AN ITERATIVE METHOD TO CALCULATE TRAFFIC LOSSES IN GRADING SYSTEMS

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ABSTRACT

An approximate method to calculate the probability of congestion in traffic systems is suggested. A system is divided into a number of subsystems, each of which is analyzed by use of the equations of state. The interaction between the subsystems is taken into account by perturbing Poisson traffic. This traffic appears as coefficients in the sets of equations and must be determined iteratively. The method is tried out on a straight grading. The results are compared with simulation experiments and calculations based on the Equivalent Random Theory, the usefulness of the method is thereby positively confirmed.

1. INTRODUCTION

The behaviour of a traffic system may principally be determined by means of the equations of state. An analytical solution of these equations is, however, only available for very simple systems. Numerical solutions are often prohibited because of the staggering number of system states which have to be defined. In this paper an approximate method is suggested. This method makes it possible to calculate numerically the traffic losses in rather complex gradings.

Section 2 deals with some theoretical aspects connected to traffic calculations. Section 3 outlines the algorithm used for a straight grading, and the results are discussed in Section 4.

The principle evaluated is not constricted to the calculation of traffic losses in gradings. The possibility of adopting the algorithm to other traffic systems should stimulate further research.

2. HOW TO ESTIMATE THE PERTURBING POISSON TRAFFIC

We consider a loss system consisting of two independent traffic sources offering calls to one outlet. The holding time distribution of the calls is assumed to be negative exponential with mean holding time $T = 1.0$. The input process from source $i$ is completely described by a function $\phi_i(s)$. $\phi_i(s)$ is the probability that the interarrival time $T_i$ between two succeeding calls from the source is greater than $s$.

$$\text{Prob}(T_i > s) = \phi_i(s)$$  \hspace{1cm} (1)

The mean number of calls $Y_i$ per time unit from source $i$ is then given by

$$\frac{1}{Y_i} = \int_0^\infty \phi_i(s) \, ds$$  \hspace{1cm} (2)

As the two input processes are independent, the total input process is defined by

$$\phi(s) = \frac{Y_1 Y_2}{Y_1 + Y_2} \int_0^\infty \phi_1(x) \, dx + \phi_2(s) \int_0^\infty \phi_1(x) \, dx$$ \hspace{1cm} (3)

The mean number of calls from source 1 carried by the outlet, $z_1$, is given by

$$z_1 = \int_{s=0}^s Y_1 Y_2 \int \phi_2(s) \int e^{-x} \phi_1(x) \, dx \, ds$$ \hspace{1cm} (4)

Source 2 is now replaced with a Poisson traffic source. The calling intensity $\lambda$ of this source is determined by the condition that the outlet should carry the same number of calls from the Poisson source as from source 2 in the original system.

$$z_2^* = z_2$$ \hspace{1cm} (5)

Using formulae analogous to (4), this equation may be rewritten.
The assumption is that the traffic carried and traffic lost from source 1 may be found to a high degree of accuracy, if instead of studying the original system we analyze a system with a Poisson source, which fulfills equation (5), replacing source 2. The estimate on the traffic carried is then

\[
z_1^* = y_1 \cdot \int_0^\infty g(x) \phi_1(x) \, dx \quad \text{and} \quad y_2 = y_2 \cdot \int_0^\infty g(x) \phi_1(x) \, dx \quad \text{ds}
\]

The perturbing Poisson traffic is

\[
a = y_2 \cdot \frac{1-y_2}{1-y_1} \quad \text{and} \quad \frac{1-y_2}{1-y_1}
\]

Substituting this value into (7) and rearranging, we achieve full agreement between the real value \(z_1\) and our estimate \(z_1^*\). This may be shown to be valid for a general input function if Poisson traffic is generated from source 1.

Examples:

i) Source 1 is a Poisson traffic source. Source 2 generates calls with a constant interarrival time.

From (4) we find the traffic carried from source 1

\[
z_1 = y_1 \cdot \frac{1+y_1}{1+y_1} \quad \text{and} \quad y_2 = y_2 \cdot \frac{1+y_2}{1+y_1}
\]

The perturbing Poisson traffic is

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Substituting this value into (7) and rearranging, we achieve full agreement between the real value \(z_1\) and our estimate \(z_1^*\). This may be shown to be valid for a general input function if Poisson traffic is generated from source 1.

ii) The input processes are defined by the functions

\[
\phi_s(s) = c_1 \cdot e^{-a_1 s} + (1-c_1) e^{-\beta_1 s}
\]

Evaluating the formulae (4), (6) and (7) it follows that

\[
z_1 = y_1 \cdot \frac{1+y_1}{1+y_1} \quad \text{and} \quad y_2 = y_2 \cdot \frac{1+y_2}{1+y_1}
\]

When traffic from two independent sources is offered to \(n\) outlets, the intensity of the Poisson traffic source which should replace the second source is determined as follows:

Let the number of calls per unit time carried by outlet number \(k\) from source 1 be \(p_k\) and from source 2 be \(q_k\). The total number of calls carried by the outlets will be

\[
z_1 = \sum_{k=1}^{n} p_k \quad \text{and} \quad z_2 = \sum_{k=1}^{n} q_k
\]

In the system with the perturbing Poisson traffic, the traffic carried by outlet number \(k\) is named \(p_k^*\) and \(q_k^*\). The Poisson traffic intensity is hence determined by the equation

\[
z_1^* = \sum_{k=1}^{n} p_k^* \quad \text{and} \quad z_2^* = \sum_{k=1}^{n} q_k^*
\]

This condition will be used independent of the hunting method. Our estimate of the traffic carried from source 1 is

\[
z_1^* = \sum_{k=1}^{n} p_k^*
\]

3. APPLICATION ON GRADINGS

A straight grading is studied.

\[
A_1 \rightarrow \begin{array}{cccc} A_7 & A_8 & A_9 & A_4 \\ A_2 & A_3 \\ A_4 & A_5 \\ A_6 & A_1 
\end{array}
\]

A\(_1\) denotes the mean value of the Poisson traffic offered to inlet group \(i\). Sequential hunting is assumed.
We split the system into four subsystems, one for each inlet group.

For subsystem 1, for instance, $B_1$ and $C_1$ denotes perturbing Poisson traffics. These are given values such that the loading on outlets 13, 15 and 17 caused by $B_1$, is equal to the loading on the same outlets caused by $A_2$ in subsystem 2, the loading on outlet 17 caused by $C_1$ should be equal to loading caused by $A_3$ in subsystem 3 plus the loading caused by $A_4$ in subsystem 4.

In this way the interaction between the subsystems (or between the inlet groups in the grading) is taken care of. In order to calculate the traffic losses in a subsystem it is sufficient to define the state by three parameters, $j$ the number of calls in the three first outlets, $k$ the number of calls in the next two outlets and $l$ the state of the last outlet. There are 24 possible states. Let $P_i(j,k,l)$ denote the probability of subsystem $i$ being in state $[j,k,l]$. The equations of state for subsystem 1:

$$(18)\quad P_1(j,k+1,l) = \delta_{j+1} P_1(j,k,l) + \delta_{j+1} P_1(j,k-1,l) + A_1 P_1(j-1,k,l)$$

$$= (j+1) P_1(j+1,k,l) + (k+1) P_1(j,k+1,l) + A_1 P_1(j-1,k,l) + (A_1 P_1(j-1,k,l) + B_1 P_1(j,k-1,l)) + (A_1 P_1(j-1,k,l) + C_1 P_1(j,k-1,l))$$

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We have used Kronecker's $\delta$-function

$$\delta_{m,n} = 1 \quad \text{if } m=n$$

and defined

$$P_i(r,s,t) = 0 \quad \text{if } r,s,t < 0$$

$$\text{or } r>3, s>2, t>1$$

The equations to determine the perturbing Poisson traffic are:

$$B_1(1-\frac{1}{k+1}) P_1(j,2,1) = A_1 \sum_{k=0}^{\infty} P_2(3,k,l)$$

$$C_1(1-\frac{1}{k+1}) P_1(j,k,1) = A_1 P_1(j,2,0) + A_1 P_1(j,2,0)$$

Analog equations are constructed for the three other subsystems. Numerical results are evaluated and compared with calculations based on the Equivalent Random Theory and simulation experiments in Figure 2.

4. CONCLUSION

The critical point of the algorithm discussed is the set of equations (5), (16), (21) and (22) determining the values of the perturbing traffics. A proper theory dealing with this should be developed. At this state the equations are indifferent to the hunting method in the system, and I have a feeling that this should not be the case. On the other hand, the method is in principle approximate, the gain from refining parts of it might therefore be fictions.

The numerical results found on the straight grading are very good. To attain more confidence, the method must be applied to other and more complex systems. This should cause no great difficulty, since the principle used is of a general nature. For instance, a grading with $g$ inlet groups, availability $k$ and $N$ outlets is to be split into $g$ subsystems, each with a maximum of $g$ perturbing Poisson traffic sources and with a maximum of $2^k$ equations of state. Very often the splitting results in subsystems of lesser complexity. An attempt to write a computer program which calculates the traffic losses in a general grading construction will be made.

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