CONGESTION STUDIES IN AN INCOMPLETE LINK SYSTEM

Bjørn Løken
Norwegian Computing Center
Oslo, Norway

ABSTRACT

The traffic handling capacity of a link system is heavily dependent on the hunting method. In this paper the influence of rotary hunting and stepwise hunting on the congestion in an incomplete link system is analyzed. It is found that these two hunting methods give a higher congestion than purely random hunting. The reason is that the occupations are not uniformly distributed in the system.

Of special interest is the point loss which is found to increase considerably compared with random hunting.

INTRODUCTION

The switching networks in modern telephone systems are often built as multistage link systems. Calculation of congestion in a link system is usually carried out by application of the principles laid down by Jacobsen [1]. He gives the following well known expression for the congestion:

\[ E = \sum_{p=0}^{m} G(p) \cdot H(m-p) \]  

(1)

The traffic, the system structure and the hunting method must be taken into account when selection of traffic distributions \( G \) and \( H \) is made.

In this paper we will discuss the influence of rotary hunting and stepwise hunting on the congestion in an incomplete link system. For the sake of simplicity, we will limit our discussion to a system with only two stages as seen in fig. 1. Fig. 1 shows only those devices which can be used in a connection between a certain A-inlet row and a certain route in C. The group of \( m'f \) B-links is divided into \( m \) subgroups of \( f \) links each.

\[
\begin{array}{cccccccc}
A & B & m'f \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
f & f & f \\
C & o & o & o & q \\
\end{array}
\]

Fig. 1. Part of an incomplete link system.

The two hunting methods under study are defined as [2]:

Rotary hunting. The hunting is done in a definite cyclical order from a randomly chosen starting point.

Stepwise hunting. Every group of \( f \) links is hunted through successively. The sequence between the \( m \) hunting steps is assumed to be random. Note that the method of hunting within each subgroup will have no influence on the congestion.

In the following discussion, we will consider the full availability group consisting of the B-stage.
2. ANALYSIS OF ROTARY HUNTING

The hunting is started by choosing a device at random. If the chosen device is already occupied, we search linearly forward in the group until a free device is found — searching circularly past the end of the group to the beginning, if necessary. We say that a collision has occurred when the starting point is already occupied. The occupations tend to cluster together after a few collisions have occurred, giving long chains of occupied devices. The reason is that a free device following an occupied device (or devices) has a greater probability of being occupied than a free device surrounded by other free devices (see example below). The occupations are accordingly not spread uniformly in the system.

Clustering of occupations implies that the probability of congestion in a certain group will be higher than in the case of purely random hunting. This is easily seen because, at an arbitrarily chosen point in time, the occupations are concentrated in some groups while others are almost free from occupations. The loading in the system fluctuates between the subgroups in some sense in the direction of hunting.

Example.

device no 1 2 3 4 5 6 7 8 9 10

Fig. 2. A full availability group consisting of 10 devices. Filled devices symbolize occupied devices.

Consider the full availability group in fig. 2. The direction of hunting is from left to right. There are three free devices — no 3, 8 and 9. They have a probability of being occupied given by 0.4, 0.5 and 0.1, respectively.

Of special interest is the loss, or point loss, occurring when a particular inlet in the first stage (A) is to be connected to a particular outlet in the last stage (C). The point loss is equal to the congestion in a subgroup.

Application of mathematical tools developed for purely random hunting should accordingly be used with great caution when rotary hunting is used.

3. ANALYSIS OF STEPWISE HUNTING

In this case the hunting is done in two steps; — 1) selection of a subgroup and — 2) selection of a free device within the chosen subgroup. The nature of the hunting is seen to distribute call attempts uniformly across the m subgroups without regard to the traffic situation in the stage. The occupations are not uniformly distributed in the system.

We now turn to the problem of calculating the congestion by Jacobaeus' method on the assumption of stepwise hunting.

The traffic distribution in the B-stage is usually chosen according to the ratio between n and m f (Erlang-, Engset- or Bernoulli distribution). We want to calculate the probability distribution for the number of completely occupied subgroups. As in the case of purely random hunting among the m·f B-links, this distribution cannot easily be derived from the above mentioned distributions. Instead we shall take advantage of the given hunting method to derive the state distribution for each individual subgroup and from these deduce the wanted distribution.

When stepwise hunting is used we may state the following assumptions about the system:

a) Interaction between different subgroups is limited to situations where calls are overflowing the first chosen subgroup due to lack of free connection possibilities to the route.

b) Overflow traffic from the subgroups is equally distributed over all subgroups. This implies that it will be sufficient for our purpose to regard each subgroup as if it were acting independently. Further, on the assumption that the total traffic A offered from the n inlets is much greater than f, we may assume that each subgroup is offered a Poisson traffic A f from the n inlets.

From these assumptions it follows that the distribution in each subgroup is approximately given by an Erlang distribution. The congestion in the subgroup is then given by Erlang's first formula E f (A f ).

Since the subgroups are assumed to be independent of each other and to have the same probability of congestion, it is reasonable to assume that the Bernoulli distribution

\[ G(p) = \binom{m}{p} E_f(A_f) p^p (1-E_f(A_f))^{m-p} \]  

is a valid description of the wanted distribution.

Assuming Erlang distribution in the route, random hunting and traffic B offered to the route, we can obtain the total congestion (from (3))

\[ E = \sum_{p=0}^{m} \binom{m}{p} E_f(A_f) p^p (1-E_f(A_f))^{m-p} \]  

\[ E = \frac{E_f(A_f)}{p} \]  

In calculating the point loss in our system, we only regard the f B-links which may be used to connect to the given outlet. The congestion will then be approximately given by

\[ E = E_f(A_f) \]  

This is a rather surprising result. We have obtained the point loss by regarding the system as a full availability group consisting of f devices offered a Poisson traffic A f .

An estimate of the value of A f is given by

\[ A_f = \frac{\frac{f}{m} \cdot (1+E_f(A_f))}{m} \]  

4. EXAMPLES

The accuracy of the approximate expressions derived in section 3 are checked against simulation experiments. The results obtained from a number of simulations on a full availability group consisting of the A- and B-stage are given in tables below. The C-stage in the link system is assumed to consist of an unlimited number of outlets. The system is offered both traffic generated according to a Bernoulli distribution and to a Poisson distribution.
Four different hunting methods are examined in the simulation experiments.

Case 1: Purely random hunting.
Case 2: Rotary hunting.
Case 3: Stepwise hunting obtained by the use of purely random hunting between the m groups.
Case 4: Stepwise hunting obtained by the use of rotary hunting between the m groups.

The simulation results are, in addition to the equations in section 2, also compared with the theoretical probability distributions for the number of completely occupied subgroups in the case of purely random hunting.

On the assumption of Bernoulli distribution in the group we have

\[ G(p) = \binom{m}{f} p^f (1-p)^{m-f} \]  
\[ a = \frac{A}{m^f} \]

The point loss is given by

\[ E = a^f \]  
\[ (7) \]

On the assumption of Erlang distribution in the group we have [3]

\[ G(p) = \binom{m}{f} p^f (1-p)^{m-f} \cdot |H(p; x = f, y = f)| \]

where

\[ H(x) = \sum_{i=x}^{\infty} \frac{i!}{m^f} \frac{A^j}{j!} \]

\[ P(i) = \frac{m^f}{\sum_{j=0}^{\infty} \frac{A^j}{j!}} \]

\[ (8) \]

The probability that x devices, specified in advance, are all occupied. The point loss is given by

\[ E = \frac{1}{m} \cdot \sum_{p=1}^{m} p \cdot G(p) \]  
\[ (9) \]

Table 1. Theoretical and simulated results, n=30, m=6, f=5 and A=15.4 Erl. The offered traffic is generated according to a Bernoulli distribution.

<table>
<thead>
<tr>
<th>State</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1:</td>
<td>0.808</td>
<td>0.171</td>
<td>0.021</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(6):</td>
<td>0.805</td>
<td>0.178</td>
<td>0.016</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 2:</td>
<td>0.659</td>
<td>0.276</td>
<td>0.060</td>
<td>0.005</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(7):</td>
<td>0.564</td>
<td>0.359</td>
<td>0.070</td>
<td>0.008</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 3:</td>
<td>0.569</td>
<td>0.337</td>
<td>0.083</td>
<td>0.011</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(8):</td>
<td>0.574</td>
<td>0.339</td>
<td>0.079</td>
<td>0.008</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 4:</td>
<td>0.651</td>
<td>0.290</td>
<td>0.054</td>
<td>0.005</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(9):</td>
<td>0.661</td>
<td>0.259</td>
<td>0.067</td>
<td>0.011</td>
<td>0.002</td>
<td>0.001</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 2. Theoretical and simulated results, n=30, m=6, f=5 and A=10.3 Erl. The offered traffic is generated according to a Bernoulli distribution.

<table>
<thead>
<tr>
<th>State</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1:</td>
<td>0.834</td>
<td>0.143</td>
<td>0.021</td>
<td>0.002</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(6):</td>
<td>0.836</td>
<td>0.138</td>
<td>0.021</td>
<td>0.004</td>
<td>0.001</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Case 2:</td>
<td>0.868</td>
<td>0.253</td>
<td>0.051</td>
<td>0.011</td>
<td>0.001</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>eq(7):</td>
<td>0.859</td>
<td>0.266</td>
<td>0.069</td>
<td>0.010</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Case 3:</td>
<td>0.655</td>
<td>0.290</td>
<td>0.054</td>
<td>0.005</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>eq(8):</td>
<td>0.651</td>
<td>0.290</td>
<td>0.054</td>
<td>0.005</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>Case 4:</td>
<td>0.661</td>
<td>0.259</td>
<td>0.067</td>
<td>0.011</td>
<td>0.002</td>
<td>0.001</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 3. Theoretical and simulated results, n=27, m=6, f=5 and A=14.1 Erl. The offered traffic is generated according to a Poisson distribution.

<table>
<thead>
<tr>
<th>State</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1:</td>
<td>0.006</td>
<td>0.006</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(6):</td>
<td>0.006</td>
<td>0.003</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 2:</td>
<td>0.002</td>
<td>0.005</td>
<td>0.004</td>
<td>0.004</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(7):</td>
<td>0.002</td>
<td>0.005</td>
<td>0.004</td>
<td>0.004</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 3:</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(8):</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 4:</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.017</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4. Theoretical and simulated results, n=30, m=6, f=5 and A=10.3 Erl. The offered traffic is generated according to a Poisson distribution.

<table>
<thead>
<tr>
<th>State</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1:</td>
<td>0.038</td>
<td>0.003</td>
<td>0.003</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(6):</td>
<td>0.033</td>
<td>0.001</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 2:</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(7):</td>
<td>0.002</td>
<td>0.001</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 3:</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>eq(8):</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Case 4:</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Note. The simulated state distributions over the m f devices are in example 1 and 2 a Bernoulli distribution and in example 3 and 4 an Erlang distribution. This is in accordance with the assumptions, of course.

The agreement between equation (2) and (4) and the simulation results is seen to be good both in the case of Bernoulli input and Poisson input. Both rotary hunting and stepwise hunting give less favourable state distributions than in the ideal case of purely random hunting.
CONCLUSION

It would appear from this congestion study that it is extremely important to take the hunting method into account when congestion in link system is calculated. This is especially true when individual selection or routes with high losses are involved.

REFERENCES


   L.M. Ericsson.


ACKNOWLEDGEMENT

The author wishes to thank Mr. Tryggev Thornc at Standard Telefon- og Kabelfabrik A/S, Oslo, for his help and advice in the preparation of this paper.