The statistical problems arising at traffic measurements processing and at simulation of traffic systems are discussed.

The first part of the paper contains a short review of results devoted to verifications of basic traffic theory assumptions, namely on Poisson call flow, exponential holding time distribution, a comparison of accuracy of various BH definitions, a reliability of service criteria, in particular load variations, overflow loads, repeated calls.

The second part covers variance reduction approaches: a comparison of accuracy of loss probability definitions including a substituting the random variables for their averages, using of EB formula, Student's criterion, sample variance by Benes.

The third part discusses a problem of the continuity of solutions for service systems, i.e., whether small variations of basic distributions to induce considerable errors in grade of service.

The advancement of telephone and telegraph communication has called into being the teletraffic theory, which presently turned into a very interesting investigation area with serious yields into practice. Many of its constituents, such as the point process theory, the queuing theory - have formed self-contained scientific branches, having important applications far beyond the scope of the telecommunication theory and practice. Statistical methods were used for telephone networks calculations as early as on the eve of our century. Systematic efforts in this direction, however, were initiated by K. Erlang. Since than many attempts were made to verify those laws encountered in communication practice. Naturally that along this way statistical methods have been penetrating into the traffic theory.

We thank cordially the Technical Committee of the Congress for invitation to make a report. This offers us an exclusive opportunity to express some of our viewpoints on several points of teletraffic theory, that need further study.

I. STATISTICAL PROBLEMS ARISING BY TRAFFIC MEASUREMENTS.

I.I. Verification of the basic assumptions of teletraffic theory.

Statistical verification of both the basic assumptions and its conclusions has accompanied the teletraffic theory development during all its existence. Originally, the idea about the call flow as a Poisson process was not a theoretical proposition, but rather an empirical fact based on comprehensive statistical data. The assumption that Erlang formula retains the appearance regardless of the holding time distribution function has appeared from observations and preliminary trials. During almost forty years this hypothesis had been inspiring prominent scientists for finding its strict proof.

In the last few years excellent traffic measurements were carried out for checking the basic assumptions of traffic theory as well as for proving the service criteria, in particular, the recommendations of the International Consultative Committee for Telegraphy (CCITT). Say for instance about fundamental traffic measurements arranged in Switzerland [1], Finland [2], Denmark [3] and reported on the Sixth International Teletraffic Congress (ITC-6). Below we shall dwell on these and other materials of ITC-6 and discuss some recommendations of CCITT.

A review paper by Hayward and Wilkinson [4] gives good data about correlation between theory and measurements; it confirms Poisson character of the call flow; shows a rather satisfactory correspondence of exponential distribution to the holding time measurements; illustrates such complicated problems of the traffic theory as the effects of load variations, of repeated call attempts; discusses a relationship of these problems and practical tasks of estimating the grade of service according to traffic measurements.
Wilkinson[3] gives numerical illustrations of the se questions, in particular, coincidence with the first Erlang formula, which corresponds well to historically known fact that loss probability on a full available group, serving Poisson call flow, is determined by Erlang formula irrespec-
tively of a type of holding time distribution (Sevastyanov[6]). Nevertheless a deviation of holding time distribution from exponential one have been observed and studied in detail by Hakho[2], and in his opinion the Weibull distribution \( \exp \{ -(r-y)^{\alpha}/\alpha \} \) is to be preferred.

1.2. Load measurement at the busy hour (BH)

Telephone networks are designed so as to provide a required quality of service at the BH. Measurements have shown (LeGall[7], Lotze[5]) that the busiest period falls on the time between 9 and 11 a.m., when a rather high and stable load is observed (of course, for residential districts a busiest period should be chosen within evening hours).

Various statistics measured in a busiest period may be used for evaluation of BH traffic. Let us compare some three statistics:

1. Mean load during a definite hour (for example, from 10 to 11 a.m.) over a rather long observation period.
2. According to the classical method (Gehme[9]) three hour period (to be shifted by 15 min) are made during the busiest period and maximum value among five measured means \( y_i \), \( i = 1, 2, 3 \), indicates the BH. Then, such maximum values over days are averaged on the whole measurement period.
3. According to the method recommended by CCITT[10] measurements may follow the classical method but with another mode of averaging: a mean value for the first hour (9.00-10.00 a.m.) is calculated over the whole measuring period, then calculation is made for the second hour (10.15-11.15 a.m.) and so on for the fifth hour (10.00-11.00). A maximum among five means is taken as a BH traffic.

Compare load values obtained by various method. Suppose that \( n \) is a number of measurement days, \( y_i \), \( y_j \); load estimate by \( i \)-method, \( i = 1, 2, 3 \). Let the real mean load be equal \( \bar{y} \), and real variance of the load values is \( \sigma^2 \).

According to definition

\[
\begin{align*}
\bar{y}_1 &= \frac{1}{n} \sum_{i=1}^{n} y_i, \\
\bar{y}_2 &= \frac{1}{n} \max \{ y_1, y_2, y_3 \}, \\
\bar{y}_3 &= \max \{ \frac{1}{3} \sum_{i=1}^{n} y_i \}.
\end{align*}
\]

Calculations (Gehme[9]) show that

\[
\bar{y} + 0.564 \sigma^2 / n \leq \bar{y}_1 \leq \bar{y} + 1.13 \sigma^2
\]

where the estimate from the above is obtained as an average value of the five independent measurements participants, having normal distribution, with a mean value \( \bar{y} \), and standard deviation \( \sigma \). The estimate from below is accordingly an average value over two measurements (from 9 to 10 and from 10 to 11 a.m.). From \( \bar{y}_1 \) definition it follows that \( \bar{y}_1 \sim \bar{y} \) in the sense of mathematical expectation. Comparison of two other definitions gives \( \bar{y}_2 > \bar{y}_3 \), which may be proved strictly. With \( n \) given we obtain from (4) that

\[
\bar{y} - 0.564 \sigma^2 / n \leq \bar{y}_2 \leq \bar{y} + 1.13 \sigma^2
\]

Due to 2nd method, independent of \( n \)

\[
\bar{y} > 0.564 \sigma^2 / n \leq \bar{y}_2 \leq \bar{y} + 1.13 \sigma^2
\]

(\( \sigma \) affects only the variance of \( \bar{y}_2 \)
).}

From (5) and (6) we find that with \( n > 4 \), defined from the equation \( 0.564 \sigma^2 / n = 1.13 \sigma^2 / n \), with \( \bar{y}_2 \sim \bar{y}_3 \). Therefore

\[
\bar{y}_2 > \bar{y}_3 \sim \bar{y}_2 - \bar{y}_3
\]

Hence we draw a conclusion that the CCITT recommended method gives some overestimated BH load value as compared to real value. Determination of this overestimate requires statistical investigation of the load distribution and, especially, of \( \bar{y} \)-value. Of course the approving itself of this overestimate is outside the scope of teletraffic theory.

1.3. On service criteria.

According to CCITT recommendations[11] two conditions are to be fulfilled for international teletraffic

\[
\begin{align*}
\Pr ( A_{30} ) &< 0.01, \\
\Pr ( A_5 ) &< 0.07
\end{align*}
\]

where \( A_{30} \) and \( A_5 \) are the averages of the 30 and 5 highest BH traffic flow values during last year (i.e. 360 days). \( \sigma \) is the loss probability. Putting \( A_{30} \) and \( A_5 \) instead of \( A_{30} \) (the average of 360 BH's) was introduced for a more sensible response to traffic variations. However, if we consider statistical nature of \( A_{30} \) and \( A_5 \), estimates the above recommendations would hardly be justified, because statistical conclusions made over 30 or only 5 measurements are less accurate than over 360. In the last case a mean square deviation equals \( \sigma^2 / n = 0.053 \sigma^2 / 360 \) while in two first cases \( \sigma^2 / 30 \) and \( \sigma^2 / 5 \), and besides evidently \( \sigma^2 / 360 < \sigma^2 / 5 \).

Therefore \( A_{360} \) estimate is at least 3.5 times and \( A_5 \) estimate 8 times less sensitive to the mean load variations than \( A_{360} \). Requirement (7) having the form of two inequalities makes the variance of conclusions even more.

To prove inequalities \( \sigma^2 / 360 < \sigma^2 / 5 \) we adduce corresponding inequalities for quantities \( A_{30} / 21^2 / 12^2 A_{50} / 12^2 \), which indirectly conform with \( A_{30} / A_{50} A_5 \). For Gaussian curve, i.e. for normal distribution with mean value of zero and variance of a unit mean square deviations of these quantiles are accordingly \( 0.066, 0.079, \), 0.177 (See Kendall and Stuart[12]).

From statistical viewpoint preference should be given to statistics in the form \( \Pr ( A_{30} ) < \beta \) (for example \( \beta = 0.001 \)) or at least in the form

\[
\alpha \Pr ( A_{30} ) < \beta \]

instead of requirements (7) because this inequality has less variance than (7).

The discussed question on inequalities accuracy is pertinent to an unsolved problem: how to define the grade of service? Importance of this question was emphasized by Wright at the ITTC-C.A. Alldin [13,14] considers as necessary to supplement conditions (7) by a requirement of their precise accomplishment and he also states the need in their further modifications to take into account the load variations and its tendency to grow.

A more complicated criterion would have been devised for this purpose, of a type

\[
\sum \alpha_i \bar{y}_1 < \gamma
\]

where \( \bar{y}_1 \) may be understood as measured parameters \( A_{30} / A_{50} A_5 \). As the result of several studies on the "fair"-ness of the "loading" factor, considering the effect of parameters on a total quality criterion. The available results of traffic measurements [1-3] are adequate for proving the criterion of (8) type.

The use of criterion (7) requires solving a sta-
tistical problem: how to pass from known values of $\mathbb{A}(A_d)$ and $\mathbb{E}(A_s)$ to other system parameters, for instance to $\mathbb{H}(A_{se})$, and vice versa how to estimate $\mathbb{T}(A_{se})$ and $\mathbb{E}(A_{s})$ by known $\mathbb{A}(A_{se})$.

1.4. Load variations.

Load variation in communication network is a matter of common knowledge:

1. Variations during day-time.
2. Variations from day-to-day in the load values and BH times.
3. Systematic increase may be during some week days.
4. Seasonal variations and others.

A telephone system design without considering these load variations may lead in practice to losses several times more than were specified. It is because the average losses considering load variation are more than loss function values at the mean load point $\eta$ since the function of loss probability is convex within low losses range.

To avoid this situation a rated load $y = y_0$ is often recommended instead of $y$ (Livshits [15]), Karlsson [16]) where $\sigma^2$ variance of $y$, h a factor $> 0.1$, if $h = 2$, then the 95% upper confidential load level is to be taken instead of $y$. The choice of $y_0$ however should be connected with the following statistical considerations which ignorance may result in $\frac{y_0}{\eta}$ greatly exceeding $\frac{y}{\eta}$:

1. If $y$ is understood as a number of offered calls than in case of poison load (without repeated calls) as it is known $\sigma^2_y = \frac{y^2}{12}$ because the mean value and variance are coincided for poison call flow.
2. If $y$ is understood as a carried load as it is known $\sigma^2_y = \frac{y^2}{3}$ but also by random holding times. Hence $\sigma^2_y = \frac{y^4}{12}$ More precise it may be said that for first kind poison load $\frac{y^2}{3} > \sigma^2_y > \frac{y^4}{12}$.

The upper limit is reached by observing the carried load in an infinite trunk group and $\sigma^2_y$ is near this meaning at a small loss probability for any system.

It is noted that Bretschneider [17] has drawn attention to the necessity of using relation $\frac{y^2}{3} = \frac{y^4}{12}$ at traffic measurements.

Benel [18] investigated this question in more detail. He has shown that for full evaluative group with losses

$$D(M_r(t)) = 2\frac{m^2 y^2}{e^{\frac{m}{\eta}} - 1 + \frac{m}{\eta}} \left(\eta^2 + T^2\right)^2$$

(9)

where

$$M_r(t) = \int_0^t (M(t) - x(t)) dt$$

$\{x(t)\}$ a functional which is defined over Markov process $x(t)$ describing the action of group, $m$ and $\sigma^2_y$ mean value and variance of a busy lines number, $\sigma^2_y$ - variance of function $x(t)$ over stationary distribution.

Instead of (9) one can take

$$D(M_r(t)) \approx 2\frac{m^2 y^2}{m} \frac{1}{T}$$

(10)

in a particular case $\frac{y^2}{3} / \frac{m}{\eta}$ and in case of an infinite trunk group $\frac{y^2}{3} / \frac{m}{\eta} = 1$ and $\frac{m}{\eta}$ whence we have the required approximation $\frac{y^2}{3} / \frac{m}{\eta}$.

From the above some conclusions may be drawn:

1) The deviation from the first type poison load may be discussed if $\frac{y^2}{3} / \frac{m}{\eta}$
2) To facilitate dimensioning of trunk groups with load variations additional tables should be available besides the existing ones in which load variations $\sigma^2_y$ would be considered, for instance by normal low and with different variances. This would have provided the more proven dimensioning than using a rated load $y + \sigma^2_y$, selected as it is known (Livshits [15], Karlsson [16]) from a requirement not to surpass maximum losses in majority of BH (90-95%). Similar proposal was expressed by Le Gallery [7], Longley [19].

The above mentioned considerations need to be taken into account in cases when due to measuring the carried load $y$ conclusions are made that the call flow intensity $\lambda$ has increased by some value $\Delta \lambda$. Similar relations between averages of these values were derived by Oberto [20].

For using them in practice the same relations between variances of $\lambda$ and $y$ are to be taken into account.

Finding the unbiased estimates of $\lambda$ and $y$ is a useful statistical problem considered by Descloux [21].

1.5. Calculations with overflow traffic.

According to CCITT document [22] overflow traffic coming to the secondary group and obtained by adding $\lambda_1$ loads lost on primary groups is recommended to be taken with a peakedness coefficient $\frac{y}{\eta} = \frac{\lambda_1}{\eta}$

(11)

where $\xi$ the peakedness coefficient for $\lambda_1$ load.

In [22] a table of $\xi$ coefficients is offered. For example $\xi = 1.15$ for a load lost on one line, $\xi = 1.34$ for two lines etc.

A problem of summing flows is associated with limit theorems for Poisson process which lay in the basis of the teletraffic theory (Gnedenko [23]), in particular with Grigolini's theorem [24] containing the conditions of Poisson flow origination by summing nonordinary and nonstationary flows.

A calculation of losses in two gradings (Fig.1) is performed to check usefulness of formula (11) and limit theorem.

$$\lambda \lambda \lambda \lambda \lambda \cdots$$

Fig.1.

In Table 1 precise values of loss probability $\lambda$ with two approximations are enlisted (with negative errors in brackets): in first case it is supposed that the summary flow lost on individual lines is a Poisson flow with a total intensity $n\lambda^2$ $(\lambda = \lambda + \lambda_1)$ and in the second case the same but due to [22] the value $1.175\eta$ is taken instead of $\eta$.

Table 1.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Poisson</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.0519</td>
</tr>
<tr>
<td>0.0392(-22%)</td>
<td>0.0487(-6%)</td>
</tr>
<tr>
<td>0.278</td>
<td>0.00949</td>
</tr>
<tr>
<td>0.00501(-49%)</td>
<td>0.00728(-26%)</td>
</tr>
<tr>
<td>0.109</td>
<td>0.000094</td>
</tr>
<tr>
<td>0.00033(-5%)</td>
<td>0.000097(-7%)</td>
</tr>
</tbody>
</table>

Conclusions.

1) The Poisson approximation is insufficiently accurate despite of a rather big number of flows (in b case there are 8 flows).
2) The approximation in accordance with CCITT recommendations is more accurate but it may be made more accurate taking into account a dependence of $\xi$ from $\lambda$ (or $\eta$), this dependence is included in Wilkinson-Bretschneider's equivalent random method. (The later method needs computer programs for solving direct and inverse equations with fractional number of lines.)
1.6. Repeated call problem.

Statistical traffic measurements show that a call loss in practice are often many times more than specified and achieve many percents instead of the promise what in its turn leads to appearing repeated calls. According to Rakho [3] only 67% of calls are successful. Similar data are given by Hayward and Wilkinson [4]. In the Bell System simple procedure is assumed that the number of calls offered equals the number of carried calls plus the blocked attempts.

This shows the need for cardinal revision of teletraffic theory basis changing models with losses for models with repeated calls. Fig. 2 illustrates the differences between two models: loss probabilities by Erlang formula \( \hat{\lambda}_c(A) \) for 10-trunk group is compared with a ratio of lost attempts to offered calls plus lost attempts. -intensity of repetitions, mean holding time equals to 1. Data for the model with repeated calls are taken from Jonin and Sedol's tables [25]. (An approach based on considering the model with repeated calls as a model with queueing gives approximately the same curves [26].)

The results of ITG-6 activity indicate a turning point in the repeated call problem as at the Congress the importance of this problem for CCITT was not only shown (Fratt's message, Jensen [27]), Hearsted and Range [28]) but several authors quite independently offered solutions for this problem for full available group with losses (Jonin and Sedol [29], Bretschneider [30], Snips-Sneppe [26]). Besides a method of calculating additional load due to repeated calls has been devised (Le Gall [31], Snips-Sneppe [26]). The above reports compose a primary basis for corresponding CCITT recommendations.

As to preparing recommendations there is one difficulty of statistical nature: variance of a loss probability estimate is much more for the model with repeated calls than for the model with losses, by a given call intensity. Therefore in the recommendations it is desirable to indicate not only the method of repeated call estimation but as well the required accuracy of statistical conclusions. No doubt that works of Eland [32] and Kornishchev [33] also will be useful for proving the recommendations.

2. Problems arising by statistical simulation of communication systems

Because the method of statistical simulation is used widely for engineering calculation of communication systems (Kosten [34]) it is of importance to develop some common methods for estimation the accuracy of simulation results. It is known that simulation error by certain assumptions is of the form \( \sqrt{T} \), where \( T \) the variance of a studied estimate per time unit, \( T \) time interval over which the traffic is simulated. Let us discuss the problem connected with the so-called reduction approach, i.e. with choosing the estimates having minimum sample variance and with choosing simulation methods giving minimum variance.

2.1. On accuracy of loss probability estimates.

The below discussed algorithm has been developed in advancement of investigations performed by Kosten et al. [35], Rasharini [36], Olsin [37, 38], Descloux [39], Sneps-Sneppe and Kornishchev [40, 41]. This algorithm allows to study the accuracy of different statistics determined over birth and death processes (Snips-Sneppe and Sholy [42]).

Fig. 3 shows the results of comparing between variance of various loss probability estimates on a 6-line full available group \( \hat{\pi}_c \) by time, \( \hat{\pi}_d \) by calls, three remained estimates modified by a substitution of some random values for their averages, namely, for \( \hat{\pi}_d \) estimate this is a random sitting time in states, for \( \hat{\pi}_c \) this is a random number of offered calls in the state "all lines are busy" for the estimate by load \( \hat{\pi}_d \) for is a random sitting time and a random number of offered calls in the state "all lines are busy".

Fig. 3 shows that:
1) The use of \( \hat{\pi}_c \) is more accurate than \( \hat{\pi}_d \) at low call intensity \( \lambda \) and vice versa at large \( \lambda \).
2) Modified estimates \( \hat{\pi}_d \), \( \hat{\pi}_c \) and \( \hat{\pi}_d \) are uniformly more precise than \( \hat{\pi}_c \) and \( \hat{\pi}_d \), and the most precise is \( \hat{\pi}_d \).

The same conclusions were obtained for a system with a finite number of waiting places (See also Polyak and Belov [43]) and for ideal gradings, this enables to recommend statistics \( \hat{\pi}_c \) for any system. The papers of Polyak [44], Andronov and Rosenblit [45] are also devoted to comparison of loss probability estimates. A series of Lind's papers (see [46]) adjoints to this problem as well.

2.2. Use of formulas together with simulation results.

Let us to demonstrate this approach on the Basharin-Longley-Benes (BLB) formula

\[ \hat{\pi}_c (\lambda) = \frac{\sum \hat{g}_j (\lambda) \hat{\lambda}_j - \hat{\pi}_d (\lambda) \hat{\lambda}_d}{\sum \hat{g}_j (\lambda) \hat{\lambda}_j - \hat{\pi}_d (\lambda) \hat{\lambda}_d} \]  

(12)

where \( \hat{g}_j (\lambda) \) the estimate of loss probability on a substate consisting of states with \( j \) lines busy and weak dependence \( \hat{g}_j (\lambda) \) from \( \lambda \) discovered by the results of investigations performed by Kosten et al. [35], Rasharini [36], Olsin [37, 38], Descloux [39], Snips-Sneppe and Kornishchev [40, 41].

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known but unfortunately the least accurate approach is based on the Student's criterion (a long-term realization is fractured into n independent pieces giving $T_1, \ldots, T_n$ and according to them sample estimate is estimated).

The accounting of the correlation between $\lambda_i$ and $T_i$ offers some 30% economy (Knümel [49]). Even more economy are offered estimates of the type

$$\hat{\lambda}_i = \frac{\xi_i}{\sum \xi_i} \hat{\beta}_i,$$

where $\lambda_i, \beta_i$ equally distributed and independent random variables defined over a random interval between two successive transitions at the "all lines busy" state (all the estimates for section 2.1 may be presented in this form). According to theorem of variance for function of random variables $D_{\tau}$ depends on the estimates of mean values $\alpha$ and $\beta$, variances $\sigma^2_{\alpha}$ and covariation $\text{cov}(\alpha, \beta)$. (See also Polyak [50].)

And finally we will point to the variance estimate proposed by Benes [18]

$$\text{D}_{\tau}(\pi) = \frac{\sum \xi_i^2}{m} \frac{1}{\lambda},$$

(For legend see sec.1.4.) This estimate happened to be 100 times more precise for infinite trunk group than the estimate according to Student's criterion.

A usefulness of Benes's approach to large switching systems is yet an unsolved problem. Fig.5 gives a comparison between real variance $D$ of losses probability and its approximation by Benes for a 3-line grading [51] whence a conclusion follows that such an estimate is true also for more complicated systems than full available ones.

3. ON CONTINUITY OF SOLUTIONS OF TELETRAFFIC THEORY

Over whole length of our report we were forced to point that initial distributions of call intervals, holding times are known to us not exactly. It is natural to ask ourselves about the effect of this inaccuracy on the final result. Might it be so that a rather small inaccuracy in defining, say, of holding time distribution may lead to notable errors in the estimates of service grade? The same question contained in our report we were forced to answer, devoted to separate particular problems with considerable limitations.

On the first hand a classical problem with losses has been considered by Franken [52], Borovkov [53] and on the other hand - a classical queuing problem (Kalashnikov and Tsi­tsiashvili [54], Tsitsiashvili [56], Stoyan and Kennedy [57]). All these authors used quite different approaches to the solutions: direct calculation, use of Lyapunov's stability theory, use of measure convergence.

From that already made it may be certainly make clear that an uniform proximity of distribution of losses does not guarantee close solutions, because the distribution moments may vary greatly in this case. In some instances a transfer to the proximity in matrix $L$ saves the situation. Finally a high degree of the distributions proximity at high argument values is to be provided.

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