Effectiveness Characteristics of Partly Disabled Device Groups

Hans Andersson and Kjell Strandberg
Telefonaktiebolaget L M Ericsson, Stockholm, Sweden

ABSTRACT

The paper presents formulas and methods for the determination of availability, trafficability and effectiveness characteristics of partly disabled device groups. A partly disabled device group is a group of devices, where the occupations of a subset of devices are classified as un-successful.

Certain fully accessible, randomly hunted loss, delay and combined loss-delay systems are treated. Formulas and methods are given for an iterative calculation of state probabilities. The methods, when programmed on a computer, give simple calculations to determine probability of call failure, call and time congestion, mean waiting times and other measures.

The methods presented are primarily intended for use in the evaluation of alternative telecommunication system design proposals.

1. INTRODUCTION

The introduction and application of the concepts of (system) effectiveness and trafficability of telecommunication systems is presented in ref [15]. The main purpose there is to allow suitable classification of different characteristics relating to reliability, in order to facilitate an appropriate use of various kinds of definitions of system failures, considering the effects on traffic and call handling. The same document also shows how to use the terminology for requirements specification and design analysis.

The effectiveness concept combines the various aspects of the equipment's ability to perform its functions at an instant or over a period of time with its ability to satisfy operational traffic demands over the same period of time.

Most applications of the traffic theory are directed towards the analysis of traffic handling properties of equipment without failures. The first part of the seventies, however, has shown some approaches to a joint treatment of the mentioned properties.

The treatment presents two kinds of problems, connected with what may be called reliability engineering and tele-traffic engineering. In the first case the problem is to determine the distribution of system "hardware" states (eg state probabilities, mean state duration etc). In the second case occupation state distributions, congestion probabilities, mean waiting times etc are determined. In this second case the effect of equipment parts not functioning correctly must be taken into account. This presents new problems as new "dimensions" are added: new sets of formulas have to be derived and new numerical methods have to be employed.

Device groups in modern exchange equipment are normally functionally dependent on some kind of control equipment. Added to that the equipment is usually sectioned for reasons such as power distribution, rack size etc. Thus, from a reliability point of view, the traffic carrying devices and their control equipment form a so-called hierarchical structure. A certain device group often belongs to more than one hierarchy (rack power distribution etc) making the structure even more complex.

A theoretical approach to the state distribution was made by Guseinov, [5]. Lee has presented solutions for a practical case [11], an interoffice carrier system. A similar problem is treated by Rahko et al in [13].

The theoretical treatment of effects of failed equipment on traffic, i.e. the trafficability of the equipment, seems to have been restricted to loss systems. Klimontowicz seems to be the first who has published a solution to the problem, [7]. He treated fully accessible loss systems with three different hunting principles. The numerical results, mainly shown in a diagram form, are partly based on simulations and are restricted to device groups with a comparatively small number of devices. Forys and Messerli, [3], present results for device groups with short-holding-time devices for four different hunting routines, concentrating on device groups with one faulty device. Their primary goal seems to be aimed at operational problems. Forsblad, [2], has derived a test, based on one of the results of Forys and Messerli, to facilitate detection of a short-holding-time device in a device group.

The problem with the simultaneous treatment of the failure and traffic processes has been approached by Strandberg, [3], Lee, [10], Rahko, [14], and Karlsson [6].

During the design phase of a new telecommunication system failure modes and effect analyses are often utilized. The properties of proposed designs can inter alia be expressed in terms of trafficability and effectiveness. Requirement specifications are often expressed along lines similar to those given by Strandberg in [3]. There is therefore an expressed need for quantification of availability, trafficability and effectiveness measures.

The main intention with this paper is to show results useful for the determination of such characteristics for application during the above-mentioned phases of system development.
2. DETERMINATION OF DEVICE GROUP AVAILABILITY

2.1 GENERAL

An equipment's ability to perform its functions at an instant of time or over a period of time is usually called its availability (this meaning of the term availability must not be confused with the use of the same term within teletraffic engineering, where it is a synonym to accessibility). The most frequent measure of availability is the probability of an equipment item being in a one of the "up states" (see e.g. [1]). For a device group we could define the up states as those with at least \( k \) out of \( n \) devices non-failed.

To allow determination of the value of effectiveness characteristics, the state probabilities i.e. the probabilities of having \( 0, 1, 2, \ldots, n \) out of \( n \) devices failed are of interest. In order to determine these state probabilities we must know the reliability structure of the device group (including all necessary control equipment) failure rates of devices and other relevant equipment as well as down times (determined by the maintainability of the equipment and by the maintenance support).

The simple case with devices with no control equipment, which with suitable assumptions usually leads to the Bernoulli distribution of state probabilities, is not treated here, but can be found in [13, 14] and other papers. Lee [11] has treated a practical case with control equipment. The following section presents results for a certain class of hierarchical structures.

2.2 HIERARCHICAL SYSTEMS

The hierarchical structure is of frequent occurrence in telephone systems. A hierarchical structure consists of a number of traffic handling devices controlled by centralized control systems designed according to a hierarchical principle as shown in figure 2.1.

![Figure 2.1 A hierarchical structure where](image)

- \( C_{ij} \) = control device no \( j \) at level \( i \)
- \( D_k \) = traffic handling device no \( k \) (level 3)

A failure of \( C_{ij} \) causes disablement of all traffic handling devices subordinate to that control device. A failure of this type thus causes a reduction of the trafficability of the system.

We assume that the hierarchical system is completely symmetric, which means that if \( K_{ij} = \) number of devices on level \( i + 1 \) subordinate to \( C_{ij} \), then \( K_{ij} = K_{ji} \), \( \forall i, j \). Further it is assumed that the down times and the up times of all devices are independent and exponentially distributed. We determine the steady state probabilities of this system under these conditions.

To begin with we introduce the following designations.

- \( m \) = the number of levels of the structure
- \( n_i \) = the number of devices at level no \( i \), \( i = 1, \ldots, m \)
- \( \lambda_i \) = the failure rate of a device at level no \( i \), \( i = 1, \ldots, m \)
- \( \tau_i \) = the mean down time of a device at level no \( i \), \( i = 1, \ldots, m \)
- \( X_i \) = the number of failed devices at level no \( i \), \( i = 1, \ldots, m \)
- \( S \) = the number of disabled devices at level no \( m \)
- \( x = (x_1, x_2, \ldots, x_m) \)

Further it follows from (2.1), (2.3) and (2.4) that (2.2) can be determined as

\[
\Pr[S=s/X=x] = \sum_{u \in U(x,s)} \prod_{i=1}^{m} \left( \frac{n_i}{x_i} \right)^{u_i} \left( \frac{1}{A_i} \right)^{n_i-x_i} \tag{2.5}
\]

The steady state probabilities of the number of disabled traffic handling devices are then obtained from

\[
h(s) = \Pr[S=s] = \sum_{0 \leq x_i \leq n_i} p(x,s) \tag{2.6}
\]
3. DETERMINATION OF DEVICE GROUP TRAFFICABILITY

3.1 GENERAL ASSUMPTIONS

The studied system is a fully accessible device group consisting of \( n \) devices, which are randomly hunted. The holding times are assumed to be exponentially distributed with mean holding time \( s_n \) for successful occupations, and with \( s_f \) for unsuccessful occupations. A number of devices, \( n_f \), are disabled, which implies that occupations of these devices are unsuccessful.

We further define the number of not disabled devices \( n_n = n - n_f \).

All results in this chapter are derived on the condition that \( n_n \) and \( n_f \) have constant values.

3.2 LOSS SYSTEMS

3.2.1 GENERAL

In this section we consider a loss system and assume that it can be described as a homogeneous, irreducible, continuous time parameter Markov chain, with a finite two-dimensional discrete state space \( S \).

The state space \( S \) is defined in the following way:

\[
S = \{ (i,j) : 0 \leq i \leq n_f, 0 \leq j \leq n_n \},
\]  

where \( (i,j) \) denotes the system state with \( i \) occupied disabled and \( j \) occupied non-disabled devices.

The steady state probabilities of the system are denoted \( p(i,j) \), for all \( (i,j) \) in \( S \). If we in our calculations have \( (i,j) \) outside \( S \) we conventionally put \( p(i,j) = 0 \).

The transition rate diagram is shown in figure 3.1 where \( \lambda \) and \( \rho \) represent the rates of occupation of a disabled and a non-disabled device respectively, while \( \mu \) and \( \eta \) represent the termination rates of occupation of disabled and non-disabled devices respectively.

\[
\begin{align*}
&\lambda(i-1) \quad \lambda(i-1) \quad \lambda(i) \\
&\mu(i,j) \quad \mu(i,j) \\
&\eta(i-1,j) \quad \eta(i,j) \\
&\mu(i+1,j) \quad \lambda(i,j) \\
&\eta(i,j) \quad \eta(j+1,j) \\
&\mu(i+1,j) \quad \lambda(i,j) \\
\end{align*}
\]

Figur 3.1 Transition rate diagram, loss systems.

The steady state equations, corresponding to the transition rate diagram, are:

\[
p((i,j) + (i+1,j) + \lambda(i,j) - \eta(i,j) + \mu(i,j) + p(i,j) + \eta(i-1,j) + \mu(i-1,j) + \eta(i,j) - \lambda(i,j)) = p(i,j), \quad (i,j) \in S
\]

with the constraint

\[
\sum_{(i,j) \in S} p(i,j) = 1
\]

In the general case it is very difficult to solve an equation system of this type analytically, but we are only going to consider systems with the property:

\[
p((i,j) + (i,j+1) + \eta(i,j) - \lambda(i,j) + \mu(i,j) + p(i,j) + \eta(i,j) + \eta(i,j) + \eta(i,j) + \lambda(i,j) + \lambda(i,j)) = p(i,j), \quad (i,j) \in S
\]

This means that the expected number of transitions between two neighbouring states in the long run are equal.

In this case we can easily obtain the steady state probabilities as a function of an arbitrarily chosen state probability, for example of \( p(0,0) \):

\[
p(i,j) = \frac{\prod_{k=0}^{i-1} \lambda(k,j)}{\mu(k+1,j)} \cdot \frac{\prod_{l=0}^{j-1} \eta(l,0)}{\eta(l+1,0)} \cdot p(0,0), (i,j) \in S
\]

Let us further introduce the following designations:

\[B = \text{the proportion of calls arriving when all devices are occupied (the call congestion)}\]
\[Z = \text{the proportion of calls which occupy disabled devices (and thus result in unsuccessful occupations)}\]
\[L = \text{the proportion of calls which are unsuccessful, i.e. which either meet a fully occupied group or result in an unsuccessful occupation)}\]

We thus have

\[L = B + Z\]

We also define the average calling rate, \( C \), as

\[C = \sum_{(i,j) \in S} \mu(i,j) \cdot p(i,j), \quad (i,j) \in S\]

where \( \mu(i,j) \) is the conditional calling rate in state \( (i,j) \). We now obtain for \( B \) and \( Z \):

\[B = \frac{\sum_{(i,j) \in S} \mu(i,j) \cdot p(i,j)}{\mu(0,0) \cdot p(0,0) + \mu(n_f,0) \cdot p(n_f,0)}\]

\[Z = \frac{\sum_{i=0}^{n_f-1} \sum_{j=0}^{n_n} \mu(i,j) \cdot p(i,j)}{\mu(0,0) \cdot p(0,0) + \mu(n_f,0) \cdot p(n_f,0)}\]

The time congestion, \( E \), is

\[
E = p(n_f, n_n) \cdot \left( \sum_{(i,j) \in S} \mu(i,j) \cdot p(i,j) \right)
\]

If we now restrict the termination rates to be of the form

\[
\eta(i,j) = i \cdot \eta, \quad \lambda(i,j) = j \cdot \lambda
\]

it then follows from (3.4) and (3.11) that (3.7) can be reduced to

\[
E = \mu(0,0) \cdot p(0,0) + \lambda(0,0) \cdot p(0,0) + \eta(0,0) \cdot p(0,0) + \lambda(n_f,0) \cdot p(n_f,0) + \eta(n_f,0) \cdot p(n_f,0)
\]

where

\[
E(I) = \sum_{(i,j) \in S} i \cdot p(i,j) \quad (i,j) \in S
\]

is the average number of occupied disabled devices, and

\[
E(J) = \sum_{(i,j) \in S} j \cdot p(i,j) \quad (i,j) \in S
\]

is the average number of occupied non-disabled devices.
Furthermore, (3.9) can be simplified to
\[ Z \left( \frac{1}{C} \cdot E(i) \right) \] (3.15)
or, if we observe that \( \mu_n = s_n^{-1} \) and \( \eta_n = s_n^{-1} \) and define
\[ A = E(i) \] (3.16)
\[ B = E(i) \] (3.17)
we get
\[ Z \left( \frac{1}{B} \cdot A \right) \] (3.18)
Between \( B \) and \( E \) we have the simple relation
\[ y(n,f,nn) = B \cdot E \] (3.19)

3.2.2 NUMERICAL DETERMINATION OF STATE PROBABILITIES

The steady state probabilities \( p(i,j) \), \( (i,j) \in S \), can be easily calculated by a computer as follows:

a) At the outset put e.g. \( p(0,0) = 1 \)

It should be noted that some precautions have to be made in order to avoid numerical problems. The initial state can e.g. be chosen on the basis of the expected "centre of gravity" of the state probability matrix.

b) \( p(i,j)_{i=1}^{n_f} \) is then determined by successive use of the relation
\[ p(i+1,j) = p(i,j) \cdot \left( \frac{n-f}{n(i+1,j)} \right) \quad 0 \leq i < n_f \] (3.20)
c) \( p(i,j)_{i=1}^{n} \) is then similarly calculated for each \( j \), \( 0 \leq j < n \) from the relation
\[ p(i,j) = p(i,j) \cdot \left( \frac{n-f}{n(i,j)} \right) \quad 0 \leq i < n_f \] (3.21)
d) To fulfill the requirement \( \sum p(i,j) = 1 \) it is necessary to divide each \( p(i,j) \) with \( D \), where
\[ D = \sum_{i,j} p(i,j) \] (3.22)

3.2.3 THE ERLANG CASE

In this case we have:
\[ y(i,j) = \frac{y(i,j)}{n_f} \] (3.23)
\[ C = y \] (3.24)
The expressions for \( Z \) and \( B \) are thus
\[ Z = \left( \frac{1}{E(i)} \right) \] (3.25)
\[ B = E \] (3.26)
where the calculation of \( E(i) \) is based on the transition rates
\[ \lambda(i,j) = \begin{cases} \frac{n_i}{n-1}, & \text{if } (i,j) \in S \backslash (n_f, n_n) \\ 0, & \text{if } (i,j) = (n_f, n_n) \end{cases} \] (3.27)
\[ \mu(i,j) = \begin{cases} n_f - i, & \text{if } (i,j) \in S \backslash (n_f, n_n) \\ 0, & \text{if } (i,j) = (n_f, n_n) \end{cases} \] (3.28)

3.2.4 THE ENGSET CASE

In this case we have \( N > n \) sources and the calling rate proportional to the number of idle sources, i.e.
\[ y(i,j) = (N-i-j) \cdot y, \quad (i,j) \in S \] (3.29)
The transition rates are thus
\[ \lambda(i,j) = \begin{cases} \frac{n_i}{n-1}, & \text{if } (i,j) \in S \backslash (n_f, n_n) \\ 0, & \text{if } (i,j) = (n_f, n_n) \end{cases} \] (3.30)
\[ \mu(i,j) = n_f - i, \quad (i,j) \in S \] (3.31)
\[ \eta(i,j) = j, \quad (i,j) \in S \]

3.2.5 THE BERNOULLI CASE

The same as in 3.2.4 is valid, except that now \( N=n \) and therefore \( y(n_f, n_n) = 0 \).

This gives for \( Z \) the expression (3.9) and
\[ B = 0 \] (3.32)

3.3 DELAY SYSTEMS

3.3.1 GENERAL

In this section we consider delay systems that we can assume to be possible to describe as homogeneous, irreducible, continuous time parameter Markov chains. We further assume that delayed calls are put in a single, ordered queue of maximal size \( n_q \), possibly with \( n_q = \infty \), where they are assumed to wait until they can be handled, i.e. no voluntary or forced releases.

The state space \( Q \) can be defined in the following way if we as before assume that \( S = \{(i,j): 0 \leq i \leq n_f, 0 \leq j \leq n_n\} \):
\[ Q = \{(i,j,k): 0 \leq i \leq n_f, 0 \leq j \leq n_n, 0 \leq k \leq n_q \} \] (3.33)
where \( (i,j,k) \) denotes the system state with \( i \) occupied disabled devices, \( j \) occupied non-disabled devices and \( k \) waiting calls.

The steady state probabilities of the system are denoted \( p(i,j,k) \), \( (i,j,k) \in Q \). If we in our formulas have \( p(i,j,k) \) with \( (i,j,k) \in Q \) we conventionally put \( p(i,j,k) = 0 \).

The transition rate diagram has in this case the same appearance as for the loss systems, with the exception of the states with all devices occupied, with or without waiting calls, see figure 3.2.
The steady state equations corresponding to the transition rate diagram are:

\[ \begin{align*}
& \left. \begin{array}{l}
\lambda(n_f n_0) + \mu(n_f n_0) = \gamma(n_f n_0)
\end{array} \right\} \text{if } (i,j,0) \in S \setminus \{(n_f, n_0)\}
\end{align*} \]

where \( S \) denotes the set \( S \) with element \((n_f, n_0)\) excluded.

\[ \begin{align*}
& \left. \begin{array}{l}
\mu(n_f n_0) = \gamma(n_f n_0)
\end{array} \right\} \text{if } (n_f, n_0) \in S
\end{align*} \]

with the constraint

\[ \sum_{(i,j,k) \in S} p(i,j,k) = 1 \]

As in section 3.2 we are here only going to consider systems with the property:

\[ \begin{align*}
& \left. \begin{array}{l}
p(i,j,0) = p(i,j,1) = \gamma(i,j,0)
\end{array} \right\} \text{if } (i,j,0) \in \Omega
\end{align*} \]

If (3.36) is fulfilled, we easily obtain the steady state probabilities as a function of an arbitrarily chosen state probability, for example of \( p(0,0,0) \):

\[ \begin{align*}
& \gamma(n_f n_0) = (n_f + n_0 - 1) p(i,j,0) + p(i,j,1),
\end{align*} \]

\[ \begin{align*}
& \gamma(n_f n_0) = n_f p(i,j,0) + n_0 p(i,j,1),
\end{align*} \]

\[ \begin{align*}
& \gamma(n_f n_0) = n_f p(i,j,0) + n_0 p(i,j,1).
\end{align*} \]

The average calling rate is

\[ \begin{align*}
& \sum_{i=0}^{n_f} \sum_{j=0}^{n_0} \gamma(i,j,0) p(i,j,0) + \sum_{k=0}^{n_q} \gamma(n_f n_0) p(n_f n_0, k)
\end{align*} \]

where \( \gamma(i,j,k) \) is the conditional calling rate in state \((i,j,k)\).

The proportion of calls that occupy disabled devices, \( Z \), is

\[ \begin{align*}
& Z = \frac{\sum_{i=0}^{n_f} \sum_{j=0}^{n_0} \lambda(i,j,0) p(i,j,0) + \psi \sum_{k=0}^{n_q} \gamma(n_f n_0) p(n_f n_0, k)}{\sum_{i=0}^{n_f} \sum_{j=0}^{n_0} \gamma(i,j,0) p(i,j,0) + \sum_{k=0}^{n_q} \gamma(n_f n_0) p(n_f n_0, k)}
\end{align*} \]

where

\[ \lambda(n_f n_0) = \gamma(n_f n_0) \]

If we now restrict the termination rates to be of the form

\[ \begin{align*}
& \gamma(i,j,0) = i \cdot \mu
\end{align*} \]

then it follows from (3.36) and (3.38) that (3.39) can be expressed as

\[ \begin{align*}
& E(\gamma) = \sum_{(i,j,k) \in \Omega} \gamma(i,j,k) p(i,j,k)
\end{align*} \]

is the average number of occupied disabled devices and

\[ \begin{align*}
& E(\gamma) = \sum_{(i,j,k) \in \Omega} \gamma(i,j,k) p(i,j,k)
\end{align*} \]

is the average number of occupied non-disabled devices.

Furthermore (3.40) can be simplified to

\[ \begin{align*}
& Z = E(\gamma)
\end{align*} \]

or

\[ \begin{align*}
& Z = E(\gamma)
\end{align*} \]

where \( A \) and \( B \) are defined by (3.15) and (3.16) respectively.

The call congestion is

\[ \begin{align*}
& Y(n_f n_0 n_q) = \sum_{(i,j,k) \in \Omega} \gamma(i,j,k) p(i,j,k)
\end{align*} \]

3.3.2 Numerical Determination of State Probabilities

The steady state probabilities \( p(i,j,k) \), \((i,j,k) \in \Omega\), can be calculated by computer in the same way as described in 3.2.2.
3.3.3 THE M/M/n CASE

In the M/M/n case we have

\[ y(i,j,k) = y, \quad (i,j,k) \in \Omega \]  
\[ C = n \]  
\[ n_{\infty} \]  
\[ n_{q} \]  

According to the formulas (3.47) and (3.48) we obtain the following expressions for \( Z \) and \( B \)

\[ Z = \frac{E(1)}{Y s} \]  
\[ B = 0 \]  

(3.52)

(3.53)

where the calculation of \( E(1) \) is based on the transition rates

\[ \lambda(i,j,0) = \begin{cases} \frac{n_{i} - i}{n_{i} - j} y, & \text{if } (i,j) \in S \setminus (n_{f},n_{h}) \\ 0, & \text{if } (i,j) = (n_{f},n_{h}) \end{cases} \]  

\[ \rho(i,j,0) = \begin{cases} \frac{n_{j} - j}{n_{i} - j} y, & \text{if } (i,j) \in S \setminus (n_{f},n_{h}) \\ 0, & \text{if } (i,j) = (n_{f},n_{h}) \end{cases} \]  

(3.54)

and the probability that an arriving call will have to wait is

\[ R(0) = p(n_{f},n_{h},0) \frac{C}{\alpha - y} \]  

(3.57)

The distribution of the waiting time for those calls which have to wait is obtained from

\[ \Pr[T > t | K = 0] = e^{-(\alpha - y)t} \]  

(3.58)

and the mean waiting time for those calls which have to wait is

\[ m = \frac{1}{\alpha - y} \]  

(3.59)

The average number of waiting calls is given by

\[ A = \sum_{k=1}^{n_{q}} k \cdot p(n_{f},n_{h},k) = p(n_{f},n_{h},0) \cdot \frac{Y s}{(\alpha - y)^2} \]  

(3.60)

3.3.4 THE M/M/n-(L,W) CASE

The case with limited number, \( n_{q} \), of queuing places is denoted M/M/n-(L,W) and we have

\[ y(i,j,k) = y, \quad (i,j,k) \in \Omega \]  
\[ C = n \]  
\[ n_{\infty} \]  
\[ n_{q} \]  

From the formulas (3.47) and (3.48) we now obtain the following expressions for \( Z \) and \( B \)

\[ Z = \frac{E(1)}{Y s} \]  

(3.64)

(3.65)

where the calculation of \( E(1) \) and \( p(n_{f},n_{h},n_{q}) \) is based on the same transition rates as in the M/M/R case, except that we now have

\[ \alpha(n_{f},n_{h},k) = \alpha(n_{f},n_{h},n_{q}) = m, \quad n_{q} \leq k \leq n_{q}, \quad k=1,2,\ldots, \]  

(3.66)

with a finite \( n_{q} \).

In a similar manner as in section 3.3.3 we obtain the distribution of the waiting time \( T_{w} \). This distribution is determined by

\[ R(t) = \Pr[T_{w} > t] = \sum_{k=0}^{n_{q}-1} a(k,t) \]  

(3.67)

where

\[ a(0,t) = e^{-\alpha t} \]  

\[ a(k,t) = \frac{(\alpha t)^k}{k!} e^{-(\alpha - y)t} \]  

(3.68)

The mean waiting time for all calls, \( M_{w} \), is obtained from

\[ M_{w} = \sum_{k=0}^{n_{q}-1} p(n_{f},n_{h},k) \cdot \frac{k}{\alpha} = p(n_{f},n_{h},0) \sum_{k=0}^{n_{q}-1} \frac{(\alpha t)^k \cdot k}{\alpha} \]  

(3.69)

and it follows that
The average number of waiting calls is given by
\[ n_q = \sum_{k=0}^{n_q-1} k p(n_q, n, k) \]

The probability that an arriving call will have to wait is
\[ R(0) = \sum_{k=0}^{n_q-1} p(n_q, n, k) = R_0 \]

and it follows that
\[ R(0) = \sum_{k=0}^{n_q-1} p(n_q, n, k) \]

The distribution of the waiting time for those calls which have to wait is
\[ R(x) = \int_0^x \frac{p(t)}{1 - R_0} dt \]

where \( K \) is defined as in 3.3.3, while the mean of the waiting time for calls which have to wait is
\[ \mu = \frac{R(0)}{R_0} \]

4 DETERMINATION OF DEVICE GROUP EFFECTIVENESS

A measure of effectiveness was defined in [15] as the expectation of trafficability
\[ Q = E(\Upsilon(s, A)) \]

where \( \Upsilon(s, A) \) is the trafficability when \( s \) devices are disabled and the traffic \( A \) is offered to the device group.

If the two random variables "number of disabled devices" and "traffic offered" are considered stochastically independent it was shown in [15] that the effectiveness can be expressed as
\[ Q = \sum_{s=0}^{\infty} h(s) \int_0^x \Upsilon(s, A) f(A) dA \]

where \( h(s) \) is the probability of \( s \) devices being disabled
\[ f(A) \]

if we further disregard the traffic variations and thus assume a constant value \( A = A_0 \) we obtain the following simplified expression, useful for our purpose as described in the introduction part.
\[ Q = \sum_{s=0}^{\infty} h(s) \Upsilon(s, A_0) \]


