CALL CONGESTION AND DISTRIBUTION OF NUMBER OF ENGAGED OUTLETS IN A GIVEN SPLIT OF GRADINGS WITH RANDOM ROUTING

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ABSTRACT
A formula has been constructed to calculate the call congestion and the distribution of the number of engaged outlets in a given split in gradings with random routing.

It is an interpolation formula where, for given values of availability, number of outlets and traffic offered is interpolated with respect to the interconnection number \( H \) between the following extremes:
1) a grading with \( H = \infty \), which is approximated by the ideal grading of Erlang,
2) a grading with \( H = 1 \), which is disintegrated into a number of separate full availability groups.

1. INTRODUCTION
The design of gradings with random routing (for instance for non-homing selectors), having Poisson traffic offered to them, requires an approximation formula which is precise enough for determining the call congestion (for a given traffic volume) or the permissible traffic volume (for a given call congestion).

It should moreover not be too expensive in terms of computer time to set up extensive tables, if necessary.

Because gradings with random routing are also used at the outgoing side of link networks (e.g. in the P.R.X.-system), the formula must also be applicable to calculate the distribution of the number of engaged outlets of a given split.

In the present report a formula is constructed (in part empirically), which meets the requirements fairly well.

It is an interpolation formula, with the ideal grading (Erlang's interconnection formula) at one extreme and a grading with interconnection number \( 1 \), degenerated into a group of separate full-availability groups, at the other extreme.

Notation
\( N \) - number of outlets of the grading
\( I \) - availability
\( H \) - average interconnection number
\( A \) - traffic offered
\( f(x) \) - probability that \( x \) outlets are engaged out of the \( I \) outlets of a given split.
\( w \) = \( f(I) \) - call congestion
\( s \) = number of splits.

2. Existing formulas
In the present section a survey is given of existing formulas and methods for the design of gradings. This list is not complete. Of the methods which are used for gradings with homing position only those are listed which are also applicable, with or without some modification, to gradings with random routing.

a. Formula of the modified O'dell type

\[ A(1-w) = c \frac{(1-w)^{(N-K)}}{c.\frac{v}{X} + (1-c).\frac{1}{1-w}} \]

where \( c \) is to be obtained from \( w = E_N(A_F) \).

In this formula \( c \) is a constant \( \epsilon \ (0:1) \) which depends on the type of grading and must be found empirically.

This formula is used by the Swedish Telecommunications Administration [1] for gradings with homing position.

b. Modified formula of Palm-Jacobsen (MPJ-formula)

The modification has been introduced by Lotze and Wagner [2].

The formula is:

\[ w = E_N(A_F)/E_{N-1}(A_F) \]

where \( A_F \) is to be obtained from

\[ A(1-w) = A_F \left( 1 - E_N(A_F) \right) \]

This formula does not take account of the type of grading. In order to consider also the type of grading Herrog [3] has introduced an empirical formula for the correction value \( \Delta A \).
This value is to be added to the traffic volume \( A \) that may be permitted according to the MF2-formula in order to obtain the correct value of the permissible traffic volume.

For the correction formula we have

\[
\Delta A = \frac{N-1}{N^2} (K-2)/(60 + 4K),
\]

where \( P \) is a constant which depends on the type of grading and will have to be found empirically.

\( P \) may be positive or negative; in practical gradings \( P \) is negative as a rule.

In Herroog's report values are given for \( P \), amongst other things for gradings with homing position of

\[
\text{GRAPOI}
\]

As a rule the call congestion values for a given grading

methods [4], [5], [6] are concerned with homogeneous gradings with an arbitrary interconnection number.

In practical gradings the number of splits is mostly far smaller than \( \left( \frac{H}{2} \right) \) as a result the permissible traffic volume on such gradings will be much smaller for a given call congestion than on an ideal grading.

c. Erlang's interconnection formula

This formula is concerned with homogeneous gradings with similar grading numbers.

In practical gradings the number of splits is mostly far smaller than \( \left( \frac{H}{2} \right) \). As a result the permissible traffic volume on such gradings will be much smaller for a given call congestion than on an ideal grading.

d. Methods of Eldin, Kruithof and Valenzuela

These methods [7], [5], [6] are concerned with homogeneous gradings with interconnection number 2.

e. GRAPOI method

This method [7] is concerned with homogeneous or almost homogeneous gradings with random routing and an arbitrary interconnection number.

As a rule the call congestion values for a given traffic volume will be a little too high with this formula.

The method is rather complicated and too expensive, therefore, to produce extensive tables.

2.1. Applicability of existing formulas to gradings with random routing.

The formula at a) can be made applicable by determining constant \( c \) empirically for different values of interconnection number \( H \). For method b) constant \( F \) must be found empirically or even another empirical correction formula for \( \Delta A \) may have to be found, if necessary.

These are not very attractive solutions, also because determination of constants empirically (i.e. by simulation) requires many observations.

There is the extra difficulty that the "constants" are not true constants, but approximations for what (one hopes) are quantities showing little variation.

Methods d) are limited to \( H=2 \) and are hard to generalize to arbitrary values of \( H \). Method e) for arbitrary values of \( H \) does not offer any points from which to obtain a simplified formula.

What can be done, however, is the following.

As an approximation, the ideal grading of Erlang may be considered as an extreme case, i.e. with \( H=\infty \). Because the opposite extreme, \( H=1 \), can also be calculated, an interpolation formula may be based on these two extremes. This is done in the next sections.

3. Construction of a formula for gradings with random routing.

Gradings for random routing have a homogeneous or almost homogeneous structure, in which a rather extensive skipping is applied to mix the traffic. This means that, in addition to the number of outlets \( N \) and availability \( K \), these gradings are well determined by the average interconnection number \( H \).

This makes them altogether different from gradings with homing position, which for a given average interconnection number may show a great variety of structure.

For gradings with random routing, therefore, it was tried to find an interpolation formula containing the following extremes.

1. \( H=\infty \) case. For this case the ideal grading of Erlang, for which the interconnection formula is true, was chosen as an approximation.

2. \( H=1 \) case. For this case the grading disintegrates into several separate full-availability groups, for which Erlang's formula is true. This extreme case is called disintegrated grading.

The distribution of the number of engaged outlets of a given split is represented for the extreme cases by \( f_1 \) (for the ideal grading) and \( f_2 \) (for the disintegrated grading).

For formula \( f_1 \) we have

\[
f_1(x) = \int \frac{1}{x} \frac{N}{N+1}, \quad x \geq 1
\]

where \( g(y) \) may be determined from the recurrent relations

\[
g(y) = g(y-1). y/N+1, \quad y=1, \ldots, K
\]

and the normalizing equation

\[
\sum_{y=1}^{K} g(y) = 1
\]

For formula \( f_2 \) is

\[
f_2(x) = \left( \frac{AK}{N} \right)^x \frac{1}{(N+1)^x}
\]

An interpolation formula for the \( f(x) \) distribution for a grading with arbitrary interconnection number will therefore be as follows:

\[
f(x) = \frac{1}{1} f_1(x) + \frac{1-v}{1} f_2(x), \quad v(H, N, K)
\]

where the value of \( v \) is \( \in (0,1) \). For simplicity \( v \) is called the "weight factor". Obviously, this factor must first of all be a function of \( H \).

If a weight function \( v(H) \) does not depend on \( H \) only should not lead to satisfactory precision, then \( v \) may also be made to depend on \( K \) and/or \( N_0 \). This is the extreme limit for \( v \); if \( v \) should also depend on \( K \) and/or \( x \), the "weight factor" character will be lost. In section 3.1. the choice for \( v \) will be discussed in more detail.

First we have to choose a method of interpolation.

The linear and logarithmic-linear interpolation methods have been examined. For these cases the formulas for \( f(x) \) are as follows:

linear: \( f(x) = v f_1(x) + (1-v) f_2(x) \)

logarithmic linear: \( f(x)=\text{const.} f_1(x)^{1-v} f_2(x)^v \)

where "const. " is a normalizing constant such that \( \int_0^x f(x) \, dx = 1 \).

Simulation tests have shown that the logarithmic-linear interpolation gives far better call congestion results than the linear interpolation, for the same function for \( v \). For this reason the logarithmic-linear interpolation was chosen.

A thing of minor importance is, that with the logarithmic-linear interpolation, the \( f(x) \) distribution corresponds to simpler equations of state than with linear interpolation. See section A7 of the appendix.

3.1. Choice of function for \( v \)

The \( v \) function must satisfy the following demands:

1. For \( H=1 \) \( v \) must be equal to 0.

2. For \( H=\infty \) \( v \) must be equal to 1.

3. \( v \) must increase monotonically with the rise of \( H \).

It is moreover thought undesirable for this function to contain constants which must be determined empirically.

Before trying to find possible functions for \( v \), the GRAPOI calculation method was considered for possible starting points, but these were not found.

Three functions for \( v \) have been considered, referred to as \( v_0 \), \( v_1 \), and \( v_2 \) hereafter.

The first choice for \( v_0 \) was taken to be the following function:

\[
v_0 = \frac{H-1}{H}
\]

The reason for this choice was the following.

The outlets to a given split gives access to \( K(x) \) contacts of this particular split and to \( K(H-1) \) contacts of the other splits together. The ratio of the numbers of both kinds of contacts, which have a direct effect on the state of the group of outlets concerned, therefore
is 1/(H-1).

Simulation tests have shown that function (5) will generally give too high values for v, particularly for high values of K/N. The following function was tried, therefore as second one: 

$$v_2 = \frac{(N-1)}{(N-H)}$$

For K=N (full-availability group) and H=1 we have \(v_1=1\); for K=N and H=1, v is undefined. This is acceptable, because for K=N the \(f_I\) and \(f_I\) distributions are identical.

This may be readily deduced from (1), (2) and (3).

In fig.1 the lines for \(v_1\) are constant are shown in the \((H, K/H)-plane\).

The congestion results of function \(v_1\) are far better than those of function \(v_2\). In certain areas, however, there are still distinct deviations. For small H(H<2) the function will give too high a low call congestion value. For large K/N the call congestion will be too high. In order to offset this effect, the following function was tried:

$$v_2 = \frac{(N-1)}{(N-H)}$$

This function is shown in fig.2.

For \(K/N < 1\) this function yields higher values for the call congestion than function \(v_1\) and for \(K/N > 1\) it yields lower values.

A comparison with the simulation results shows that \(v_2\) still has the same drawbacks as \(v_1\), only less so. The largest deviations, especially, have become smaller. The results will be discussed and analysed more extensively in section 5.

The interpolation formula with weight factor \(v_2\) is accepted as satisfactory precise for determining the call congestion and the \(f(x)\) distribution.

In the next section some comments are made with respect to the numerical calculation.

4. Calculation of A as a function of \(N\) for high values of \(K\) and fixed values of \(H\) and \(w\).

For \(N \rightarrow \infty\), and constant offered traffic per outlet, \(\gamma_o = K/H\), the distribution \(f_I\) for the ideal grading changes into

$$f_I(x) = \frac{1}{K} [\gamma_0(1-\gamma_1)]^x \cdot \frac{(1-\gamma_1)}{\gamma_1}$$

where \(\gamma_1\) is the value determined from

$$\gamma_1 = \frac{[\gamma_0(1-\gamma_1)]^x}{K}.$$  

How this has been derived, will be shown in section 4.2 of the appendix.

For the disintegrated grading we have for every value of \(N\): 

$$\gamma_1(x) = \frac{1}{x^N} \frac{1}{1-w}$$

For the weightfactor \(v_2\) we have

$$v_2 = \frac{(N-1)}{H}.$$ 

Hence, for a grading with arbitrary \(H\) this will provide

$$\gamma_1(x) = \frac{1}{x^N} \frac{1}{1-w} \left[ \frac{K}{(1-w)} \right]^{1/N}$$

where \(\gamma_1\) may be determined by means of (8).

For tabulating \(A\) as a function of \(N\) for fixed values of \(K, H\) and \(w\), the following method may be used:

1. Calculate \(\gamma_0\) from (9), (6) and formula \(w=f(K)\).
2. Calculate \(A\) as a function of \(N\) by means of (4) and (7) to a certain value \(N_g\) of \(N\). If \(A_g\) is the value of \(A\) corresponding to \(N_g\), the following approximation is adopted for \(N > N_g\):

$$A = A_g + (N-N_g) \gamma_0.$$
Table 1: Comparison of simulation and calculation results for the call congestion.

<table>
<thead>
<tr>
<th>N</th>
<th>K</th>
<th>( \text{H} = \text{K}/\text{N} )</th>
<th>A</th>
<th>( w_{\text{sim}} )</th>
<th>( \sigma_{\text{sim}} )</th>
<th>GRAPOI</th>
<th>( \text{w}_{\text{GR}} )</th>
<th>( \sigma_{\text{GR}} )</th>
<th>( \text{v}_1 )</th>
<th>( \text{v}_2 )</th>
<th>( \varepsilon_1 )</th>
<th>( \varepsilon_2 )</th>
<th>( \varepsilon )</th>
<th>source 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2</td>
<td>1.20</td>
<td>.40</td>
<td>.0084</td>
<td>.0020</td>
<td>.0107</td>
<td>1.15</td>
<td>.0101</td>
<td>.85</td>
<td>.0103</td>
<td>.28</td>
<td>.56</td>
<td>.56</td>
<td>[6]</td>
</tr>
<tr>
<td>36</td>
<td>8</td>
<td>1.22</td>
<td>.44</td>
<td>.00245</td>
<td>.00335</td>
<td>.00501</td>
<td>1.70</td>
<td>.0023</td>
<td>.45</td>
<td>.0026</td>
<td>.45</td>
<td>.45</td>
<td>.45</td>
<td>[6]</td>
</tr>
<tr>
<td>24</td>
<td>6</td>
<td>1.25</td>
<td>1.5</td>
<td>.0193</td>
<td>.00095</td>
<td>.0204</td>
<td>1.16</td>
<td>.0182</td>
<td>.16</td>
<td>.0196</td>
<td>.30</td>
<td>.30</td>
<td>.30</td>
<td>[6]</td>
</tr>
<tr>
<td>32</td>
<td>8</td>
<td>1.25</td>
<td>1.5</td>
<td>.0373</td>
<td>.0056</td>
<td>.0344</td>
<td>1.52</td>
<td>.0323</td>
<td>.89</td>
<td>.0344</td>
<td>.52</td>
<td>.52</td>
<td>.52</td>
<td>[6]</td>
</tr>
<tr>
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<td>1.25</td>
<td>1.5</td>
<td>.0219</td>
<td>.0070</td>
<td>.0186</td>
<td>1.47</td>
<td>.0178</td>
<td>.59</td>
<td>.0195</td>
<td>.34</td>
<td>.34</td>
<td>.34</td>
<td>[6]</td>
</tr>
<tr>
<td>40</td>
<td>10</td>
<td>2.25</td>
<td>2.5</td>
<td>.0173</td>
<td>.0032</td>
<td>.0154</td>
<td>.59</td>
<td>.0140</td>
<td>.03</td>
<td>.0152</td>
<td>.56</td>
<td>.56</td>
<td>.56</td>
<td>[6]</td>
</tr>
<tr>
<td>32</td>
<td>8</td>
<td>2.25</td>
<td>2.5</td>
<td>.0254</td>
<td>.0025</td>
<td>.0252</td>
<td>.08</td>
<td>.0230</td>
<td>.96</td>
<td>.0235</td>
<td>.76</td>
<td>.76</td>
<td>.76</td>
<td>[6]</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>2.25</td>
<td>4</td>
<td>.0087</td>
<td>.0016</td>
<td>.0110</td>
<td>1.44</td>
<td>.0100</td>
<td>.81</td>
<td>.0100</td>
<td>.81</td>
<td>.81</td>
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<tr>
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<td>2.25</td>
<td>5</td>
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<td>.0017</td>
<td>.0223</td>
<td>1.94</td>
<td>.0203</td>
<td>.76</td>
<td>.0200</td>
<td>.59</td>
<td>.59</td>
<td>.59</td>
<td>[6]</td>
</tr>
</tbody>
</table>

1) the deviation relative to the simulation result is expressed as \( \varepsilon = (w_{\text{calculated}} - w_{\text{sim}})/\sigma_{\text{sim}} \).
2) source of simulation result.
3) the GRAPOI-method is not applicable for \( N \leq 3 \).

Table 2: Frequency distribution of deviation \( \varepsilon \) of the calculated call congestion with respect to the measured call congestion, over the set of investigated cases.

<table>
<thead>
<tr>
<th>Interval of ( \varepsilon )</th>
<th>Number of cases within interval for method of calculation:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GRAPOI</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>(-5 \leq \varepsilon &lt; -4 )</td>
<td>0</td>
</tr>
<tr>
<td>(-4 \leq \varepsilon &lt; -3 )</td>
<td>0</td>
</tr>
<tr>
<td>(-3 \leq \varepsilon &lt; -2 )</td>
<td>1</td>
</tr>
<tr>
<td>(-2 \leq \varepsilon &lt; -1.5 )</td>
<td>0</td>
</tr>
<tr>
<td>(-1.5 \leq \varepsilon &lt; -1.0 )</td>
<td>0</td>
</tr>
<tr>
<td>(-1.0 \leq \varepsilon &lt; -0.5 )</td>
<td>0</td>
</tr>
<tr>
<td>(-0.5 \leq \varepsilon &lt; 0.0 )</td>
<td>10</td>
</tr>
<tr>
<td>(0.5 \leq \varepsilon &lt; 1.0 )</td>
<td>0</td>
</tr>
<tr>
<td>(1.0 \leq \varepsilon &lt; 1.5 )</td>
<td>4</td>
</tr>
<tr>
<td>(1.5 \leq \varepsilon &lt; 2.0 )</td>
<td>2</td>
</tr>
<tr>
<td>(2 \leq \varepsilon &lt; 2.5 )</td>
<td>1</td>
</tr>
<tr>
<td>(2.5 \leq \varepsilon &lt; 3.0 )</td>
<td>0</td>
</tr>
<tr>
<td>(3 \leq \varepsilon &lt; 3.5 )</td>
<td>0</td>
</tr>
<tr>
<td>(3.5 \leq \varepsilon &lt; 4.0 )</td>
<td>1</td>
</tr>
<tr>
<td>(4 \leq \varepsilon &lt; 4.5 )</td>
<td>2</td>
</tr>
<tr>
<td>total number of cases: ( \varepsilon )</td>
<td>32</td>
</tr>
</tbody>
</table>

* There is one case to which the GRAPOI-method does not apply; see note 3) of table 1.
Fig. 1: Weight factor \( v_1 = (H-1)/(H-K/N) \)

Fig. 2: Weight factor \( v_2 = (H-1)/\{H-(N/K)\} \)

Fig. 3
N=40, K=10, H=2, A=21.98

Fig. 4
N=24, K=12, H=1.5, A=12.80

Fig. 5
N=48, K=32, H=2, A=35.11

Fig. 6
N=40, K=20, H=4, A=26.00

Fig. 7
N=24, K=6, H=1.5, A=10.00

Fig. 8
N=32, K=8, H=8, A=18.00

Fig. 3-8: results for distributions \( f(x) \) of the number of engaged outlets of a given split, obtained by calculation with weight factor \( v_2 \) (---) and by simulation (----).
Fig. 9...12 Working charts for designing gradings with random routing.

- \( N \) = number of outlets
- \( K \) = availability
- \( H \) = average interconnection number
- \( W \) = call congestion for Poisson traffic
- \( A \) = offered traffic in Erlangs
The interpolation formula with weight factor \( v \), produces smaller positive deviations than \( \text{GRAPOI} \), but the negative deviations are markedly larger than for \( \text{GRAPOI} \).

The interpolation formula with weight factor \( v_2 \) is clearly better as regards the positive deviations, and the negative deviations are only a little worse than for \( \text{GRAPOI} \).

**Conclusion 1**

The interpolation formula with weight factor \( v_2 \) is quite suitable to design gradings with random routing.

### 5.2. Distribution of the number of engaged outlets of a given split.

The results for the distribution \( f(x) \) of the number of engaged outlets of a given split are shown in figs. 3-8 for a number of cases. For the calculation (full lines) use has been made of the interpolation formula with weight factor \( v_2 \). The simulation results are represented by broken lines. The two distributions appear to agree quite well on the whole; the calculated distribution tends to have a somewhat lower peak than the simulated distribution.

**Conclusion 2**

The interpolation formula with weight factor \( v_2 \) is quite suitable to calculate the distribution of the number of engaged outlets of a given split.

As a final remark it is pointed out that the normalizing constant in the interpolation formula appears to be close to 1 as a rule, in most of the investigated cases it is between 1 and 1.01, in the remaining cases between 1.01 and 1.02.

### 6. Graphs for design purposes

Figs. 9-12 show \( N \) plotted as a function of \( A \), for different values of \( K \), \( B \) and \( W \).

**Appendix**

### A.1. Interpolation type selected

The investigation into interpolation types is confined to linear and logarithmic-linear interpolation.

Simulation tests have shown that the latter method gives the most precise results. This has obviously been the main argument for the choice made. Apart from this, however, it is interesting to find out the "equations of state" corresponding to the approximations.

Let \( f(x) \) be an approximation for the distribution of the number of engaged outlets of a given split. If the quotient \( f(x)/f(x-1) \) is written in the form:

\[
\frac{f(x)}{f(x-1)} = \frac{A(x)}{x}
\]

then \( x \) represents the conditional release intensity in state \( [x] \) and \( A(x-1) \) represents an approximation for the conditional build-up intensity in state \( [x-1] \).

In other words, the approximation \( f(x) \) for the distribution corresponds to the approximation \( A(x) \) = \( f(x)/f(x-1) \).

For the conditional build-up intensity in state \( [x] \), the exact expression for the build-up intensity of the ideal grading may be derived from (1) and (2). This is a rather complicated formula, which we shall not give here.

The build-up intensity for the disintegrated grading is \( \frac{1}{N} \sum_{y=x-K}^{x} \frac{y!}{(y-x)!} \frac{(N-y-x)!}{N!} \cdot \frac{g(y)}{g(1+\delta)} \).

### A.2. Asymptotic behaviour of Erlang's interconnection formula

The distribution \( f(x) \) of the number of engaged outlets of a given split in an ideal grading is obtained from (1) and (2). If \( \omega_0 \) is the call congestion and \( \gamma \) the average total number of engaged outlets of the grading, then we have

\[
\gamma = A(1-\omega_0)
\]

We shall now consider the limit condition where \( N \to \infty \) and \( A \to N \) with a constant traffic offered per outlet, \( \rho_0 = A/N \).

We shall first prove that under the limit condition we also have \( \gamma \to \infty \).

Logarithmic linear interpolation of the distribution with weight factors \( v \) and \( (1-v) \) corresponds to logarithmic linear interpolation of the build-up intensities \( A(x) \) and \( A_D(x) \) with weight factors which depend on \( x \).

The call congestion of such a grading is

\[
\gamma = A(1-\omega_0)
\]

In connection with (A1) we have for a grading with \( K \geq 1 \):

\[
\gamma_A = \frac{\rho_0}{1+\delta} = N \cdot \frac{\rho_0}{1+\delta}
\]

which shows that for \( N \to \infty \) and \( \rho_0 > 0 \) we have \( \gamma \to \infty \).

We shall now consider the behaviour of the function \( g(y) \), i.e. the distribution of the total number of engaged outlets.

In a non-blocking system, which has Poisson traffic of intensity \( A \) offered to it, the ratio "standard deviation/average" is \( \sqrt{\frac{A}{N}} \).

In a loss system the value of this ratio is lower because the peaks are clipped by blocking. So for both systems the standard deviation to average ratio tends to zero for \( N \to \infty \).

For the grading we therefore have \( g(y) \to 0 \) for values of \( y \) which satisfy \( \frac{\sqrt{y-x}}{\delta} \), where \( \delta \) is an arbitrary small positive number.

Hence we have for (1):

\[
N \to \infty \quad f(x) = \lim_{N \to \infty} \frac{f(x)}{N!} 
\]

for all \( x > K \).

The build-up intensities for these extreme cases will further be represented by \( A(x) \) and \( A_D(x) \), respectively.

The approximations \( f(x) \) for a grading with interconnection number \( N \), obtained by linear interpolation and logarithmic linear interpolation are represented by \( f_{\text{LIN}}(x) \) and \( f_{\text{LOG}}(x) \), respectively.

In other words:

\[
f_{\text{LIN}}(x) = \sum_{y=x-K}^{x} \frac{y!}{(y-x)!} \frac{(N-y-x)!}{N!} \cdot \frac{g(y)}{g(1+\delta)} 
\]

and

\[
f_{\text{LOG}}(x) = \sum_{y=x-K}^{x} \frac{y!}{(y-x)!} \frac{(N-y-x)!}{N!} \cdot \frac{g(y)}{g(1+\delta)} 
\]

...
Since $W_1 = f_1(K)$ we have:

$$\lim_{N \to \infty} W_1 = \frac{\gamma_0 (1 - W_1^K)}{1 - W_1^K}.$$ 

From this formula we may calculate the asymptotic value of the traffic offered per line, $\gamma_0$, for a given $W_1$ and $K$.

References


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