AN ANALYSIS OF MIXTURES OF MULTIPLE BAND-WIDTH TRAFFIC ON TIME DIVISION SWITCHING NETWORKS

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ABSTRACT

This paper discusses economical and efficient configurations and channel arrangements of the switching network in a communication system which handles many kinds of information (as in a data communication system, TELEX, voice band data and facsimile, wide band data, highspeed facsimile, and data transfer between computers) carried by different transmission bandwidth channels.

Traffic characteristics are mainly calculated for such systems composed of the full availability trunk-group and of the limited availability trunk-group. If desired to efficiently exchange such information occupying different transmission bandwidth channels, it is necessary to use trunks commonly. As a typical model of the system, consider a TDM switching system. In a TDM switching system handling such information, there are two important channel assignment problems influencing traffic capacity and system implementation.

When a call on a given bandwidth arrives,

1. It can be carried only if there are sufficient idle time slots required for it. This is similar in nature to a full availability trunk-group.

2. It can be carried if there are sufficient idle time slots required for it and some other control conditions, such as time slot hunting sequence, are fulfilled. This is similar in nature to a limited availability trunk-group.

In (1), the explicit solution is given but, in (2), traffic analysis is complicated. This paper introduces a method of calculating the loss characteristics of case (2) approximately. This is a method of bringing a mismatch loss coefficient of the limited availability trunk-group into equilibrium state equations.

The results from applying this method to several systems, controlled by different time slot assignment techniques, are consistent with computer simulation. Curves are given to demonstrate the influence of traffic offered in the full availability trunk-group and in the limited availability trunk-group.

1. INTRODUCTION

Traffic analysis of a system, which exchanges many kinds of information carried by different transmission bandwidth channels, was proposed by L.A. Gippelson(1). It was further developed by M. Akiyama(2). Traffic Analysis of the TDM switching systems, loss systems in particular, was performed by H. Rondina(3) independently. In these analyses, the system was composed of the full availability trunk-group. This paper discusses mainly the system composed of the limited availability trunk-group. In a multiple bandwidth TDM switching system, in general, a wide band call requires a number of equally spaced time slots within each frame and a narrow band call requires one time slot per frame. If this system has no buffer memory (i.e. time-slot shifter), establishing connection of the wide band call often suffers from mismatches between the equally spaced time slots and idle time slots, despite an available excess of idle time slots. This case is a limited availability trunk-group. The probability of loss of a wide band call is increased by the internal blocking. On the contrary, that of a narrow band call is decreased. If the system has a buffer memory, it is free from the internal blocking and traffic capacity is increased. But, in this case, implementation and control of the system become complicated.

Being influenced by the time slot assignment technique, loss characteristics of the limited availability trunk-group are difficult to calculate.

A method of microscopically describing the limited availability trunk-group is indicated and, based on an analogy of this, an approximate method of using the mismatch loss coefficient of the limited availability trunk-group is discussed.

2. TRAFFIC CHARACTERISTICS OF THE FULL AVAILABILITY TRUNK-GROUP

List of Symbols

N: number of time slots on the highway
n: number of different bandwidth channels on the highway
NW: number of time slots in a channel of the i-th bandwidth
A: total highway traffic offered = \sum_{i=1}^{n} NW \times A_i
\lambda: calling rate of the i-th bandwidth traffic component
h: average call holding time of the i-th bandwidth traffic
N: number of sources of the i-th bandwidth traffic
B: probability of loss of the i-th bandwidth traffic
AC: amount of i-th bandwidth traffic carried
P(r_1, r_2, ..., r_n): probability of the highway which is in state \{r_1, r_2, ..., r_n\}

This section deals with the highway having a memory such as a time slot shifter. In Fig. 1, three narrow band calls and two wide band calls are in progress on the highway. (M=16, NW=4). Although idle time slots are not equally spaced, five idle time slots are available to establish connection for one more wide band call by using a time slot shifter.

Speaking more generally, when an i-th bandwidth call arrives, it can be carried if the number of idle time slots is greater than, or equal to, NW.
Fig. 1 A state of a highway occupied by three narrow band calls and two wideband calls.

2.1 WHEN THE NUMBER OF SOURCES IS INFINITE

For each traffic component $A_i \ (i=1,2,\ldots,n)$ it is assumed that:

(i) individual components are independent of each other.

(ii) traffic offered is Poissonian with mean arrival rate $\lambda_i$.

(iii) call holding times are negative exponentially distributed with mean $h_i$.

(iv) lost-calls are cleared.

When the equilibrium state has been reached, following equations for the equilibrium state probabilities are obtained.

$$
K_i = \left( N - \sum_{j=i+1}^{n} \frac{n}{\left( r_j - 1 \right) \lambda_j} \cdot \frac{A_j}{h_j} \right) / \left( N W_j \cdot r_j \right)
$$

Also, the sum of the highway state probabilities must equal one:

$$
K_i = \left( \frac{N W_i}{A_i} \right) \cdot \frac{A_i}{h_i}
$$

From Eq. (1) and Eq. (2),

$$
P(r_1, r_2, \ldots, r_n) = \prod_{i=1}^{n} \frac{A_i}{h_i}
$$

2.2 WHEN THE NUMBER OF SOURCES IS FINITE

Equations for the equilibrium state probabilities are

$$
K'_i = \left( \frac{\sum_{j=i}^{n} NW_j \cdot r_j}{NW_j} \right) / \sum_{j=i}^{n} \frac{NW_j \cdot r_j}{NW_j}
$$

$$
B_i = 1 - A_i / A_i
$$

The sum of the highway state probabilities must equal one:

$$
in(\sum_{i=1}^{n} K_i)\cdot\lambda_i = \sum_{i=1}^{n} P(r_1, r_2, \ldots, r_{i-1}, 0, r_i, \ldots, r_n)
$$

From Eq. (8) and Eq. (9),

$$
P(r_1, r_2, \ldots, r_n) = \prod_{i=1}^{n} \frac{A_i}{h_i}
$$

Since traffic offered $A_{oi}$ is mean density of originating calls measured by average call holding time $h_i$, $A_{oi} = h_i C_i$. Then

$$
B_i = 1 - \sum_{j=1}^{i-1} \frac{\sum_{j=1}^{n} \frac{NW_j \cdot r_j}{NW_j}}{\sum_{j=1}^{n} \frac{NW_j \cdot r_j}{NW_j}}
$$
Assuming that traffic offered is constant in average and $B_i < 1$, the following approximation is available.

$$A_i = (N_i - AC_i) \cdot \lambda_i h_i$$

$$\cdot \lambda_i h_i = \left\{ \begin{array}{ll} N_i - A_i (1 - B_i) & \text{if } i \neq 1, \\
N_i - A_i & \text{if } i = 1 \end{array} \right.$$  \hspace{1cm} (13)

As when the number of sources is infinite, several results calculated are shown in Fig. 4 and Fig. 5.

3. TRAFFIC CHARACTERISTICS OF THE LIMITED AVAILABILITY TRUNK-GROUP

This section deals with the highway having no buffer memory. In Fig. 1, idle time slots are not equally spaced. Despite an available excess of idle time slots, no additional wide band call can be carried. If #2 narrow band call is terminated, #3, #7, #11, #15 time slots are equally spaced and available for a wide band call. The system is a kind of limited availability trunk-group.

In this case, traffic characteristics are influenced by the time slots assignment techniques or by the order of searching available time slots.

3.1 ACCURATE ANALYSIS OF THE LIMITED AVAILABILITY TRUNK-GROUP

3.1.1 PROBABILITY OF LOSS

Traffic characteristics for the system with two mixtures of narrow and wide band traffic is discussed. For simplicity, a wideband traffic requires $m$ consecutive time slots, as shown in Fig. 6.

![Fig. 6 A wideband call requires $m$ consecutive time slots.](image)

Let $n(r_1, r_2)$ and $P_i = (a_1, a_2, ..., a_m)$ be the number of state and the individual probability that the highway is in state $(r_1, r_2)$ when the particular time slots assigned to each bandwidth call are distinguished.

$$n(r_1, r_2) = (m^{-m} \cdot r_1 \cdot r_2) / r_1 \cdot r_2 (m^{-m} \cdot r_1 \cdot r_2); \hspace{1cm} (14)$$

$$P_i(r_1, r_2) = \sum_{j=1}^{m} P_i (a_1, a_2, ..., a_m); \hspace{1cm} (15)$$

where $m$ is number of time slots in a wideband channel.

$a_1, a_2, ..., a_m$ indicates the state of 1st, 2nd, ..., $m$th time slot, respectively. (The value of each $a_j$ is 0 or 1 or $m$.)

If $N_s$ and $P_i = (a_1, a_2, ..., a_m)$ represent the total number of $n(r_1, r_2)$ and individual probability, respectively, following equations for the equilibrium state probabilities are obtained.

$$\sum_{i=1}^{N_s} \left( k_{ij} N_1 + k_{ij} N_2 + k_{ij} N_3 + k_{ij} N_4 \right) \cdot P_i (a_1, a_2, ..., a_m) = 0$$

(j = 1, 2, ..., $N_s$)  \hspace{1cm} (16)

This transition matrix is shown in Fig. 18. Also, the sum of the probabilities must equal one.

$$N_s = \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} n(r_1, r_2); \hspace{1cm} (17)$$

The system of simultaneous linear homogenous equations (16) together with Eq. (17) has a solution.

Then from Eq. (15)

$$B_1 = 1 - \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} P_i (r_1, r_2); \hspace{1cm} (18)$$

$$B_2 = 1 - \sum_{r_1=0}^{\infty} \sum_{r_2=0}^{\infty} r_2 P_i (r_1, r_2); \hspace{1cm} (19)$$

Figure 7 shows the probability of loss $B_2$ for the system controlled by the following different time slot assignment techniques.

(a) Random assignment
(b) Assignment from two fixed time slot positions
(c) Assignment from one fixed time slot position
(d) Reassignment (Full availability trunk-group)

$N_s = 4, m = 2, A = 0.8 \text{ erl}, h_1 = h_2 = 0.025 h$

Results of simulations are also shown in these figures.
3.1.2 MISMATCH LOSS COEFFICIENT OF THE LIMITED AVAILABILITY TRUNK-GROUP

Mismatch loss coefficient of the limited availability trunk-group, \( K(M,m,A_1,A_2,h_1,h_2,r_1,r_2,C) \), is defined as follows.

\[
K(M,m,A_1,A_2,h_1,h_2,r_1,r_2,C) = \sum_{i,j} P(r_1,r_2)(d_i d_j \cdots d_m)
\]  

(20)

where \( \Sigma \) represents the summation over particular states of the highway having at least one m consecutive idle time slots. \( C \) is a parameter concerned with time slot assignment technique.

From Eq. (15) and Eq. (20)

\[
0 \leq K(M,m,A_1,A_2,h_1,h_2,r_1,r_2,C) \leq 1
\]

(21)

3.2 APPROXIMATE ANALYSIS OF THE LIMITED AVAILABILITY TRUNK-GROUP USING \( K(M,m,A_1,A_2,h_1,h_2,r_1,r_2,C) \)

Analysis of the previous section is based on the microscopic state probability of the highway. When \( M \) is large, it is difficult to solve Eqs. (16) and (17) because of large matrix size. If mismatch loss coefficient of the limited availability trunk-group can be obtained approximately in the statistical equilibrium state, the probability of loss may be calculated as closely as that of the previous results. Considering the time slot occupancy on the highway statistically, the system is described by using approximated mismatch loss coefficient of the limited availability trunk-group. Equations for the equilibrium state probabilities are,

\[
K(M,m,A_1,A_2,h_1,h_2,r_1,r_2,C) \approx \frac{P(r_1,r_2)}{h_1 h_2}
\]

The sum of the probabilities must equal one.

\[
M \sum_{r_1=0}^{m-1} \sum_{r_2=0}^{m-1} P(r_1,r_2) = 1
\]

(23)

3.3 SEVERAL EXAMPLES OF THE MISMATCH LOSS COEFFICIENT OF THE LIMITED AVAILABILITY TRUNK-GROUP

Mismatch loss coefficient of the limited availability trunk-group in Eq. (22) is dependent on the time slots assignment techniques. Method of calculating the loss coefficient for the system utilizing the two time slot assignment techniques is discussed.

3.3.1 RANDOM ASSIGNMENT

As shown in Fig. 6, the start point for searching idle time slots is randomly defined for both narrow and wide band traffic, and a wideband call requires m consecutive time slots. Though random assignment is not practical in TDM electronic switching system, being free from the defacement of crosspoints, it is significant to analyze such a system as a worst case.

Calculation of the mismatch loss coefficient for this system is based on the assumption that each \( P(r_1,r_2) \) is equally likely. According to this, this coefficient is the least, and it is independent on traffic offered \( A_i \) and holding time \( h_i \).

\[
S_0 = \left( \begin{array}{c}
M - m - lr_2 \\
M - m - 1
\end{array} \right)
\]

(24)

\[
S_1 = \left( \begin{array}{c}
M - m - 1 - N_1 - N_2 \\
N_1 + N_2
\end{array} \right)
\]

(25)

Where \( k = r_1 + r_2 \)

\[
u(t) : \text{unit step function}
\]
Then
\[
K(M, m, r_1, r_2) = S_1 / S_2
\]
\[
= 1 \quad \text{when} \quad (m-1)(r_1 + r_2 - 1) < M - m r_2 - r_1
\]
\[
= 0 \quad \text{when} \quad M - m r_2 - r_1 < m
\]  
(26)

Since Eq. (25) can be written by using complementary set, Eq. (26) is simplified as follows.

\[
K(M, m, r_1, r_2)
\]
\[
= 1 - \frac{1}{S_2} \sum_{N_1=0}^{N_1(M-m-1)} \sum_{N_2=0}^{N_2} \frac{1}{M - m r_2 - r_1} \left( \frac{N_1}{r_1} \right) \left( \frac{N_2}{r_2} \right) \left( \frac{N_1 + N_2}{N} \right)
\]
\[
- \frac{1}{S_2} \sum_{N_1=0}^{N_1(M-m-1)} \sum_{N_2=0}^{N_2} \frac{1}{M - m r_2 - r_1} \left( \frac{N_1}{r_1} \right) \left( \frac{N_2}{r_2} \right) \left( \frac{N_1 + N_2}{N} \right)
\]
(27)

Equation (27) is further reduced to a simple formula

\[
K(M, m, r_1, r_2)
\]
\[
= \frac{1}{M - m r_2 - r_1} \left( \frac{K_1 + r_1 + r_2 - 1}{m} \right) \sum_{i=1}^{\min(r_1, r_2)} \left( \frac{K_i}{m} \right)
\]
(28)

EXAMPLE 1.

\[
K(12, 4, r_1, r_2)
\]

is tabulated in Fig. 9.

\[
\begin{array}{cccc}
M &=& 12 & N = 4 \\
\hline
I & 2 & 1 & 2 \\
6 & 1.000000 & 1.000000 & 0.000000 \\
5 & 1.000000 & 0.833333 & 0.000000 \\
4 & 1.000000 & 0.000000 & 0.000000 \\
3 & 0.909091 & 0.190909 & 0.000000 \\
2 & 0.666667 & 0.333333 & 0.000000 \\
1 & 0.000000 & 0.000000 & 0.000000 \\
\end{array}
\]

Fig. 9 Mismatch loss coefficient for the system controlled by random assignment technique.

EXAMPLE 2.

In Fig. 10, the probability of loss for wide band traffic B2 is shown. \((M=24, m=4, A=12 e r l, h_1 = h_2 = 0.029^h)\)

Figure 11 shows \(M=48, m=4, A=24 e r l, h_1 = h_2 = 0.029^h\)

3.3.2 ASSIGNMENT FROM TWO FIXED TIME SLOT POSITIONS

This assignment is clarified according to the following reasons. Being often encountered in a TDM switching system which has no time slot shifter, each wide band call requires an equally spaced time slot, as shown in Fig. 12.

\[
\begin{array}{cccc}
W_1 & 1 & 2 & 3 \\
W_2 & 4 & 5 & 6 \\
N & 7 & 8 & 9 \\
\end{array}
\]

Fig. 12 A state of a highway occupied by three narrow band calls and two wideband calls.

When this periodical arrangement is disturbed by narrow band calls, the probability of loss for wide band calls is increased. If each narrow band call is assigned to an equally spaced time slot in accordance with the periodical arrangement for wideband calls, the chance of channel mismatch is decreased. As the result, the process of searching idle time slots is simplified.

In Fig. 12, two wideband calls and three narrow band calls are in progress on the highway. If scanning time slots for wideband calls is in order \((1, 5, 9, 13, 2, 6, 10, 14, \ldots)\) for narrow band calls \((6, 12, 18, 24)\) and \((15, 11, 7, 3, \ldots)\), it is equivalent to the assignment from two fixed time slot positions, as shown in Fig. 13.

\[
\begin{array}{cccc}
\#1 & \#2 & \#3 & \#4 \\
W & & & \\
N & & & \\
\end{array}
\]

Fig. 13 Assignment from two fixed time slot positions.
As it is difficult to calculate the mismatch loss coefficient for such a system directly, an approximate method is discussed under following assumption. "Total traffic offered A is light compared with M. Frequency of wide and narrow band traffic being mingled with each other is rare". This assumption is reasonable because low probability of loss is practical in usual switching systems. Consider a highway in which single bandwidth traffic is transmitted, and all time slots are numbered from #1 to #M. If the start point of searching for an idle time slot is #1 time slot, traffic carried by #r time slot ac(r) and its probability are:

\[ ac(r) = (B_r - 1) A \]
\[ P_r = \frac{ac(r)}{A} \]  

where \( A \): offered call
\( r \): number of time slot
\( B_r \): probability of loss for the system with offered call A and number of available time slot r.

Figure 14 shows \( P_r \) for the case \( A=2.7 \text{ erl, 1.5 erl, 0.75 erl} \). This treatment may be applied to the calculation of the mismatch loss coefficient \( K_m(m,r_1,r_2,A) \) for two mixtures of narrow and wide band traffic. In this approximation, \( K \) is independent of \( A_1, h_1, \) and \( h_2 \). Let \( S_0 \) be the number of ways in which \( r_1 \) narrow band calls can be distributed into \( M-m_2 \) time slots.

\[ S_0 = \binom{M-m_2}{r_1} \]  

The probability of each time slot occupancy, \( P(i) \) \((i=1,2,\ldots, S_0)\), is defined as follows using \( P_r \) in Eq. (30)

\[ P(1) = P_1 P_S - P_r (1-P_r) \]  
\[ P(2) = P_1 P_S - P_r (1-P_r) P_r (1-P_r) \]  
\[ \vdots \]  
\[ P(i) = (1-P_r) (1-P_r) \]  

\[ K_m(m,r_1,r_2,A) = \frac{1}{S_0} \sum P(i) \]  

where \( \sum \) represents the summation over particular \( i \) states of time slots having at least one \( m \) consecutive idle time slots.

**EXAMPLE 1**

\[ K(m=4, r_1=2, r_2=0, A=0.4 \text{ erl}) \] is calculated.

\[ P_1=0.71429 \quad P_2=0.23166 \quad P_3=0.08690 \quad P_4=0.00044 \]

The mismatch loss coefficient calculated by using Eq. (20) is compared with that of the approximated by using Eq. (34) in Fig. 15.

**EXAMPLE 2**

Figure 16 shows the probability of loss for wide band traffic B2. \((M=24, m=4, A=12 \text{ erl})\)

In these figures, probabilities of full availability trunk group for wide band calls B2 are also shown for comparison. Approximated values are within the limits of the section in which reliability is ninety five percent.

**4. TRAFFIC SIMULATION FOR THE SYSTEM OF TWO MIXTURES OF NARROW AND WIDE BAND CALLS**

Traffic simulator used in this paper is not a Markov type, but a general type in which arrival and departure times of narrow and wide band traffic are given separately. In the figures of the probability of loss, symbol "•" denotes the section in which reliability is ninety five percent. Symbol "o" denotes mean value. In this experiment, 33,000 calls in all are originated per experiment and initial 3,000 calls in transient state are eliminated. The probability of loss \( B_i (i=1,2,\ldots, 10) \) is calculated in every 3,000 calls.

Assuming that \( B_i \) is independent and is normally distributed with mean \( B \) and variance \( S \), the section in which
Fig. 17 Probability of loss for the system controlled by assignment from two fixed time slot positions.

reliability is $100(1-\tilde{\theta})$ percent is:

$$\bar{S} \pm t_{1-\tilde{\theta}} \frac{s}{\sqrt{k-1}}$$  \hspace{1cm} (35)

where $t_{1-\tilde{\theta}}$ is the value of Student $t$ distribution with freedom $(k-1)$ at point $100(1-\tilde{\theta})$ percent.

5. CONCLUSION

In multiple bandwidth TDM switching systems, a kind of limited availability trunk-group is practical from the standpoint of implementation and easy control. To calculate loss characteristics of such a system, a method of using the mismatch loss coefficient of the limited availability trunk-group proved to be available in comparison with the results of computer simulation. Although traffic capacity of the limited availability trunk-group is less than that of the full availability trunk-group, a certain time-slot assignment technique minimizes the difference. Calculation is performed only for a two bandwidth TDM switching system, but it can also be performed for multiple bandwidth TDM switching systems.

6. REFERENCES

(1) L.A. Gimpelson,


(2) T. Yamaguchi, M. Akiyama,


(3) H. Rondina, E. Aro, R. Sweet,

"Traffic Analysis of Multiple Bandwidth TDM Switching Systems Utilizing Different Channel Hunting Techniques", Conference Record of 1965 IEEE International Conference on Communications.
Fig. 18 Transition matrix for the system controlled by assignment from two time-slot positions.

Fig. 5 Probability of loss for the system with three mixtures of narrow and wide band traffic.