COMMON CONTROL PERFORMANCE OF SWITCHING SYSTEMS IN HIERARCHICAL NETWORKS

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ABSTRACT

In common control switching systems (C.S.) for toll offices the call processing is operated by common resources.

A method of evaluating and comparing common controls (C.C.) is described considering the service performance, i.e., the intensity and the frequency of the traffic peaks saturating the C.C.

INTRODUCTION

A common control switching system consists of two major parts: a switching network and a common control. The service performance of a C.C. depends both on C.S.' own characteristics and its input process, The operating times of the C.C. and the internal blocking of the switching network are assumed to be negligible in comparison with similar quantities in the network.

In the input process the external conditions of the C.S. are synthetized, i.e.,
- the matrix of the traffic interests (i.e., the mean and the variance of the offered traffic from every incoming trunk group to every destination)
- the network (i.e., the size of every outgoing trunk group)
- the hierarchical alternate routing plan.

A parameter containing specific information on the input process is introduced in order to allow an evaluation criterion of the service performance of the C.C.

The operating times - both in wired program and stored program C.C. - are so thin that an evaluation of properties of the C.C. is described inadequately by ordinary criteria for waiting systems (i.e., waiting probability, waiting - time distribution). (1)

The parameter is calculated by an equivalent system for the C.C. in which the characteristics of the C.S. are resumed by the following factors:
- architecture of the C.C.
- interconnection strategy

The input process feeds this system.

INPUT PROCESS

The input process is determined by approximating the traffic to the first two moments. These two moments are evaluated by using the Equivalent Random Theory.

In a hierarchical alternate routing plan, every traffic flows have one or more alternate routes (outgoing trunk groups) - ordered in successive choices - to their destination. Moreover, if two or more traffic flows have in common an alternate route, they will also have in common the successive alternate routes. The network, the matrix of traffic interests and the alternate routing plan determine the traffic offered to every outgoing trunk group.

The process of the service requests to the C.C. is the superposition of the processes of the starting instants for the search of a free outlet in each outgoing trunk group. Therefore every call originates one or more service requests and every service request is addressed to the relating outgoing trunk group.

It is assumed that this pooled process is characterized by the mean (\( \mu \)) and the variance (\( \sigma^2 \)) of the corresponding
birth and death process associated with the mean holding time (D) of the outlets.

The approximation introduced by the E.R.T. of B.I. Wilkinson allows the calculation of the variance of the offered and carried traffics for each outgoing trunk group. Therefore it is possible to calculate the mean M and variance V of the traffic offered to the primary and secondary trunk group (x, x’ ) and also the variance of the carried traffic of the primary trunk group (x). We have:

\[ V = \text{cov}(x, x') + V_c, \]

from:

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\[ \text{cov}(x, x') = \text{cov}(x, x) + \text{cov}(x, x'), \]

The service request process which has peak traffic characteristics (V > M) is assumed to be approximated by the regenerative process – overflowing from a primary trunk group with negative exponential service and mean holding time D – characterized by the values M and V.

THE EQUIVALENT SYSTEM

Given the input process, in order to compute the above said parameter, it is necessary to determine the death process associated with the C.C.'s own characteristics. The variable of this process is the holding time of the C.C.

Let f(t) be the d.p.f. of the holding time of the C.C. for a service request with mean value T.

In C.C. with wired or stored program the function f(t) depends not only on the service requests process but also on the following factors:
- the architecture of the C.C.
- the interconnection strategy

The architecture of the C.C. is defined by two factors:
- the functional structure, i.e., the functions of every type of resources
- the organization of the resources, i.e., the number of resources for every type and their scheduling.

For every service request the interconnection strategy is set up by the criterion for hunting a free outlet (if one) in the outgoing trunk group and by the criterion for hunting a possible path through the switching network to the selected outlet.

A service request is featured by the instant of its birth and by the outgoing trunk group it is on. These features and the interconnection strategy individualize the use sequencing of the resources (processing suite) for the service request processing, according with the C.C. functional structure.

The possible processing suites are clustered according to their types in order to carry out the processing alternatives for a service request.

The interconnection strategy and the external conditions of the C.C. determine the vector whose components v_k weigh the pr. of each alternative.

We assign to every service request a processing suite according with the above said vector. This scheme and the organization of the resources determine the holding time of the C.C. for a service request.

The function f(t) and the quantity T are evaluated by the method G.E.R.T. of the flow diagrams for a stochastic network. (2) By this method the w-function for the whole C.C. is determined.

The w-function for a branch in a stochastic network is defined by:

\[ w = p \cdot \mathcal{L}[g(t)] \]

where:
- p is the conditional probability of carrying out the branch having an execution time with d.p.f. g(t)
- \( \mathcal{L}[g(t)] \) indicates the laplace transform of g(t).

Let N be the greatest number of service requests that the C.C. can process simultaneously. We introduce an equivalent system for the C.C. with N servers with waiting, having d.p.f. f(t) of holding times.

The regenerative process described above is the input process to this equivalent system. By studying this simplified system it is possible to evaluate the probability distribution P(i) of the number of service requests simultaneously present in the C.C. (processed by the C.C. or waiting in the input buffer).

THE EVALUATION CRITERION

The parameter C:

\[ C = \sum_{i=1}^{\infty} (i - N) \cdot P(i) \]

evaluates the service performance of the C.C. in accordance with the frequency and the intensity of the traffic peaks saturating the C.C.

The parameters C and T vary with the same trend when the above listed factors change. Because of the prevalence in C of the characteristics of the input process, the variations of C and T are not proportional.

APPLICATION TO A SYSTEM

We show how the input process and the characteristics of the C.C. affect the variations of the parameters C and T.

![Fig. 1](image-url)
processes characterized by the same values for \( M \) and by different values for \( V \). The considered network consists of 39 trunk groups of type a) and 25 groups of type b).

This network is compared with other two networks: the first (network a) consists of trunk groups of type a) and the second (network b) of groups of type b).

For this comparison we have used a birth rate for service requests of 50/sec.; with \( D = 100 \) sec.

Thus the variances \( V \) are for the considered network and the networks a and b.

Architecture of the C.C. The C.C. processes the service requests which the \( N \) control units (C.U.) extract from the input buffer.

A C.U. requests the intervention of the following units:
- one translator for alternate routing (T) which provides the information on the alternate routing plan and the interconnection strategy.
- \( n \) hunters (H) which hunt for an outlet in one section of an outgoing trunk group.
- \( n \) markers (M) which effect the interconnection between an inlet and an outlet of the switching network.

These units exchange information with the C.U. by means of a BUS, requiring the intervention of a BUS assigner unit (B).

Fig. 2

The intervention of T, H, M are shown in fig. 2 with the times in msec.

In examining an outgoing trunk group a C.U. employs a constant time of 51.2 msec. in addition to the interventions of T, H and M.

Processing suites of the service requests. Assuming negligible the internal blocking, the processing suites may be expressed as follows:

\[
\begin{align*}
\text{P}_1 & : T^*(2) M^*(1) H^*(1) M^*(1) \\
\text{P}_2 & : T^*(2) H^*(2) M^*(1) \\
\text{P}_3 & : T^*(2) M^*(2) T^{*i(1)} H^*(5) M^*(2) \\
\text{P}_4 & : T^*(2) M^*(2) T^{*i(1)} H^*(7) M^*(2) \\
\end{align*}
\]

where \( k \) is the number of interventions of each unit (T, H, M) and the subscripts of T distinguish the intervention of the 1st, 2nd, 3rd type of the alternate routing translator.

The weight \( P_{ik} (k = 1, 2, \ldots, 6) \) is given by the probability of service for a request on the \( k \)-th section explored in the outgoing trunk group with loss B.

Fig. 3

Fig. 3 shows the call distribution \( Q(i) \) for the pr. of occupation on the three trunk groups and the corresponding weights \( P_{ik} \) given by:

\[
P_{ik} = R_k - 1 - R_k, \quad B = Q(m)
\]

where

\[
R_k = \sum_{i=1}^{m} \frac{m,k}{m} \frac{m}{i}
\]

if \( m \) is the number of junctions on the trunk group.

The weights \( P_{ik} \) of the network are obtained by summing up the weights of the component trunk groups according to their relative traffic interests.
Determination of \( f(t) \). Fig. 4 shows the respective flow diagrams for a scanner and for the units B, T, H and N with the following notations:

- \( T_X \) : holding time of waiting time on the unit \( X \) (\( X = B, T, H, N \))
- \( X' \) : residual holding time of the unit \( X \).
- \( W_X \) : w-function associated with \( T_X(t_X) \).
- \( Q_B \) : pr. that the occupation of B is originated by the unit \( Y = T, H, N \).
- \( Q_i \) : pr. that the occupation of \( T \) is of \( i \)-th type (\( i = 1, 2, 3 \)).
- \( X' \) : conditional pr. that the unit \( X \) is occupied at the instant of a service request.
- \( \beta_X \) : conditional pr. that the unit, connected with a step of the scanner of the unit \( X \), is waiting when a service request arrives on the scanner.

\( P \), \( Q \), and \( I \) are obtained through the weights \( p \).

The \( W \)-functions for each unit and each scanner are derived from graphs in fig. 4.

The global \( W \)-function \( W \) of the holding time is deduced from the processing suites described above. For example, for a scanner with \( k \) steps:

\[
W_X^{(k)}(s) = \frac{1}{k} \frac{s^{k-1}}{(s-1)^2} \left( \frac{1}{\beta_X} \right)
\]

from which the moments \( \mu_X^{(k)}(t_X^{(k)}) \) can be derived:

\[
\mu_X^{(k)}(t_X^{(k)}) = \frac{1}{k} \left( \frac{1}{\beta_X} \right) ^k \frac{1}{\mu_X^{(k-1)}} \cdot \mu_X^{(k-1)}(t_X^{(k-1)})
\]

The d.p.f. \( f(t) \) allows the straight evaluation of the equivalent system of the C.C.

The determination of \( f(t) \) needs the inversion of the \( w \)-function \( W \).

This has not yet been done.

**CALCULATION OF C.**

In order to give an only analytical method the distribution \( P(i) \) is replaced by that of the number of service requests falling in \( T + t \) where \( t \) is the mean waiting time in the input buffer for a service request. The latter distribution is given by the regenerative process theory.

**Fig. 5**

Fig. 5 shows the flow diagram for the \( w \)-function \( W \) of the waiting time in the input buffer.

\( \alpha \) indicates the conditional pr. that all C.U. are engaged at the instants of a service request and \( \beta \) indicates the conditional pr. that a service request is not served at a termination of C.U. occupation.

\( T \) indicates the residual period of the duration which separates two successive events of occupation of a C.U. for service requests when the C.U. is in saturation. From the above flow diagram there results:

\[
t = \alpha \left( \frac{1}{N} \cdot \beta + \int_0^\infty \left( \frac{1}{t} \right) \Phi(t) dt \right)\]

where:

\[
\Phi(t) = \int_0^\infty f(t) dt
\]

**CONCLUSIONS**

The calculation of the mean waiting time \( t \) needs the determination of the d.p.f. \( f(t) \).

In order to make the calculation easier, the distribution of the number of service requests is determined referring to the time length \( T \) instead of \( T + t \).

The mean holding time \( T \) has been computed by an estimate of \( \alpha \) assuming arrivals at random for service requests on the unit \( X \).

The estimate of \( \beta \) is:

\[
\beta = \frac{1}{k} \mu_X^{(k)}(t_X^{(k)})
\]
The calculation — with \( N = 8 \) and \( n = 3 \) — gives the following values of \( T \) for the considered network, the network \( a \) and the network \( b \) respectively:

\[
112.84, \quad 104.28, \quad 137.77.
\]

### Table 1

<table>
<thead>
<tr>
<th>U.E.</th>
<th>B</th>
<th>T</th>
<th>H</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANALYTICAL RESULTS</td>
<td>0.7052</td>
<td>0.2821</td>
<td>0.1201</td>
<td>0.3295</td>
</tr>
<tr>
<td>SIMULATION RANDOM TRAFFIC</td>
<td>0.5910</td>
<td>0.2823</td>
<td>0.1233</td>
<td>0.3305</td>
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<tr>
<td>SIMULATION PEAKED TRAFFIC</td>
<td>0.7070</td>
<td>0.2817</td>
<td>0.1251</td>
<td>0.3110</td>
</tr>
</tbody>
</table>

Table 1 shows the loads per unit of the C.S. obtained with calculation and simulation with 100,000 service requests in the considered network.

From simulation we have \( T = 110.56 \) (random input) and \( T = 113.12 \) (overflow input).

The deviations from the calculation are to be ascribed to the estimate hypothesis on \( \alpha_X \) and \( \beta_X \).

Thus the following values of the parameter \( C \) where obtained:

\[
0.231, \quad 0.197, \quad 0.672.
\]

These reveal the forecast unproportionality of \( C \) with \( T \).

The approximations introduced in the calculation of \( C \) have to be evaluated comparing these values with the ones obtained by the equivalent system.

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