Traffic Engineering with Programmable Pocket Calculators

Gerhard Bretschneider
Siemens AG, FL KOM, München, West Germany

ABSTRACT

Programmable pocket calculators are shown to be a practical tool for the traffic engineer. The author, who has programmed the basic formulas of traffic theory for these calculators, demonstrates their usefulness by explaining some of the operating instruction charts.

INTRODUCTION

Hitherto only tables and graphs have been available for traffic engineering practice. These contain therefore the required data for preselected sets of values only. With the invention of programmable pocket calculators such as the HP 65 or the TI SR-52 the traffic engineer is now able to obtain the required function values in an instant for any combination of parameters whenever and wherever necessary. Moreover instant access to both computer and formulas enables him to compute either up or down, so allowing him freedom of action in decision-making.

The author has already programmed some of the principal formulas for the HP 65 calculator, specifically:

- the two Erlang formulas for full-availability trunk groups in loss and delay systems with an infinite number of sources
- the Engset formula for the loss in full-availability trunk groups with a finite number of sources
- the modified Palm-Jacobaeus formula (due to Prof. Lotze) used by the Deutsche Bundespost for calculating limited-availability trunk groups
- the Erlang interconnection formula for idealized limited-availability trunk groups
- the equivalent random theory for calculating the loss of overflow traffics.

In developing these programs close attention has been devoted to flexibility and operational ease. As an illustration, the structure and use of the programs for the Erlang formula for delay systems and, as a more complicated example, the Erlang interconnection formula will be discussed in some detail.

GENERAL

The keys of the type HP 65 programmable pocket calculator can be pressed sequentially to store arithmetic operations, storage and readout instructions as well as relational, branching and interrupt instructions both in the calculator and on magnetic tape.

Once such a tape has been recorded the calculator is ready for the stored program to be run.

To run the program, the keys for starting the program and entering the input parameters must first be pressed; in some cases an interrupt key R/S may have to be pressed in between. After a renewed start the calculator computes the desired function and displays the result. If necessary, further values can be calculated and displayed after pressing additional keys.

For running a program the user needs, in addition to the appropriate magnetic tape containing the stored program, an instruction card showing which keys have to be pressed, what parameters have to be entered and what results will be displayed and when. For the traffic theory programs to be treated here we will therefore always give the contents of the corresponding instruction cards.

ERLANG DELAY FORMULA

The Erlang delay formula is the internationally most widely used formula for calculating full-availability trunk groups for delay systems. It assumes exponentially distributed inter-arrival times and holding times. If A denotes the traffic offered (in erlangs) and N the number of trunks in the full-availabel serving group, then the probability of delay for an incoming call is

$$P(>0) = \frac{\frac{N}{N-A} \cdot \frac{A^N}{N!}}{\sum_{K=0}^{N-1} \frac{A^K}{K!} + \frac{N}{N-A} \frac{A^N}{N!}}$$

The probability that the waiting time will exceed $x \cdot t_m$, where t_m = mean holding time, is defined

$$P(>x) = P(>0) \cdot e^{-(N-A)x}$$

INSTRUCTION CARD

Step	Instruction	Input (unit)	Key	Display
1	Enter program			
2	Enter traffic offered	A(erl)		
3	Start program		A	1.00...0
4	Enter number of trunks	N		
5	Calculate probability of delay		R/S	probability P(>0)
6	Enter given waiting time in multiples x of mean holding time	$x(t_m)$		
7	Calculate probability of waiting time being exceeded		R/S	P(>x)
	Automatic return to step 6			
8	Calculation with number of trunks increased by 1 (automatic return to step 5)		B	N+1
9	Calculation with number of trunks reduced by 1 (automatic return to step 5)		C	N-1

Steps 8 and 9 can be repeated as often as desired and in any sequence.

REMARKS

The automatic return to step 6 allows the successive calculation of the probability for various waiting times being exceeded within a few seconds. Keys B und C (steps 8 and 9) can be used to determine the probability of delay with trunk groups successively increased or reduced by 1 without the need for a complete recalculation. This saves calculation time and increases flexibility in practical applications.

ERLANG INTERCONNECTION FORMULA

For calculating the probability of loss for limited-availability trunk groups Erlang introduced the interconnection formula, which has not so far found practical application because it involves very laborious calculations. Although it is based on the assumption of idealized trunk groups it could be used both as a standard method of calculation as well as a basis for a method adapted to real gradings. Thus far only a single table of this function has been published (Tabellen zu A.K. Erlangs Interconnection-Formel, Siemens & Halske AG, Berlin/München 1961).

If the traffic offered is denoted A, the number of trunks in the group N, the availability K, and using the abridgments

$$\varepsilon(\mu) = \frac{\binom{\mu}{K}}{\binom{N}{K}}, \quad \sigma(\mu) = 1 - \varepsilon(\mu) \text{ and } D(\mu) = \prod_{r=0}^{\mu-1} \sigma(r)$$

the probability of loss B can be expressed

$$B = \frac{\sum_{\mu=0}^{N} \varepsilon(\mu) D(\mu) \frac{A^\mu}{\mu!}}{\sum_{\mu=0}^{N} D(\mu) \frac{A^\mu}{\mu!}}$$

INSTRUCTION CARD

Step	Instruction	Input (unit)	Key	Display
1	Enter program			
2	Start program		A	1.00...0
3	Enter traffic offered	A(erl)	R/S	2.00...0
4	Enter number of trunks	N	R/S	3.00...0
5	Enter availability	K		
6	Calculate probability of loss		R/S	probability of loss B

Automatic return to step 3

REMARKS

The time it takes to calculate the probability of loss with this formula using present-day programmable pocket calculators is still rather long. For N = 70 and K = 10 the calculation of each submitted load value with the HP 65 requires, for instance, about 3 min. This program however makes such values accessible practically for the first time.

REFERENCE

Bretschneider, G.

Die Verwendbarkeit programmierbarer Taschenrechner für die Verkehrsplanung von Nachrichtenvermittlungssystemen.

Nachrichtentechn. Z 28(1975) pp. 386 to 388