FULL AVAILABILITY ONE-WAY AND BOTH-WAY TRUNK GROUPS
WITH DELAY AND LOSS TYPE TRAFFIC, FINITE NUMBER OF TRAFFIC SOURCES,
AND LIMITED QUEUE LENGTH

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ABSTRACT
An exact method was developed for the calculation of full availability single stage trunk
groups with delay type and loss type traffic supposing random input, exponentially distribut-
ed holding times, finite number of traffic sources and a limited queue length. For both-
way trunk groups a closed relation may be given for the state probabilities if the average hol-
ding times of delay type calls and loss type calls are equal, and in this case the effect of
the engaged called subscriber may also be taken into account. For one-way trunk groups, and for
both-way trunk groups with different average holding times, in the lack of a closed relation
the state probabilities should be determined numerically from the relevant linear equation
system. The values of the system characteristics as the function of the ratio of the aver-
age holding times approximate a limiting value, and the extent of the changes is the biggest in
the vicinity of equal average holding times. The effect of the engaged called subscriber is the
increase of the traffic handling capacity of the system, supposing, that in the same time
repeated cell attempts are excluded.

INTRODUCTION
Several telephone switching systems exist where the same trunk group may be used by queueing
and non queueing subscribers. For example, in PABX’s extensions can be authorized to different
categories and as a consequence one group of extensions can make outgoing calls with queueing
while the other one can make them with losses, or e.g. in one of two telephone exchanges con-
ected to each other by a both-way trunk group there is a possibility of waiting in case of
congestion while in the other one this possi-
bility is not given, etc.

The common characteristic of these systems is that the trunk group is loaded by traffic from
two groups of sources authorized to different service, and furthermore a certain number of
waiting places are assigned for queueing type calls. The average holding time of the conver-
sations may be uniform throughout the whole system, but it may also be different according
to the origin of the call.

For the case of one-way trunk group, Poisson input, and exponential holding time COHEN inves-
tigated the system [1] supposing an unlimited number of waiting places and full availability.
The calculations of TIEFFER [2] are related with an identical system but with limited avail-
ability trunk group. A limited number of sources was examined by CAPPETTI [3] with combined
delay type and loss type traffic per source. One possible variation of the system inves-
tigated by COHEN for a limited number of sources/ The different average holding times of
the types of calls make the calculations complicated as it was already mentioned in [1].
The characteristics of the loss type calls are exactly calculated by PRATT [4] and supposing
a Poisson input he gives asymptotic solutions for extreme holding time conditions.

Both-way trunk groups were dealt with by HERZOG [5] and RUBAS [6]. A limited number of traffic
sources, and loss type traffic were supposed. The calls were supposed to have identical
average holding time with exponential distribu-
tion and the effect of the engaged condition of
the called party was also taken into account.

In the following we shall deal with both-way trunk groups serving loss type and delay type
calls, and the effect of different average holding times will be examined both for one-way
and both-way trunk groups. On the basis of the results information may be received about the
influence of the different holding times on the system characteristic.

1. TRAFFIC MODEL

The N_l and N_2 limited numbers of traffic sources which originate the delay type calls
and the loss type calls, respectively, offer a so called pure chance traffic of type 2/P22/.
The average call rates of the free traffic sources are \( \alpha_1 \) and \( \alpha_2 \), respectively, the
traffic sources are independent from each other, and the free traffic sources originate calls randomly. According to this the free time of the individual traffic sources has a negative exponential distribution $1/\alpha i$ with $i = 1, 2$ average time duration.

Both types of calls have an exponential holding time distribution where the average holding time $\tau_{at} = 1/\mu_i$ and $i = 1, 2$, that is the average terminating rate is $\mu_i$. The holding times of the calls are independent from each other and from the condition of the system.

2. EXAMINED SYSTEMS

The offered traffic is handled by fully available $n$ trunks. It is supposed that $N_1 > n$ and $N_2 > n$. The trunks are randomly occupied by the arriving calls.

If all $n$ trunks are occupied the arriving loss type call attempts are lost and they leave the system without any reaction. The delay type calls can wait for a free trunk in case of congestion. There are altogether $M$ waiting places in the system $/N_1 \& N_2/$. If all the waiting places are occupied already, the delay type calls are also lost. They will also leave the system without reacting to it, and their holding time is also zero.

The waiting calls are served in the order of their arrival /FCFS queueing discipline/.

2.1. ONE-WAY TRUNKS /SYSTEMS 1A AND 1B/

The traffic sources $N_1$ and $N_2$ use the trunk group in the same direction /see Fig. 1/.

Fig.1. One-way trunk group carrying delay-type and loss-type traffic

The called party is located outside the system. The system is in state $/j, k, m/$ if $j$ delay type calls and $k$ loss type calls are handled at a time, and $m$ number of delay type calls are waiting. In this state there are $N_1 - j$ free traffic sources in the first, and $N_2 - k$ in the second group. The designation of the system is 1A if $\mu_1 = \mu_2$, and 1B if $\mu_1 \neq \mu_2$.

This is an another possible variation of the problem discussed by Cohen [1] for the case of POTS.

2.2. BOTH-WAY TRUNKS /SYSTEMS 2A, 2B AND 2C/

The traffic sources $N_1$ and $N_2$ use the trunk group in the opposite direction /Fig. 2/.

The two groups of the traffic sources set up conversations to each other. The called party is located within the system, it should be regarded busy during the conversation, and it does not originate calls.

In state $/j, k, m/$ there are $N_1 - j - k - m/$ free traffic sources in the first, and $N_2 - j - k$ in the second group.

In both-way trunk groups the unsuccessful calls caused by the engaged condition of the called party may significantly influence the system. There are two possibilities if the called party is engaged:

a. The call is connected up to the engaged called party in the called exchange /switchboard position/.

b. The call is lost.

In variant a. the effect of the engaged called party may be disregarded, see [5]. However, the simplified calculation has an approximation character in this case, because until the release of the called party there is a passive traffic source only on one side of the connection as opposed to the above mentioned case.

The calls forced to waiting join the queue independently from the condition of the called party. When their handling starts either the connection is established or one of the above cases a. or b. is realized.

The designation of the system is 2A for the case a. if $\mu_1 = \mu_2$, and 2B if $\mu_1 \neq \mu_2$, and it is 2C for the case b. if $\mu_1 = \mu_2$.

3. STATE SPACE, EQUATIONS OF STATE

The common characteristic of the systems under examination is that the fate of the delay type calls and the loss type calls should be followed individually.

3.1. SYSTEMS 1A, 1B, 2A AND 2B

The states and possible transitions are shown as an example in Figure 3. The transition coefficients are only marked in the Figure, they are shown in detail in Table 1.

Fig.3. State space and transition coefficients
Table I. Transition coefficients of Systems 1A, 1B, 2A and 2B.

<table>
<thead>
<tr>
<th></th>
<th>(\lambda^{(i)}_{j,k,m})</th>
<th>(\lambda^{(2)}_{j,k,m})</th>
<th>(\mu^{(i)}_{j,k,m})</th>
<th>(\mu^{(2)}_{j,k,m})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>([N_{1}-(j+m)]\alpha_{s})</td>
<td>((N_{2}-k)\alpha_{2})</td>
<td>(j\mu)</td>
<td>(k\mu)</td>
</tr>
<tr>
<td>1B</td>
<td>([N_{1}-(j+m)]\alpha_{s})</td>
<td>((N_{2}+k)\alpha_{2})</td>
<td>(j\mu)</td>
<td>(k\mu)</td>
</tr>
<tr>
<td>2A</td>
<td>([N_{1}+(j+k+m)]\alpha_{s})</td>
<td>([N_{2}+(j+k)]\alpha_{2})</td>
<td>(j\mu)</td>
<td>(k\mu)</td>
</tr>
<tr>
<td>2B</td>
<td>([N_{1}+(j+k+m)]\alpha_{s})</td>
<td>([N_{2}+(j+k)]\alpha_{2})</td>
<td>(j\mu)</td>
<td>(k\mu)</td>
</tr>
</tbody>
</table>

Table II. Transition probabilities of Systems 1A, 1B, 2A and 2B.

In the examined systems only those states are possible for which \(j \leq 0\), \(k \leq 0\), \(j+k \leq 0\), \(0 \leq m \leq M\) and, further, if \(j+k < n\) then \(m = 0\).

The systems are examined in the state of statistical equilibrium. In this case the state probabilities are independent from the time. For stationary state probabilities the transition Kolmogorov equation has the following form:

\[
\sum_{s} q(v,s)P(s) = \sum_{s} q(s,v)P(v) = 1
\]

[1a]

Where \(P(s)\) is the stationary state probability of state \(s\), and \(q(v,s)\) is the transition coefficient belonging to the transition from state \(s\) to state \(v\). The above homogeneous linear equation system is supported by the normalizing relation.

In the following the stationary probability of the state \(j,k,m\) is designated by \(P(j,k,m)\).

According to Figure 3, equation system \(1.1/\) will take the following general form:

\[
\begin{align*}
\sum_{s} P(s) &= 1 \quad \text{(1b)} \\
\sum_{s} q(s,v)P(s) &= \sum_{s} q(v,s)P(v) \quad \text{(1a)} \\
\end{align*}
\]

The transition coefficients belonging to the impossible states are 0.

The closed form solution of the equation system \(1/2/\) is practically impossible because of the change of the state transition characteristics along the line connecting states \(j,k,0/\), \(j+k=n/\).

However, the equation system may always be solved as the number of unknowns is limited.

Practically useful closed relation may only be derived for the Systems \(2A\) and \(2C\) because the state space may be simplified in these cases. For the determination of the state probabilities of systems \(1A\) and \(1B\) the method of successive overrelaxation may be used. The method has been used for the solution of state equation systems for a long time, for example [7], its theory is well developed [8].

3.2. STATE PROBABILITIES OF SYSTEM 2A

The state probabilities of system \(2A\) may be expressed in closed form. In this case the state space of Figure 3 may be simplified, it is sufficient to take into account states \(i,m/\), \(i=j+k\) instead of states \(j,k,m/\). In this case:

\[
P(i,0) = \frac{1}{N} \sum_{s=0}^{N} \left( N_{1}^{i} + N_{2}^{i} \right) P(i,0)
\]

\[1 \leq i \leq n\]

\[
P(n,m) = \frac{1}{N} \sum_{s=0}^{N} \left( N_{1}^{n,i} + N_{2}^{n,i} \right) P(n,m)
\]

\[1 \leq m \leq M\]

\[
P(0,0) = \frac{1}{N} \sum_{i=0}^{N} \frac{P(i,0)}{P(0,0)} + \sum_{m=1}^{M} \frac{P(n,m)}{P(0,0)}
\]

3.3. STATE SPACE AND EQUATIONS OF STATE OF SYSTEM 2C

The effect of the engaged condition of the called party is taken into account with the method employed in [5] and [6]. It is supposed that the arriving call may be directed with equal probability to the individual members of the opposite group, therefore, the probability of arriving to an engaged called party is \(1/N_{s}\), where \(s = 1,2\) if \(1\) trunks are engaged. If free trunks exist the probability that the connection will be established is only \(1-N_{s}/N_{s}\), and the probability that the call will be lost is \(N_{s}/N_{s}\).

If every trunk is engaged the delay type calls join the queue independently from the condition of the called party. The condition of the called party should be taken into account at the start of the call set-up process. Every waiting call has only a probability \(1-1/N_{s}\) of being established.

As a limiting case it could happen that every waiting call is directed to an engaged called party, and so the state changes from state \(i,m/\) directly to state \(i,0/\). The state space is shown in Figure 4, and the transition coefficients are given in Table II.

The stationary state probabilities of System 2C may be presented in the following form:

\[
P(i,0) = K(i,0)P(n,M) \quad 0 \leq i \leq n-1
\]

\[
P(n,m) = K(n,m)P(n,M) \quad 0 \leq m \leq M
\]

and further

\[
P(n,m) = \sum_{i=0}^{n-1} K(i,0) + \sum_{m=0}^{n} K(n,m)
\]

[4]
4. DETERMINATION OF THE SYSTEM PARAMETERS

In the possession of the stationary state probabilities \( \pi_{j,k,m} \), the delay and loss characteristics of the individual systems may be determined. These are the following:

a. Delay type calls

\[ A_1 \] - offered traffic,
\[ B_1 \] - cell congestion,
\[ P(\tau) \] - probability of waiting,
\[ t_{w/m} \] - average waiting time of the served calls as the ratio of the average holding time,
\[ D(\tau) \] - the complementary distribution of the waiting time of the calls not lost.

The waiting time distribution was determined in the case \( \mu_1 = \mu_2 \) only. For the case \( \mu_1 \neq \mu_2 \), the waiting time distribution may be calculated by a "direct" method [cf. SEGAL [9] and STORMER [11]]. In the lack of the general equation for the convolution of exponential distributions with different parameters, this method was successfully employed up to the value \( M=4-5 \) only.

The waiting time distribution may also be determined by the SYNSKI method [which was used and further developed by KUHN [10], and which was also used by SEGAL [9]], if the eigenvalues of the matrix belonging to the differential equation system of the waiting process are determined.

b. Loss type calls

\[ A_2 \] - offered traffic,
\[ B_2 \] - call congestion.

The equations of the characteristic values are given in simplified forms. The detailed equation may also be reproduced by the use of the additional values given below.

As a common characteristic, \( B_{tot} \) - total cell congestion was also calculated.

4.1. ONE-WAY TRUNKS, SYSTEMS 1A AND 1B

\[ A_1 = \frac{\alpha_1}{\mu_1} (N_1 - Y_1 - Y_w) \]  \hspace{1cm} (6)
\[ B_1 = \frac{(N_1-M)}{N_1 - Y_1 - Y_w} - \frac{Y_w}{N_1 - Y_1 - Y_w} \]  \hspace{1cm} (7)
\[ P(\tau) = \frac{N_1 - m)}{N_1 - Y_1 - Y_w} - \frac{(N_1 - m)}{N_1 - Y_1 - Y_w} \]  \hspace{1cm} (8)
\[ t_{w/m} = \frac{\alpha_1}{\mu_1} (N_1 - Y_1 - Y_w) - (N_1 - m) - (N_1 - m) \]  \hspace{1cm} (9)
\[ D(\tau) = \frac{D(\tau + \tau_m)}{N_1 - Y_1 - Y_w} - (N_1 - m) - (N_1 - m) \]  \hspace{1cm} (10)

only if \( \mu_1 = \mu_2 = \mu, \tau = \tau_m \mu \)

\[ A_2 = \frac{\alpha_2}{\mu_2} (N_2 - Y_2) \]  \hspace{1cm} (11)
\[ B_2 = \frac{(N_2 - m)}{N_2 - Y_2} \]  \hspace{1cm} (12)
In the equations 3/13 the following designations were used.

The sums of state probabilities:

\[
P(n) = \sum_{j=0}^{n} P(j,n-j,m)
\]

\[
P(n,M) = \sum_{j=0}^{n-M} P(j,n-j,M)
\]

The served delay type and loss type traffic:

\[
Y_i = \sum_{i=0}^{n} \sum_{j=0}^{i} j P(j,i-j,0) + \sum_{i=0}^{M} j P(j,n-j,m)
\]

\[
Y_2 = \sum_{i=0}^{n} \sum_{j=0}^{i} (i+j) P(j,i-j,0) + \sum_{i=0}^{M} j P(j,n-j,m)
\]

The served delay type traffic, if \( n = M \):

\[
Y_{1M} = \sum_{j=0}^{n} j P(j,n-j,M)
\]

Total waiting traffic, and the waiting traffic if \( n = M \):

\[
Y = \sum_{j=0}^{n} m P(j,n-j,m)
\]

\[
Y_{2M} = M P(n,M)
\]

4.2. BOTH-WAY TRUNKS, SYSTEMS 2A AND 2B

\[
A_1 = \frac{\alpha_1 B_1}{A_1 + A_2}
\]

\[
B_1 = \frac{N_1-n-M}{N_1-Y-Y_w} P(n,M)
\]

\[
D(p) = \frac{(N_1-n)(P(n)-D(n,M))-(Y_w-Y_{w1})}{N_1-Y-Y_w}
\]

\[
t_w = \frac{\alpha_1}{(N_1-Y-Y_w)-(N_1-n-M)D(n,M)}
\]

\[
P(t) = \sum_{m=0}^{\infty} \frac{(N_1-n-m)P(n,M)}{(N_1-Y-Y_w)-(N_1-n-M)D(n,M)} e^{-\tau}
\]

\[
\tau = \frac{1}{\alpha_1}
\]

only if \( \alpha_1 = \mu_1 = \mu_2 \) with state probabilities being valid for the simpler system 2A.

\[
A_2 = \frac{\alpha_2}{\mu_2} (N_2-Y)
\]

\[
B_2 = \frac{N_2-n-Y}{N_2-Y} P(n)
\]

\[
B_{tot} = \frac{A_1B_1 + A_2B_2}{A_1 + A_2}
\]

The designations are the same as in Point 4.1. and further \( Y = Y_1 + Y_2 \) designates the total served traffic.

4.3. BOTH-WAY TRUNKS, SYSTEM 2C

The characteristic value for the estimation of the traffic handling capacity of the system is - as before - the call congestion caused by the engaged condition of all trunks or that of all waiting places. The call congestion caused by the engaged condition of the called party influences the handled and waiting traffic, and it effects the other characteristics through these.

Because of this the offered traffics \( /A_1, A_2/ \), the call congestions \( /B_1, B_2/ \), and the total call congestion \( /B_{tot}/ \) may be calculated by equations 14, 19, 20, 16, and 21, respectively.

As, - according to our suppositions - to join the queue is independent of the condition of the called party, the waiting call will know the condition of the called subscriber only at the start of the call set-up. Therefore, the characteristics of every waiting call are the same. Equation 17 is further valid for the relative average waiting time.

The distribution of the waiting time is influenced by the lost calls caused by the engaged condition of the called party. The cell arriving in state \( /n_m/ \) will wait until the release of the first cell. From the \( n \) calls before it, \( s \) may be lost because of the engaged condition of the called subscriber. In this case the cell should wait only for the release of \( m-s \) calls instead of \( m \) calls. For the waiting calls the probability of directing a cell to an engaged party is \( P(n,M) \). Taking into account the possible variations, the probability of the loss of \( s \) waiting calls is:

\[
\left( \frac{n-1}{N_2} \right)^s \left( 1 - \frac{n-1}{N_2} \right)^{m-s}
\]

According to the above the distribution of the waiting time is the following:

\[
P(t) = \sum_{m=0}^{\infty} \frac{(N_1-n-m)P(n,M)}{(N_1-Y-Y_w)-(N_1-n-M)D(n,M)} e^{-\tau}
\]

\[
\tau = \frac{1}{\alpha_1}
\]

The designations are the same as before.

5. RESULTS

5.1. THE EFFECT OF THE HOLDING TIME DIFFERENCE ON THE SYSTEM CHARACTERISTICS

If the holding times of the delay type calls and loss type calls are different the system characteristics change as compared to those in case of equivalent holding times both in Systems

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The change of the characteristics may be derived from the change of the state probabilities. A simple example is given in Figure 5. The character of the different holding times is identical for systems 1B and 2B. In the following we present the system \( n=2, M=2, N_1=N_2=15, 2B \) as an example.

![Fig. 5. State probabilities as a function of \( t_{m1}/t_{m2} \)](image)

The parameter for the curves is the holding time ratio \( t_{m1}/t_{m2} \). On the basis of Figure 6, showing the change of the two characteristics it may be seen, that the curves with parameters \( 50 \) and \( 0.02 \) may be regarded as asymptotic limiting values.

![Fig. 6. The dependence of \( B_1 \) and \( P(t) \) from \( t_{m1}/t_{m2} \)](image)

On the horizontal axis of Figure 6, such a scale was employed which emphasizes the asymmetrical character of the results. The change of the characteristics is very definitive in the range \( 10 \leq t_{m1}/t_{m2} \leq 100 \), out of this range the curves horizontally approximate the limiting value. The character of the curves is identical for every characteristic value. The curves are less steep on the right hand side. The position of the point belonging to the value \( t_{m1}/t_{m2} = 1 \) is shifted downwards between the two limiting values especially in case of small \( B_1 \) and small \( t_w/t_m1 \).

To demonstrate the extent of the changes the two limiting values together with the value belonging to \( t_{m1} = t_{m2} \) are given in Tables III, IV, and V.

### Table III. Extreme values of system characteristics, \( A_1=2 \) erl, \( A_2=1 \) erl

<table>
<thead>
<tr>
<th>( t_{m1}/t_{m2} )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_{tot} )</th>
<th>( P(t) )</th>
<th>( t_w/t_m1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.203</td>
<td>0.811</td>
<td>0.598</td>
<td>0.440</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.232</td>
<td>0.782</td>
<td>0.536</td>
<td>0.520</td>
<td></td>
</tr>
<tr>
<td>0.02</td>
<td>0.273</td>
<td>0.750</td>
<td>0.459</td>
<td>0.597</td>
<td></td>
</tr>
</tbody>
</table>

### Table IV. Extreme values of system characteristics, \( A_1=0.5 \) erl, \( A_2=0.25 \) erl

<table>
<thead>
<tr>
<th>( t_{m1}/t_{m2} )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_{tot} )</th>
<th>( P(t) )</th>
<th>( t_w/t_m1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.00278</td>
<td>0.155</td>
<td>0.0536</td>
<td>0.151</td>
<td>0.0499</td>
</tr>
<tr>
<td>1</td>
<td>0.00499</td>
<td>0.156</td>
<td>0.0554</td>
<td>0.149</td>
<td>0.0876</td>
</tr>
<tr>
<td>0.02</td>
<td>0.01780</td>
<td>0.158</td>
<td>0.0648</td>
<td>0.136</td>
<td>0.1715</td>
</tr>
</tbody>
</table>

### Table V. Extreme values of system characteristics, \( A_1=0.3 \) erl, \( A_2=0.1 \) erl

<table>
<thead>
<tr>
<th>( t_{m1}/t_{m2} )</th>
<th>( B_1 )</th>
<th>( B_2 )</th>
<th>( B_{tot} )</th>
<th>( P(t) )</th>
<th>( t_w/t_m1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.000403</td>
<td>0.0535</td>
<td>0.0137</td>
<td>0.0528</td>
<td>0.183</td>
</tr>
<tr>
<td>1</td>
<td>0.000651</td>
<td>0.0538</td>
<td>0.0139</td>
<td>0.0525</td>
<td>0.291</td>
</tr>
<tr>
<td>0.02</td>
<td>0.003000</td>
<td>0.0550</td>
<td>0.0160</td>
<td>0.0509</td>
<td>0.0576</td>
</tr>
</tbody>
</table>

If \( B_1 \) is little \( B_2 \) increases with the decrease of \( t_{m1}/t_{m2} \) as it was determined by PRATT for the unlimited queue [4]. If \( B_1 \) in not negligible the decrease of \( t_{m1}/t_{m2} \) results in the increase of \( B_1 \) and the decrease of \( B_2 \). The extent of the change of the characteristic values is different as compared to the case \( t_{m1}/t_{m2} = 1 \), the changes of \( B_1 \) and \( t_w/\lambda_m1 \) are the biggest.

### 5.2. THE EFFECT OF THE NUMBER OF WAITING PLACES

In the examined systems \( B_1 \) and \( B_{tot} \) decrease, \( B_2 /P(t) \) and \( t_w/t_m1 \) increase with the increase of \( M \). The values of all characteristics gradually approximate the value being valid for the unlimited queue. In Figures 7. and 8. the results are shown for the system \( n=20 \), \( N_1=N_2=15, 2A \).

### 5.3. THE EFFECT OF THE ENGAGED SUBSCRIBERS

For equal offered traffic the handled traffic is less in system 2C then in system 2A. Namely,
6. CONCLUSIONS

An exact method was developed for the calculation of full availability single stage trunk groups with delay type and loss type traffic supposing random input, exponentially distributed holding times, finite number of traffic sources and limited queue length. For both-way trunk groups a closed relation may be given for the state probabilities if the average holding times of delay type calls and loss type calls are equal and in this case the effect of the engaged called subscriber may also be taken into account. For one-way trunk groups, and for both-way trunk groups with different average holding times, in the lack of a closed relation the state probabilities should be determined directly from the relevant linear equation system. For solving the equation system the method of successive overrelaxation was employed which - by the use of computers - has good effectiveness in case of big systems too.

Fig. 7. Call congestion of delay-type calls as a function of $A_1$ and $M$.

Fig. 8. Call congestion of loss-type calls as a function of $A_1$ and $M$.

The examinations revealed that the different average holding times for the two types of calls influence the call congestion and relative average waiting time of the delay type calls mainly, first of all in case of little losses. At the same time the call congestion of the
loss type calls changes relatively slightly. The values of the system characteristics as the function of the ratio of the average holding times approximate a limiting value and the extent of the changes is the biggest in the vicinity of equal average holding times. There are further studies required in connection with the distribution of the waiting time.

Because of the lost calls caused by the engaged condition of the called subscribers the handled traffic decreases in the examined both-way trunk group. Seemingly, this results in the increase of the traffic handling capacity of the system as compared to the system making possible the waiting for the called subscriber. However, because of the bigger ratio of the lost calls it could be a question whether we have a real picture of the system if at the same time we disregard the effect of the repeated call attempts.

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