. 321
Author: J W COHEN

Question by V E BENES
Your references to Brownian motion and to the lack of
mathematical tidiness in queueing remind me that great
simplification and clarity have resulted recently in dif-
fusion and control theory from the description of invol-
volved stochastic processes by their Radon-Nikodym
derivatives with respect to simpler, well-understood
processes like the Brownian motion. Now, it is a
"folk-theorem" that the Poisson process is to traffic
theorists what Brownian motion is to physicists. So
similar methods should work in traffic theory. This is
indeed so: at Berkeley, Prof. Eugene Wong and his
students are applying the martingale results of Cathérine
Doleans-Dade to queuing and control problems. I call
the delegates' attention to these developments, and pre-
dict that they will tidy up large parts of the theory, and
provide ways of solving control, estimation, and hypo-
thesis-testing problems.

Answer
Unfortunately I do not know the approach to queuing
problems of Wong. I am grateful to Dr. Beneš for
taking my attention to this method; it looks indeed very
promising.

PAPER No. 322
Author: W KRÄMER

Question by C GRANDJEAN
First of all I would like to compliment Prof. Cohen for
his excellent paper and emphasize the fact that it con-
tains an extensive and very useful bibliography. Such a
list of course cannot be complete and I didn't find any
reference to a paper which I believe is worth mentioning:
that is R. Syski's paper given at the Lisbon NATO con-
ference a few years ago and devoted to an excellent pre-
sentation of Pollaczek's work. Do you agree?

Answer
Yes, I do. Note that I referred to the proceedings of the
Lisbon meeting; but could, unfortunately, not enter into
more details.

Question by C GRANDJEAN
Besides telephone switching or message switching there
are other areas in which queueing theory are applied,
such as road traffic, air traffic, .... Some kind of
cooperations would certainly benefit to everybody.
Could you comment about that?

Answer
People active in the fields you mention know about the
results of teletraffic theory. There is indeed some co-
operation. However, this needs time, since for instance
in road traffic theory the experience with theoretical
models is still very limited.
as a subsystem or part of them.

Furthermore I suppose (but this has to be proved) that e.g., the correlation of two single stage waiting times of a call is a maximum for 2 subsequent stages.

Nevertheless further investigations for systems with an arbitrary number of stages are just under work, which yield - though regarding these dependencies - approximate results and have to be checked with simulations.

The last part of my answer refers also to a remark given in my paper, which states the extension of the results of chapters 2 and 3 to a system with several parallel queuing systems in the second stage.

The general system (fig. 1) may represent a simple data switching centre with e.g., preprocessing and selection of a certain outgoing server or trunk group with certain probability.

The authors of the preceding paper have shown that of a model with infinite high input rate is very important. Do the authors have any information about the overflow probability for the present model with finite input rate, for finite as well as infinite number of sources.

For the design of the buffer capacity comparison of the overflow probability for the present model with that for the instantaneous arrival model, we have no information. But, when the message length is short or the transmission rates are high, their difference may be small.

Question by C. Grandjean

In data networks, the same message has sometimes to be transmitted towards several terminals (multiple address). Such a message is then stored in the memory until the last transmission is completed.

a) Is it the case in the applications you have considered?

b) Do you think multiple address has a significant impact on the calculation of the memory size?

c) Do you have recommendations on how to take it into account?

Answer

a) Your paper puts stress on the bursty nature, or step by step nature of message arriving process. Then, in our paper, it is assumed that each arriving message has only one address. For the multiple address messages, some modifications are required.

b) The design method of the buffer capacity depends on how to handle multiple address messages. For instance, in the case that the original message is stored in the buffer of the originating office and its copies are transmitted one by one to the addressed destinations, the buffer holding time of the multaddress message becomes larger. Therefore, this greatly affects the calculation of the memory size, when multiple address messages are many.

c) In the model I, we can take into account the effect of multiple address messages by considering their waiting times. That is, in the model I must be modified for multiple address messages and the traffic offered over the output lines must be increased because of the copies of multiple address messages.

Question by L. Kosten

The authors' Model II is nearly the same as the one in a paper I am going to present at the Tel Aviv TIMS Meeting. The only difference is that I took the number of input lines to be infinite as well as the buffer size. The authors' method needs the knowledge of all eigenvalues of matrix A as far as I see. Upto what number of N do they think their method may be used in practice?

Answer

Thank you very much for your useful information. The maximum number of N used in practice is limited by computer run time. I think, in practice, N may be limited to 10 or so. Therefore, it is desirable to obtain the approximate formula for the overflow probability. I think this is a future study.

PAPER No. 324

Author: R. A. Schassberger

Question by J. W. Cohen

Author advocates the phase method and a limiting procedure to arrive at results for general distributions. Indeed, if the limits can be calculated, the method is easy to apply; however, rather laborious. I have the feeling that if his approach yields results, a little more sophisticated analysis will yield the same results with much less labour. Does the author have counter examples to the latter statement?

Answer

The answer is no: I do not have any examples of this kind. I am not sure, however, that there aren't any. For instance, the M/G/1 example as presented in my paper might well qualify. Another example might be provided by the GI/M/n queue - n parallel servers facing recurrent input - for which time dependent distributions can easily be worked out using this method.

As Dr. Cohen points out, the method is easy to apply, and I consider this its main feature; it has a very wide range of applications requiring at the same time very
little sophistication. And once again regarding the labour involved; during the time other people might have to sit and think about a more sophisticated method, I might simply sit down, use my method, perhaps fill a good deal of pages, but might very well end up as the winner in time.

Question by C GRANDJEAN
Are your results valid if the distribution function \( F(t) \) (or \( B(x) \)) is not continuous? Does it cover for instance the case of a discontinuity at \( t = \) mean holding time for constant holding time systems?

I understand that it can be solved by considering such a distribution function as the weak limit of a phase-type distribution. Is that correct? If yes, what is the phase-type distribution to be used?

Answer
The results given in this paper as well as in other works I have published are valid for any d.f. concentrated on \((0, \infty)\). I would like to refer to the papers by Kennedy and by Whitt regarding this question. Both of these papers contain errors of crucial nature. Very recently, however, Whitt has written a report in which faultless arguments are presented. Now suppose a distribution function \( F(t) \) is concentrated on some value \( x > 0 \), i.e. it has a jump there of amount 1. Which is the phase-type distribution to be used? Well, the answer is provided by the theorem quoted on page 1 of my paper. Choose some \( \mu > 0 \). Then you get

\[
F(t) = E^t \mu(t), \quad \text{where } k \text{ depends on } \mu \text{ (there is just one term in the sum that is non zero).}
\]

Question by C GRANDJEAN
You define for your simulation a particular model of arrival process using two Poisson processes. Could you describe it a little more?

Answer
One can think of the analytic model as involving two Poisson processes of different arrival rates; the transition between these is controlled in the analytic model by the state (i.e. busy or not) of the server. In the simulation this strong coupling between server state and arrival rate is removed by introducing a third Poisson process. The arrival times of events in this third process are those at which transitions between the two input processes of different arrival rates are allowed. The average inter-arrival time of the third process forms the horizontal axis of fig. 3. While motivated by the presently existing inability to satisfactorily analyse the specific physical processes mentioned in the introduction, the intent was development of a generally useful result for clustered arrivals. When a customer "bawks", he refuses to wait for the server and will not enter the waiting line until the server is free.