SOME BINARY CONNECTING NETWORKS AND THEIR TRAFFIC CHARACTERISTICS

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ABSTRACT

Binary connecting networks with switching units having two inputs, two outputs and two states (σ-elements) have recently become an object of intensive studies. These networks are mainly known as capable of operating in simultaneous connection mode when all necessary connections are established simultaneously. In this case a two-sided full-available non-blocking network with N inputs and N outputs requires approximately $N \cdot \log_2 N$ σ-elements. The connection process, however, follows a specific, sometimes very involved procedure. One of these procedures is discussed in this paper. Less known is the fact that the network under consideration is capable of operating in ordinary connection mode when separate inputs are connected to outputs and disconnected from them at random time points. It is shown theoretically and experimentally (by way of computer simulation) that in the case of moderate traffic (about 0.5 erlang per input or less) and an appropriate structural scheme an infinitely small blocking probability can be obtained for the system at sufficiently large N. Of some interest is also a switching unit with 2 inputs, 2 outputs and 3 states (σ-element) which may prove useful in the synthesis of one-sided connecting networks; in some cases it can be used to make system structures consistent with pre-specified traffic characteristics.

1. INTRODUCTION

To date, switching technology has accumulated a vast experience in the designing and application of various switching devices (relays, selectors, crossbar connectors, reed-relay devices, etc.). These devices are widely used in numerous telephone exchange systems and other switching facilities. A relay-like bistable connecting unit is of interest due to its simplicity and the possibility to treat it as a 2x2 cross-bar switching unit. The capabilities of such binary elements, or σ-elements by A.E. Joel's definition [1], are not yet used to the full extent. This paper is aimed at describing some switching systems based on binary elements and analysing traffic characteristics of these systems.

The paper deals with the two principal modes of operation of the switching systems: simultaneous switching mode when all connections are established simultaneously by a certain rule, and ordinary switching mode when separate inputs are connected to and disconnected from outputs at random time points.

2. STRUCTURES OF BINARY CONNECTING SYSTEMS

A switching unit with 2 inputs and 2 outputs is given diagrammatically in Fig.1a. The switching unit contains 4 crosspoints, each of which can be either closed or open. Therefore theoretically the switching unit can be in one of the 16 possible states, twelve of which are shown in Fig.1, where black circles indicate closed crosspoints. The remaining 4 states with 3 closed points (in various possible combinations) are equivalent to the state 11 and consequently not shown in Fig.1. Depending on the chosen design of a switching device, some of the states indicated in Fig.1 or all of them can be used. The most interesting case is that of a 2x2 switching unit having only two states, namely 1j and 1k. Designating the first state by "0" and the second state by "1" we obtain a binary element which is symbolically shown in its two states in Fig.2. The application
of such elements is largely attributable to their use as basic elements of multistage systems. A regular stage-linking technique for multistage connecting systems is described by V. E. Beneš in [2]. As applied to a binary connecting system, the technique can be described in the following way (Fig. 3). Let us have two stages of binary switching units with an arbitrary but equal number of switching units in each stage and let \( i \) (where \( i = 1, \ldots, N \)) be the ordinal number of an output belonging to the preceding stage and \( j \) (where \( j = 1, \ldots, N \)) be the ordinal number of an input belonging to the succeeding stage. Then the number \( j \) of the input which is to be connected to an output \( i \) is defined by the following formula:

\[
\begin{align*}
& j = 2i - 1, \quad 0 < i \leq N/2, \\
& j = 2i - N, \quad N/2 < i \leq N.
\end{align*}
\]

We shall consider only those networks whose stages are linked in accordance with the regular law given by formula (1) or the law symmetrical to the regular one. If inputs and outputs of the system are numbered from top to bottom, a permutation of inputs at the outputs will be obtained for each state of the system. It can be also stated that each network involves a certain set of permutations. Let us discuss several networks.

2.1. \( N = 2^s \), where \( s \) is the number of stages in the system all stages being connected in accordance with law (1). This is the minimum network which can be characterized as full-available in the initial state (when there is not a single connection in the network). However, even the first connection reduces the availability of the input conjugated with the engaged one (i.e., connected to the same switching unit) by one half due to the blocking situations occurring in the network. Consequently, the system of this type is capable of operating in ordinary switching mode only if the traffic is rather low and losses are permitted. In this case the loss probability for the system can be in principle calculated if the requests for connections between each input and each output are assumed to form a Poisson flow with uniform intensity and the holding time is assumed to be exponentially distributed. The problem is simplified due to the fact that there exists only one connecting route between each input and output of the system. As there is no need to choose connecting routes, formulation of stochastic Markovian process equations and their solution are somewhat simplified. The calculation of losses even for small-size systems requires, of course, the assistance of quick-acting computers.

2.2. Let us add to the system considered in Section 2.1., a \((S+1)\)-th stage which is connected to the system in accordance with law (1). If all binary elements of the system obtained are in the state "0", the system has an identical permutation accomplished at the output, i.e. inputs and outputs with the same ordinal numbers are connected.

2.3. Let the switches of the system 2.2. be able to produce two more states, 1b and 1l, in addition to the states 1j and 1k. If this system is considered without the first stage, the resultant \( s \)-stage system will have the following permutation at the input:

\[
1, \frac{N}{2} + 1, 2, \frac{N}{2} + 2, \ldots, \frac{N}{2}, N.
\]

The system enables disjoint batch connections of each input to several (from 1 up to \( N \)) outputs provided the connecting process is determined in space, that is inputs to be connected are taken in increasing order of numbers of permutation (2) while outputs are taken in the order of identical permutation numbers (i.e. from top to bottom). In other words, input \( i \) can be connected to several first outputs, input \( \frac{N}{2} + 1 \) to the next free outputs and so on: in so doing some inputs with higher numbers of permutation (2) may remain unconnected.

2.4. \( N = 2^c \); the first \( 2^c \) stages are connected in accordance with the law (1), the following \( 2^{c-1} \) stages are connected symmetrically to the previous stages with respect to the stage numbered \( 2^{c-1} \). This system was described by V. E. Beneš [2]. Among other systems with connecting routes passing through each stage, this is the minimal system having all possible permutations realized. An example of this system is given in Fig. 5. The system under consideration is a full-available non-blocking system operating in simultaneous connection mode. At the same time this system is a non-blocking ordinary connecting system with possible reroutings. It can be easily seen that the two definitions are equivalent. Indeed, if a system enables simultaneous switching of any set of connections, it will be able to operate in the ordinary connection mode with reroutings: blocking situations in establishing a new connection can be avoided since it is possible to reroute simultaneously all the existent connections along with the new one. And conversely, should it be a non-blocking system operating in ordinary
switching mode with reroutings, any given set of connections may be produced, if connections are made one after another using reroutings, if required.

2.5. \( N = 2^2 \); all stages are connected in accordance with law (1). This network differs from the previous one only in the stage-connecting technique; however, this difference is appreciable when the system operating in the ordinary connection mode is used. The system described in Section 2.4, allows all possible permutations, but in fact any one of these permutations is only possible if a special route-selecting procedure is used. The system operated in the ordinary connection mode with connections occurring one after another at random time points involves many blocking situations whose probability is rather high with increased traffic. The system under consideration in which all stages are linked from formula (1) is characterized by the optimum connecting graph which minimizes the blocking probability [3]. Therefore, in the analysis of binary systems operating in the ordinary connection mode it is important to determine the blocking probability just for this system. This problem is discussed in Section 5 of the present paper.

2.6. Concentrator. A concentration (or expansion) network is a network with some partially engaged binary elements, i.e. with only one input (or output) used. The simplest example of such a network is a contact tree with \( N = 2^8 \) inputs and one output (and vice versa). This tree consists of \( s \) stages and each binary element has one unengaged output; therefore the number of switching units is reduced by one half for each consecutive stage until it reaches 1 for the stage numbered \( s \). When adding a second output, it is impossible to connect it directly to the given output switch because of blocking situations. The contact tree in this particular case can be built up only from the inputs end. Therefore, the second output must be included in a separate tree with \( s-1 \) stages, using the first stage as a coupling between the two trees. In the case of four outputs, the first two stages will be common for both trees, etc.

The considered examples do not encompass all potentialities of the binary connecting systems. Several modifications of simultaneously connecting networks with partially used connecting routes in each stage of the network (networks with incomplete stages) [4], and sorting networks discussed in [4, 5], among other things, are entirely omitted from this paper.

3. SOME REMARKS ON BINARY ELEMENTS REALIZATION

A binary connecting system in a broad sense is a connecting system based on devices with two outputs available for each input (switching device with structural parameter 2). Depending on the practical realization, the broad class of networks satisfying this definition can be divided into several subclasses. For example, all the structures described in Section 2 with the exception of 2.3, can be, generally speaking, produced using electromagnetic relay switching contacts whose connection diagram is given in Fig. 4a.

Of some interest may be special binary switches, some of which are considered below.

3.1. In the process of operation of binary connecting systems constructed from relay contacts as shown in Fig. 4a, it should be often remembered through which binary element the connections pass, in order to avoid breaking previously connected circuits during switching. Therefore each bistable element controlling the switching contacts is to be provided with another bistable element memorizing the established connections. One of the possible methods is given in Fig. 4b [6].

The top of this diagram shows supervisory circuits performing selection and route-retentive functions. The lower part of the diagram shows operating circuits (single-channel variant) which are used for information transmission. Letters \( P_1 \) and \( P_2 \) stand for two operating relay windings which control the contacts \( p_1, p_2 \) of the operating circuits, letters \( K_1 \) and \( K_2 \) stand for two windings of the supervisory relay fixing the binary elements in the "ON" state. Intersecting routes are blocked by the following contacts of these relays: \( K_1, K_2, p_1, p_2 \).

3.2. To construct a connecting network with batch switchings (see Section 2.3.), a binary element may be required which makes possible the states \( 1h, 1i, 1j \) and \( 1k \) given in Fig. 1. This element can be obtained, for example, by using contacts of two relays as shown in Fig. 4c. It is easy to see that if both relays are de-energized, the state \( 1i \) is produced; when relay \( A \) is drawn in, the state \( 1h \) is obtained; when relay \( B \) is drawn in, the state \( 1k \) is produced; when both relays are drawn in, the state \( 1j \) is obtained.

3.3. Alongside with the binary connecting element with 2 inputs, 2 outputs and 2 states shown in Fig. 4a, a similar connecting element with 3 states may be of practical importance. Apart from the states "Input 1 - output 1, input 2 - output 2" and "Input 1 - output 2, input 2 - output 1", the element can produce
This paper presents only the control procedure for a symmetric system with complete stages, and to a binary element, the number of which is given in Fig. 5. The idea of this procedure is set forth in [8], where it is used to prove full-availability of the universal automation switching unit. Let us number the network inputs arbitrarily using a permutation of numbers \( i_1, i_2, \ldots, i_N \), then let us number the network outputs using a permutation of \( j_1, j_2, \ldots, j_N \) so that the input and output to be connected have identical numbers. Any two numbers belonging to inputs of the same binary switching unit will be called conjugate inputs following [8], and any two numbers belonging to outputs of the same binary switching unit will be called corresponding outputs. When constructing a binary system it is evident that at the output of the first stage switch conjugate numbers can be either transposed or not. Similarly corresponding numbers at the input of the last stage can be arranged in the same way as at the output or transposed. Let us construct a sequence of numbers beginning from \( i_1 \), in such a way that every even number is a number corresponding to the preceding member of the sequence, while every odd number is conjugate. These conditions define the sequence unambiguously as follows:

\[
\begin{align*}
&i_1 = \underbrace{i_{11}}_{u} \quad i_2 = \underbrace{i_{12}}_{l} \quad i_3 = \underbrace{i_{13}}_{u} \quad i_5 = \underbrace{i_{15}}_{l} \quad \ldots \quad i_{2k+1} = \underbrace{i_{12k+1}}_{u} \\
&\begin{array}{c}
i_{12} = j_{12} \quad j_3 = j_{13} \quad j_4 = j_{14} \quad j_{2k+2} = j_{12k+2} \\
u \quad l \quad l \quad u
\end{array}
\end{align*}
\]

The sequence is terminated by the number starting from which the sequence members begin repeating themselves. If a number \( i_m \) is not contained in the sequence, another sequence beginning from \( i_m \) is constructed in the same way. It is easy to see that different sequences have no common members. If these sequences do not contain all the numbers \( i_1, i_2, \ldots, i_N \) a third sequence is constructed and so on until all the numbers \( i_1, i_2, \ldots, i_N \) are included in some sequence.

Let us label each member of the sequence by the letters "u" or "l" depending on the switch output ("upper" or "lower") to which this number belongs. It is obvious that each pair of corresponding numbers contains both the upper and lower outputs. The following procedure allows to determine the states of the switching units of the last stage: transposition must be performed for those switches of the last stage to which the letter sequence "l" - "u" corresponds, the switches conforming to the sequence "u" - "l" remain in the initial state. By so doing, the permutation obtained for the last but one stage is such that any two conjugate numbers are placed in the different parts of the permutation. Let us consider two conjugate numbers \( i_1 \) and \( i_{1+1} \) belonging to the permutation given at the input. The same numbers are placed in the different parts of the permutation of the last but one stage. If \( i_{1+1} \) belongs to the upper part of the permutation of the last but one stage, the indicated

\[
\begin{align*}
&i_1 = i_{11} \quad i_2 = i_{12} \quad i_3 = i_{13} \quad i_5 = \cdots \quad i_{2k+1} = i_{12k+1} \\
&\begin{array}{c}
i_{12} = j_{12} \quad j_3 = j_{13} \quad j_4 = j_{14} \quad j_{2k+2} = j_{12k+2} \\
u \quad l \quad l \quad u
\end{array}
\end{align*}
\]
conjugate numbers are to be transposed. In this way the states of the first stage switching units are determined. The permutations of the second and the last but one stages are such that their upper and lower parts contain the same numbers. As a result, we face a problem similar to the initial one, but with permutations of length \( N \). To determine the states of all switching units up to and including the central stage switches, the above procedure is to be repeated several times.

As the network inputs and outputs are enumerated by ordinal numbers in practical applications, determination of the corresponding numbers is somewhat specific. To illustrate the proposed procedure, let us consider the example given in Fig. 5, where inputs and outputs are numbered from top to bottom from 1 to 8. The list of required connections can be defined, for example, as a permutation indicating the numbers of outputs to which inputs are to be connected in the order of their natural numbers. Let the sequence 4, 6, 8, 1, 3, 2, 7, 5 be a permutation of this kind (it is shown in brackets in Fig. 5 against the numbers of the corresponding inputs). Another way to define the same list of requests is shown in Fig. 5 at the output of the network, where the permutation of inputs which are to be connected to outputs numbered in the order of natural numbers is given.

To solve the problem, one can use one of the bracketed permutations. Let us, for example, consider the last one.

The following pairs are conjugate numbers:
- 1 and 2, 3 and 4, 5 and 6, 7 and 8,
- while 4 and 6, 5 and 1, 8 and 2, 7 and 3
are corresponding numbers.

Let us construct a sequence beginning from 1:

\[
\begin{align*}
1, & \quad 5, \quad 6, \quad 4, \quad 3, \quad 7, \quad 8, \quad 2, \quad 1, \\
\text{u u l u l u u u l}
\end{align*}
\]

If this sequence does not contain certain numbers, one of them is taken and a new sequence is formed, and so on until all numbers occur in one or another of the sequences.

It can be easily seen that the first three sequences due to the alternation of the "l" - "u" pairs require crossing connections in corresponding switching units of the last stage.

Let us now consider the conjugate numbers. Considering the penultimate stage outputs we find that number 1 from the output permutation is transposed to the upper part of the network (shown by thin lines) while number 2 is transposed to the lower part of the network (shown by thick lines). The state of the first switching unit of the first stage fits this distribution and thus remains unchanged. The same is true for numbers 3 and 4 belonging to the second switching unit of the first stage. The position of number 5 at the output of the penultimate stage is such that the number is transposed to the lower subnetwork, while number 6 goes to the upper subnetwork. To keep this distribution unchanged, the conjugate numbers at the output of the third switch of the first stage are to be transposed. The same is valid for numbers 7 and 8.

It should be noted that to determine the states of switches of the first stage it is not necessary to analyse the conjugate numbers distribution at the output of the penultimate stage; it is enough to repeat the construction of a sequence for the bracketed permutation given against inputs numbers. In this case the pairs 1 and 2, 3 and 4, 5 and 6, 7 and 8 remain conjugate numbers pairs 4 and 6, 8 and 1, 3 and 2, 7 and 5 are corresponding numbers.

Let us construct a sequence beginning from 4 (this is an important condition to be met to avoid a mistake in "sign" when defining the switches' states):

\[
\begin{align*}
4, & \quad 6, \quad 5, \quad 7, \quad 8, \quad 1, \quad 2, \quad 3, \quad 4, \\
\text{u u l u l u u u l}
\end{align*}
\]

As may be seen from this sequence, the alternation of the "l" - "u" pair for the pairs (5, 7) and (2, 3) requires reswitching of the corresponding switches.

The procedure results in transforming the permutation of length 8

\[
\begin{align*}
1, & \quad 2, \quad 3, \quad 4, \quad 5, \quad 6, \quad 7, \quad 8, \\
4, & \quad 6, \quad 8, \quad 1, \quad 3, \quad 2, \quad 7, \quad 5
\end{align*}
\]

into two permutations of length 4:

\[
\begin{align*}
1, & \quad 6, \quad 3, \quad 8, \quad 2, \quad 5, \quad 4, \quad 7, \\
4, & \quad 2, \quad 8, \quad 5, \quad 6, \quad 3, \quad 1, \quad 7
\end{align*}
\]

for which the procedure is repeated.

5. TRAFFIC CHARACTERISTICS OF BINARY ORDINARY CONNECTION SYSTEMS

Fig. 6 shows a binary connecting system with minimum blocking probability. However, the blocking probability analysis is far from being a simple problem. A probabilistic graph model proposed by C.Y. Lee [9] has played an important role in the development of blocking probability.
calculation techniques for connecting systems with complex structures. The problem is considerably simplified if consistent networks having identical switching units in all stages are used. Proceeding from the Lee model, N. Ikeno [10] proved the existence of consistent connecting systems constructed from n x n switching units with the blocking probability tending to the value

$$B = 2 x_0 - x_0^2,$$

where $$x_0$$ - the minimum root of the equation

$$x_0 = (p+q x_0)^2$$

within the interval (0, 1), p - the probability that a graph edge is engaged (i.e. average traffic per link), and q=1-p, if the size of the network increases and the traffic per link is moderate. The importance of formula (3) in the teletraffic theory is related to the possibility of substantiation of the Lee model.

5.1. The Lee Model and its Applications

The probabilistic graph describes the connecting network structure with sufficient adequacy. The difficulty is however in the choice of a joint distribution of the probabilities that graph edges are engaged which would take account of the actual distribution of this value in real connecting networks. The Lee model treats this probabilities as independent. This assumption allows to considerably simplify calculations but actual probabilistic features of the system are not taken into account. It is obvious that the existence of an idle input in any switching unit implies the existence of at least one idle output. The model that takes into account this fact is worked out by I.V. Koverninsky. The Koverninsky model treats the joint distribution of the probabilities that edges outcoming from one vertex are engaged as depending solely on whether the edge entering this vertex is idle or not and being independent of the states of other edges. It is clear by intuition that the accuracy of the Lee model increases with increased the size of switching units forming the system. The case where n = 2 is the most unfavourable as regards the application of the Lee model. Therefore, to analyse the blocking probability of the system under consideration we shall use the Koverninsky model, whose essence becomes clear if the basic concepts of the Ikeno's work [10] are briefly discussed.

The latter deals with a class of multistage connecting networks where a set of interconnecting paths between any arbitrary "input-output" pair can be interpreted as a graph consisting of a pair of simple trees with outputs connected in a random manner. A t-stage simple tree (Fig. 7) corresponds in this case to the s-stage network (as=t). Then, the blocking probability of the network can be estimated from that of the graph, which can be easily calculated if we confront the idle outputs of both trees. Thus, the problem amounts to the determination of the mean number of idle outputs in a simple tree, given the probability p that an edge is engaged.

The Lee model graph is based on the assumption that edges are engaged independently, therefore it can be assumed that the edge engagement probability for edges outcoming from the same vertex are given by the binomial distribution function

$$P(k) = \binom{n}{k} \cdot p^k \cdot q^{n-k}.$$  (4)

To determine the edge engagement probability at the output of the t-th stage, the method of generating functions can be used to advantage. The generating function of distribution (4) is as follows:

$$G_t(x) = \sum_{k=0}^{n} P_t(k) \cdot x^k = (p+qx)^n.$$  (5)

The distribution function $$G_t(k)$$ for the output of the t-th stage is arrived at by t-fold iteration (5), as a result of which the expectation of the idle edges number at the output of the t-th stage is found as follows:

$$E(t) = G_t(1) = n^t q^t.$$  (6)

Confronting the idle outputs of both trees, the following formula for the blocking probability is obtained:

$$B = 1 - (q^2n)^{t-1},$$  (7)

which tends to 1 when $$p > \frac{n - \sqrt{n}}{n}$$ and $$t \to \infty$$. When $$p \leq \frac{n - \sqrt{n}}{n}$$, formula (3) is derived.

5.2. The Koverninsky Model

As may be inferred from Section 5.1., the Lee model does not take into account the specific probabilistic features of operation of actual connecting networks. Considering that one input of the switching unit is idle and the rest are engaged in a mutually independent manner with the probability p, the probability that the switching unit has k idle outputs and n-k engaged outputs is

$$P_2(k) = \binom{n-k}{k} \cdot q^{k-1} p^{n-k}.$$  

The generating function of the conditional probability distribution for the number of idle edges outcoming from the same vertex will be as follows:

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\[ G_2(x) = \sum_{k=0}^{n} P_2(k) \cdot x^k = x(p + qx)^{n-1}. \]  

To generalize the two above cases, the following model can be used. The assumptions are that the distribution of the edge engagement probabilities for edges outgoing from one vertex (provided the latter has an idle incoming edge) has an arbitrary generating function \( G(x) \) and edges having no common vertex are engaged independently of each other. Proceeding from these assumptions Koverninsky deduced the following bounds for the minimum blocking probability \( B_{\text{min}} \):

\[ B_{\text{min}} > (1 - \frac{\left( G'(1) \right)^2}{n})^{t-1}, \]

where with \( G_1'(1) < \sqrt{n} \), and \( t \to \infty \) the probability \( B_{\text{min}} \to 1 \).

In the case of the Lee model this leads to formula (7) and for the Koverninsky model the following is obtained:

\[ G_2'(1) = n(1-p)+p, \]

from which follows \( p > \frac{n-\sqrt{n}}{n-1} \) and \( B_{\text{min}} < 1 - \left( \frac{(n-\sqrt{n})^2}{n} \right)^{t-1} \to 1 \).

Consequently, if the mean number of idle edges outgoing from the same vertex (\( G'(1) \)) is less than \( \sqrt{n} \), the blocking probability approaches 1 as \( t \) increases.

If the mean number of idle edges outgoing from the same vertex is \( \sqrt{n} < G'(1) < n \), the blocking probability tends to the value

\[ \lim_{t \to \infty} B = 2x_0 - x_0^2. \]

Since \( G(x) \) is a polynomial with non-negative coefficients, all derivatives of the function within the interval \((0, 1)\) are non-negative and its diagram is a monotonically increasing curve with a downward convexity. Furthermore, for any \( x \) \((0 < x < 1)\)

\[ \lim_{t \to \infty} G(G \ldots G(x)) = x_0, \]  
\( x_0 \) is the minimum root of the equation \( x_0 = G(x_0) \) within the interval \((0, 1)\).

For the Lee model, \( x_0 \) is the minimum root of the equation \( x_0 = (p+q \cdot x_0)^{n-1} \), which, when

\[ p < \frac{n-\sqrt{n}}{n} \]  
\( (\text{following from the condition} \ G'(1) > \sqrt{n}) \), leads to formula (3).

For the Koverninsky model, the minimum root of the equation \( x_0 = (p + q \cdot x_0)^{n-1} \) is equal to zero.

\[ \lim_{t \to \infty} B = 0. \]

In the particular case where \( n = 2 \) for the Lee model, we have \( x_0 = \frac{p^2}{q} \) and, consequently,

\[ \lim_{t \to \infty} B = \frac{E}{q} (2 - \frac{p}{q}) \]  
when \( p < \frac{2\sqrt{2}}{2} \)

and

\[ \lim_{t \to \infty} B = 1 \]  
when \( p > \frac{2\sqrt{2}}{2} \)

For the Koverninsky model, we have the following in this case:

\[ \lim_{t \to \infty} B = 0 \]  
when \( p < \frac{2\sqrt{2}}{2} \)

and

\[ \lim_{t \to \infty} B = 1 \]  
when \( p > \frac{2\sqrt{2}}{2} \)

The corresponding curves are shown in Fig. 8. The above holds for the majority of networks belonging to the class investigated. However, it has been shown in [3] that the network illustrated in Fig. 6 has an optimum graph of connecting routes for which the blocking probability assumes the minimum value. The worst network of the class in terms of the blocking probability is given in Fig. 5.

5.3. Preferable Sizes of Switching Units

The diagrams shown in Fig. 8 can be plotted as well for other values of the structural parameter \( n \). It can be readily seen that with increasing \( n \) the permissible traffic per link increases while the difference between the critical values of the two models decreases. However, unduly increasing the structural parameter results in growing number of crosspoints required.

To analyze the relationship between the number of crosspoints required and the structural parameter, let us first consider that part of the network which corresponds to the tree illustrated in Fig. 7. The total amount of crosspoints in this network is \( T = N \cdot m \cdot t \), and the mean value of the traffic carried is \( A = p \cdot N \) erlang. It means that the network contains on the average \( N - A \) or \( N \) idle inputs where new calls occur. On the other hand, the average number of outputs available for a given input is \( G'(t) \) and, in any case, not less than \((1-B)N-A\). To connect an input to the required output, \( \log(1-B)(N-A) \) selections are to be performed. The total number of selections for \( A \) connections is \( \log(1-B)(N)N(A) \).

Denoting the mean number of crosspoints per selection by \( C \), we have

\[ C = \frac{T}{A \log(1-B)(N-A)} = \frac{T}{A \log(1-B)(\frac{A}{p} - 1) A \log A} \]

It is convenient to use this formula for estimat-
ing the relationship between the number of cross-points per input and the value of the structural parameter. Furthermore, one can prove [10] that this relationship holds both for the part of the network and for the whole network.

For the Lee model, in accordance with formula (6) we have \((1-B)N-A \leq n^p q^t\), from which

\[
C = \frac{n \cdot n \cdot \log n q}{p \cdot n \cdot \log n q}.
\]

Analysing this relation N. Ikeno [10] found the optimum value of the structural parameter \(n\) to be 5.

For the Koveminsky model

\((1-B)(N-A) \leq (nq+p)^t\),

from which it follows that

\[
C = \frac{n}{p \cdot \log (nq+p)}.
\]

Fig. 9 illustrates the curves plotted by formula (9). These curves show that when the values of the structural parameters vary in the range from 2 to 8, the number of crosspoints per input changes insignificantly, the optimal value being \(n=4\). However, since the curves are plotted asymptotically by formula (9), they are, strictly speaking, true only for \(t \to \infty\). The difference between the theoretical values given in Fig. 9 and the actual values depends on the number of stages. Thus, asymptotically, the number of crosspoints per input for \(n=2\) and \(n=8\) is almost the same. However, for a practical network with 8 stages and dozens of thousands of lines, it is hardly probable that the minimum number of crosspoints per input will amount to the value determined in Fig. 9 for the network based on 8 x 8 switches. As far as binary connecting networks are concerned, the asymptotic minimum of the value \(C\) can be attained in an actual network more easily since binary networks of medium and large scale configurations contain several dozens of stages.

5.4. Simulation Results

Traffic characteristics of a binary connecting network operating in ordinary connection mode were checked experimentally using simulation techniques. In so doing, the number of inputs/outputs \(N\) was assumed to vary in the range from 4 to 252 and varying values of traffic per edge were taken [17]. The resulting blocking probabilities versus \(N\) for different values of traffic per link are shown in Fig. 10. It is seen from this figure, for example, that the curves corresponding to traffic per link below 0.25 erlang cross the ordinate 0.005 with \(N\) less than 1,000. Proceeding from this fact it can be assumed that the number of binary elements per erlang corresponds to the amount of elements per each 4 inputs and outputs. These calculations show that the number of crosspoints per input is very close to the theoretical minimum value even for \(N = 1,000\).

6. SOME MODIFICATIONS OF BINARY CONNECTING NETWORK

The structure of a switching unit with 3 states described in Section 3.3 (\(J\)-element) looks very promising for constructing a medium capacity connecting network with pronounced attraction between individual connecting lines. To the author's knowledge, the capabilities of structures involving \(J\)-elements have not yet been investigated theoretically, though there are some operational connecting systems designed from empirical considerations. Fig. 11, for example, shows the diagram of a switching unit for short wave transmitting antennas [12]. This unit is capable of connecting 4 inputs (transmitters) to 12 outputs (antennas). The system consists of 16 switching units each of which, unlike Fig. 4d, is a specially designed three-position switch. In the diagram (Fig. 11) graph edges stand for crosspoints while non-intersecting chains of such edges stand for connecting routes. This structure is capable of connecting the most attracted inputs and outputs along the shortest paths. Apparently, such structures may find application in switching units of wire communication systems.

REFERENCES


Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10.

Fig. 11.