Structuring of a Telephone Network with Alternative Routing but with no Hierarchization of the Centres

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1. INTRODUCTION

1.1 This study is part of a more general one intended to compare the costs of an international network with a given grade of service between any pair of international centres in two distinct situations:

(i) where the network is designed with systematic use of alternate routing and without regard to any particularism stemming from national considerations;

(ii) where a network is based on the sum of the decisions peculiar to each of the countries it connects, the decisions being guided by the desire on the part of each country to maximize the financial return derived from its international service (assuming always that the prescribed grade of service is maintained).

These two policies for planning the international network, one based on the "common good" which leaves frontiers and individual national interests out of account (as advocated in the CCITT Recommendations), and the other based on a sum of national interests which may be conflicting, are likely to lead to very divergent results in terms of the structure of the network. A comparison between the North American network and the international network in Europe is instructive in this respect. In the former, which is governed by a planning control authority, alternative routing has been in general use for more than 25 years and provides a quality of service much appreciated by users. In the international European network, the cases where international transit switching is used for alternative routing are extremely rare, to the detriment - we shall not enlarge on this point - of service quality . . . .

1.2 The general study mentioned above was intended as a theoretical study, and so a sufficiently representative model of an international network has been used for the sake of mathematical convenience. The model of the international network is represented by a square matrix with five lines and five columns (i.e. 25 countries). It is described in more detail in Annex 1.

The cost elements used for the economic calculations are the basic tariff components (expressed in gold francs) defined in CCITT Recommendation D.300 R.

1.3 One specific problem arose as soon as it came to defining the structure of the network to be designed according to the "common good" theory (see (i) in 1.1). Should this network be given a hierarchical structure from the outset? Or should there be no hierarchization of the centres in the beginning?

The second alternative was chosen. This approach is indeed more in keeping with the situation of the international network, for which there is no central authority that imposes its views, and with the aim of an impartial theoretical study which should not start by giving precedence to any of the centres (capitals) in countries which are all, from an international standpoint, equal States.

Admittedly the present text of CCITT Recommendation Q.13 does place the centres (the CTs of that Recommendation) in a hierarchical order, but so far these provisions have not by any means been closely followed. There is, moreover, (particularly in terms of intercontinental traffic) a contradiction between:

- the hierarchization stipulated in existing Recommendation Q.13 (drafted in 1964);
- the clause in Recommendation E.250 (New system for accounting in international telephony), drafted in 1968, which acknowledges (paragraph 12.3) the right of the Administration of origin to choose the first transit centre towards which it will route traffic on the basis of economic criteria (the "price requested").

In this connection, a revision of Recommendation Q.13 is to be adopted for study by the Vth Plenary Assembly of the CCITT (New Question H/XIII - Document AP VI-No. 34, page 37) and the present theoretical study could well contribute in this context towards clarifying certain points concerning the structuring of a non-hierarchical network.

2. HYPOTHESES ON WHICH THE STUDY IS BASED

2.1 The traffic between centres in the network is taken to be one-way and we refer to orientated pairs of centres I-J, the traffic from I to J being represented by A_{iJ}.

2.2 The groups of circuits linking the pairs of centres are:

- either first choice groups (also known as "first choice route") which may overflow at the group originating centre;
- or last choice groups ("last choice routes") which cannot overflow.

2.3 For the outgoing traffic of a given centre, the choice of an overflow group (if there is to be one) will be limited to a single group. This condition is almost essential for a convenient study of the network as a whole and it reflects the situation which exists normally at the centres of the international network. Expressed in more mathematical terms, this rule will be represented as follows:

"For any orientated pair of centres I→J, there will be (as long as the I→J group is not last choice) at centre I a single transit centre \( i \) to which overflow may occur."
2.4. The alternative routing of a call may involve several overflows in succession (one per originating or transit centre).

The problem of alternative routing will thus be dealt with on a comprehensive basis which takes into account the complexity of international traffic handling. It is worth while to explain the need to provide for the possibility of several subsequent overflows; this may be done as follows:

Let us take the case of a call from I to destination J, which overflows and is routed to a first transit centre K₁. Once it has reached K₁, the call is merged with the calls originating in K₁, and there is no way of distinguishing it from them (signalling codes do not include any "indicator" specifying the international centre of origin of a call or the route which it has already followed). At centre K₁, if the group K₁J is not last choice, our call originating in I and having reached K₁ may receive an overflow routing to a second transit centre K₂. The same process will be repeated once it has reached K₂.

In a non-hierarchical network, the possibility of several overflows in succession will make it necessary to ensure that a call from I does not, over a route determined by several successive overflows, make a complete circle and find itself back in I before reaching J.

3. GUIDELINES FOR THE STUDY

The basic hypotheses mentioned in 2 could, for the purpose of our study, be formulated in four guidelines (abbreviated to GL):

GL 1 A first choice group can only overflow to a last choice group which, by definition, cannot overflow.

GL 2 The loss probability for the whole of the traffic routed over a last choice group is the maximum value adopted to define grade of service (assumed loss probability, taken as 6% in our study).

GL 3 In any centre, overflow is on an exclusive basis, a single transit centre being determined at each group originating centre for each orientated pair of centres.

GL 4 For each pair of terminal centres, the criterion adopted to determine the transit centre consists in concentrating the overflow traffic to the maximum possible extent over some groups.

The four guidelines GL 1 to GL 4 actually reflect the existing reality very closely with regard to the organization of the developed national networks. In any case, since the traffic under consideration is busy hour traffic there would be little point in trying to improve the quasi-optimum solution obtained.

4. PROCEDURE FOR STRUCTURING THE NETWORK

The aim is to dimension correctly each group between any pair of centres so as to minimize the total cost of the system while observing the grade of service standard fixed at the outset. This process can be divided into three phases.

4.1. In the first phase, a transit centre K to which the traffic IQ might overflow is determined for each orientated pair of centres IQ. The determination of K depends on two criteria:

(i) the observance of the grade of service standard set for IQ traffic;

(ii) the cost ratio between the direct route IQ and the alternative route passing through K.

4.2. In the second phase, the quality (first or last choice) of each IQ group is set according to two parameters:

(i) the total traffic flow likely to be routed over IQ;

(ii) the cost ratio between the direct route IQ and the alternative route.

4.3. In the third phase, the following is calculated:

(i) the loss probability for each first choice group (i.e., the overflow probability of that group);

(ii) the number of circuits for each group (first or last choice);

(iii) the resulting costs.

The calculations in the third phase are based, with regard to (i), on the equalization of the marginal costs between direct and alternative routing and, with regard to (ii), on the Erlang formula and Wilkinson's theory.

5. FIRST PHASE OF THE STUDY: CHOICE OF A TRANSIT CENTRE FOR A RELATION BETWEEN A PAIR OF TERMINAL CENTRES

This involves determining, for each orientated pair of centres, a single transit centre, the choice is governed by guideline GL 4 outlined above.

5.1. First, we estimate the "potential overflow" on the basis of the well-known fact that in practice overflow is advantageous only on low-traffic groups. For this purpose, the practical rules of Table 1 are used for assessing the "potential overflow traffic":

<table>
<thead>
<tr>
<th>Generated traffic A</th>
<th>Estimated potential overflow traffic</th>
</tr>
</thead>
<tbody>
<tr>
<td>A ≤ 8 E</td>
<td>A/2</td>
</tr>
<tr>
<td>8E &lt; A ≤ 16 E</td>
<td>√A</td>
</tr>
<tr>
<td>16 E &lt; A</td>
<td>0</td>
</tr>
</tbody>
</table>

These estimated values have been determined by calculations involving simple cases and should naturally be regarded as very rough approximations.

5.2. Second, for each orientated pair of centres IQ, we make an initial choice of 4 transit centres (K₁, KII, KIII, KIV) likely to be selected as transit centre K for the relation IQ. This choice is based on economic considerations relating to routing costs (expressed as "annual charges"); the parameter used is the cost ratio R advocated in CCITT Recommendation Q.89 and defined as follows:

\[ \text{cost of one additional circuit on alternative route } \text{IQ} \]

\[ \text{cost of one additional circuit on direct route } \text{IQ} \]

For alternative routing to be economic, the additional distance entailed by the alternative route should not be too great; in other words the
The procedure followed with regard to the routing, through successive overflows of a call to its destination centre is fully in conformity with the principle of "far-to-near sequence" advocated in CCITT Recommendation Q.13 (in 5.2.3 (b)).

5.5 Annex 2 contains the diagram of the iteration process performed by the computer to determine each transit centre K to be associated with any oriented pair IJ.

6. SECOND PHASE OF THE STUDY. QUALITY OF A GROUP : FIRST CHOICE GROUP OR LAST CHOICE GROUP?

In principle, the first choice groups are groups with a small number of circuits. To determine whether the traffic on a group can overflow at the centre which is its originating terminal, two criteria are applied:

- the grade of service criterion,
- an economic cost ratio criterion.

6.1 GRADE OF SERVICE CRITERION

6.1.1 Let us ascertain what grade of service is provided when there is overflow, first taking the simplest case of a first choice A-B, with triangular alternative routing using two sections AC and CB in tandem, both being last choice groups* (Figure 1).

Let us consider a pair (I, J). There are two alternatives:

- either I and J are immediate neighbours; it may then be considered that the traffic from I to J will be too heavy for alternative routing;
- or I and J are not immediate neighbours. If d (I, J) expresses the distance between two centres I and J, we then have a centre K such that:

\[ d (K, J) < d (I, J) \]

which can serve as transit centre for the group IJ. For the call to go in a circle the transit centre for the overflow traffic of (K, J) would have to be I. However:

- either K is next to J and we have seen that the calls from K to neighbouring J do not overflow.
- or K is not next to J and we then have L, such that d (I, J) > d (K, J) > d (L, J), and the transit centre selected for the groups (K, J) will be L.

And so on; a transit centre T next to J will inevitably be selected and the call will therefore reach J without again passing through I.

Remarks:

(1) If the network had a less regular structure, it would be useful to provide a computer subroutine to check that no alternative path included a closed loop.

(2) The procedure followed with regard to the routing, through successive overflows of a call to its destination centre is fully in

* On the overflow routes of the network there is always a traffic relation where the only permissible overflow involves two last choice groups in tandem. Owing to the layout of the overflow centres, the alternative routing of a call from A to B brings the call nearer to B at every stage. The call will thus arrive at a transit centre K where the traffic generated to B is sufficiently heavy for the group (K, B) to be a last choice group (see remark in 5.4). But the call has to reach K by a last choice circuit since it is overflow traffic.
Taking $P'_{AB}$, $P_{AC}$ and $P_{BC}$ at their maximum value $q$, we obtain:

$$P'_{AB} = q = P_{AB} \left[ 1 - (1-q) (1-q) \right]$$

hence:

$$P_{AB} = \frac{q}{2q - q^2} = \frac{1}{2} \left[ 1 - \frac{1}{1 - q^2} \right] \quad 1 - 0.05 \# \frac{1}{2}$$

We thus find that, so far as maintaining the grade of service standard between A and B is concerned, the overflow $B_{AB}$ may lie between 6% (if the overflow was ≤ 6%, the group AB would actually be calculated as a last choice group) and roughly 50% of the traffic offered from A to destination B.

The value 50% will also serve as an approximation of the potential overflow traffic in the evaluation made in Table [1] for the case of low-traffic relations (≤ 8 Erlangs).

6.1.2 In the case of alternative routing with a more complex route owing to successive overflows, we can refer back to the triangular situation mentioned in 6.1.1. Let us take the case of such a route ACDB (Figure 2):

![Diagram of ACDB and AB routes](image)

The groups AC and CD to which groups AB and CB respectively overflow must be last choice groups. Group DB is so by hypotheses when we extrapolate from the reasoning based on the situation described in 6.1.1.

A call coming from A and reaching C or a call originating in A must have the grade of service $q$ and all the routes, CA (direct) or CDB (alternative), may be assimilated to a hypothetical last choice with loss probability $q$.

The conclusions drawn from 6.1.1 with regard to the loss probability $P_{AB}$ of group AB (i.e. its overflow $B_{AB}$) thus remain valid.

6.2 ECONOMIC CRITERION

Provided the constraint $B_{AB} \leq 0.5$ is observed, the only criterion determining the flow of traffic which should be overflowed from AB is thus the economic criterion. This criterion is based on the equalization of marginal costs. In the situation in Figure 1, the marginal cost of an Erlang sent directly over group AB will be equalized with its cost when it passes first over group AC and then CB. It should be noted that this procedure, in the case of Figure 2, results in the cost of routing an Erlang over group CB being equal to that obtained over groups CD and DB.

For each first choice group (pair I-J), the first phase of our study led us to choose a transit centre K which is therefore associated with IJ.

We shall calculate the cost ratio $R$ (costs expressed as "annual financial charges") already mentioned in 5.2. The value of $R$ for group IJ (and for centre K associated with the pair IJ) should be as low as possible (i.e. as near to 1 as possible) for a significant advantage to be gained from overflow.

Moreover, it is well known that, the lower the traffic $A_{IJ}$, the greater are the advantages of overflow.

To select from all the centres of the network under study the pairs IJ which must be the first to be considered as warranting connection by first choice groups, we shall thus use a product $X$ of the formula

$$X = \left( \frac{R}{1} \right)^m \cdot (A_{IJ})^n$$

simplicity's sake, that the values $m$ and $n$ are equal to 1, so that:

$$X = B_{IJ} \cdot A_{IJ}$$

$A_{IJ}$ being the estimated traffic between IJ (traffic taking into account potential overflow on IJ). We shall proceed by iteration as follows:

(a) we shall calculate $X$ for any pair IJ. The group(s) with the lowest value(s) for the produce $X$ will be established as first choice group(s) if this is compatible with the existing structure of the network. We then determine, in accordance with 111.2, the overflow routes for the group(s) established as first choice, and these routes are then classified as last choice, provided that the compatibility mentioned above is maintained;

(b) the products $X$ of the groups whose choice has already been established are then withdrawn from the list and the process outlined in (a) is repeated until the quality (first or last choice) compatible with the network structure has been determined for all the groups.

A diagram of these operations is given in Annex 3.

7. A computer programme has been set up for the calculations involved in all the phases referred to in Section 4; this programme can be used for an economic cost comparison of the international networks. Written in Fortran, the programme was initially set up on the computer belonging to the Polytechnique Fédérale de Lausanne as a student project and then transferred to the ITU computer, on which it is now operational.

ANNEX 1

DESCRIPTION OF THE MODEL NETWORK

1. The model network, a geometrical approximation of the European network, is a matrix with five lines and five columns. Each box in the matrix represents one country (25 countries). At the centre of each box, there is an international exchange (an international "centre"). Each box is assumed to have a side of 500km, "standard unit of length", enabling to use the CCIT international accounting rates of Recommendation D.301 K for cost evaluations (rates fixed per 100 km crow-flight distances between international centres).
2. The traffic between "countries" (boxes of the matrix) is defined by a relationship between:

\[ K_I \]

A

\[ A_{ij} \]

traffic (busy hour traffic) to be carried between box "i", "j" and box "KI";

\[ K_I \]

distance between international centres in the middle of each box;

As a rough approximation (but one which corresponds fairly realistically to the existing European network and traffic situation), the law \( A = f(A) \) given by the following table is used for the model network.

**ANNEX 2**

**FLOW CHART FOR PHASE 1**

Iteration process for determination of the K transit centre (only one) which may be associated with each group IJ

<table>
<thead>
<tr>
<th>( \Delta^2 )</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 &lt; \Delta^2 &lt; 2 )</td>
<td>500 Erlangs</td>
</tr>
<tr>
<td>( 2 &lt; \Delta^2 &lt; 5 )</td>
<td>60 Erlangs</td>
</tr>
<tr>
<td>( 5 &lt; \Delta^2 &lt; 9 )</td>
<td>8 Erlangs</td>
</tr>
<tr>
<td>( 9 &lt; \Delta^2 &lt; 32 )</td>
<td>1 Erlang</td>
</tr>
</tbody>
</table>

**ANNEX 3**

**FLOW CHART FOR PHASE 2**

Iteration process for determination of the quality (first or last choice) of each group

1. Draw up the matrix of potential overflows from pairs IJ:
   - For any pair IJ, compute the estimated value of the potential overflow from IJ (according to \( A_{ij} \) and Table 1).

2. Overflows from pairs IJ:
   - For any pair IJ, select 4 transit centres \( K_{1J}, K_{2J}, K_{3J}, K_{4J} \) which may be associated with IJ (according to the R cost ratio).

3. Draw up the matrix of overflows to pairs IK:
   - For any pair IK, assess the maximum potential traffic which may be handled through the IK group.

4. Determine the maximum value in the above matrix. It occurs for a pair IJK.

5. Determine the centre L such that IJK admits IJK as overflow group.

6. Fix K as transit centre for the group IJK.

7. Correct the traffic matrix by elimination of overflow traffic to be discarded.

8. Are there any pair of centres for which the overflow transit centre has not yet been determined universally?

9. Compute for each pair (IJ) the product \( K_{IJ} : A_{IJ} \).

10. Draw up the corresponding matrix.

11. Determine the pair (I, J) for which X is maximum.

12. Can we fix IJ as a first choice group?

13. Can IJ overflow over the group IJK? K being the transit centres associated (see Phase 1) with the IJ group.

14. Fix IJK as first choice group and IJK as last choice group.

15. Fix IJ as first choice group and IJK as last choice group.

16. Delete X (IJK) and X (IK) from the above matrix.

17. Are all the groups IJ fixed as first choice or last choice groups?

18. STOP