ABSTRACT

This paper is concerned with the optimisation of the method, where the data collected from a previous measurement plays an important part in the measurement-dimensioning cycle.

As any traffic measurement is only an estimate of the offered traffic, further usage of the data will result in errors, directly caused by the sampling process. If the sampling variance is known, this imprecision can be allowed for in the dimensioning process. This adjustment represents an additional cost penalty attributed to the practicality of traffic measurements.

In this paper, it is assumed that the cost of the traffic study is linearly related to the duration of the measurement. The cost penalty is shown to be approximately inversely proportional to the time spent in conducting the experiment. If we consider the measurement phase as part of an investment, clearly, by choosing the traffic measuring parameters, the return on investment can be maximised.

The above concept is applied to the occupancy measurement of a first choice high usage route in a simple triangular alternative routing pattern. The traditional network cost minimisation technique and full availability working are assumed.

Results obtained indicate that, from some route parameters, one can estimate the optimum time duration for a traffic study, such that the measurement costs equal the cost penalty. In general, optimum occurs where:

\[
\frac{d(\text{measurement cost})}{dt} = -\frac{d(\text{cost penalty})}{dt}
\]

1. INTRODUCTION

Any traffic measurement is an estimate of the average traffic flow. In principle, for a given technique, it is possible to estimate the sampling variance and other statistical parameters related to the measurement. Calculations based on the measurement will therefore be imprecise. In practice, this error is allowed for by an adjustment to the dimensioning process. This adjustment is a cost penalty arising from the practicality of traffic measurements. Generally for a given technique, doubling the duration of measurement halves the sampling variance and roughly doubles the measurement costs. In return, the reduced error decreases the cost penalty by allowing more accurate prediction of circuit requirements.

If the policy of a measurement-upgrading technique is adopted, then the measurement must be considered as part of the capital investment which can be optimised, such that no more effort should be spent in data acquisition than necessary.

For simplicity, this concept is applied to the measurement of a high usage route in a simple triangular alternative routing pattern. The traditional network cost minimisation and full availability with direct access are assumed.

Results obtained thus far indicate that given some route parameters it is possible to estimate the optimum time of a traffic study such that the return on investment for a measurement-dimensioning cycle can be maximised.

2. RESUMÉ OF SOME BASIC TOOLS

Consider the simplest triangular alternate routing pattern

![FIG.1: A SIMPLE ALTERNATE ROUTING PATTERN](image)

The assumptions, definitions and notations used in Pratt's paper will be retained in this discussion (Ref.1). Some of the related definitions used here are as follows:-

Consider a route of N circuits, offered traffic A and carried traffic \( Y = (A-a) \), where a is the traffic lost or overflowing.

The route congestion is \( G = a/A \) (1)

(i) Marginal occupancy, \( H = \frac{2a}{N N_A} \) (2)

(ii) Marginal capacity, \( B = \frac{2a}{N N_E} \) (3)

For the link 1,

\[ N_i = \text{number of circuits provided for I-J traffic} \]
\[ C_i = \text{Cost per circuit} \]
\[ A_i = \text{Traffic offered} \]
\[ a_i = \text{Traffic overflowing} \]

For the simple case of Fig.1, the traditional optimisation equation is,

\[ \frac{C_1}{N_1} = \frac{C_2}{N_2} + \frac{C_3}{N_3} \] (4)

Further, for a full availability direct access route, a simple practical relation between \( A, N \) and \( H \) can be shown to be:-

\[ H = A \left( E_N(A) - E_{N+1}(A) \right) \] (5)

where \( E_N(A) = \frac{A^N}{N!} \sum_{i=0}^{N} \frac{1}{i!} A^i / N^i \), the Erlang Loss Formula. (6)
Note that \( H \) is also referred to as the Cost Factor.

3. DIMENSIONING COST PENALTY

Consider a traffic study conducted to determine the traffic on the high usage route. If the measurement period is sufficiently long then the standard deviation of the error, \( \sigma \), on the measured traffic \( Y \), will be negligibly small. Subsequent dimensioning using the traditional optimisation method will result in a near "perfect" solution, such that the cost of this network is at a minimum.

If the direct route is dimensioned on the estimated mean offered traffic \( A \) with an appreciable error \( \sigma \), then the overflow to the final route will be different from that calculated by an amount (see appendix A).

\[
\Delta U = U(A+S) + U(A-S) - 2U(A)
\]

where \( U(A) = A, E_N \) \( A \), the overflow traffic for \( N \) ccts

and

\[
U(A+S) = (A+S), E_N (A+S)
\]

\[
U(A-S) = (A-S), E_N (A-S)
\]

The expected cost of this extra unpredicted traffic is

\[
\frac{(\Delta U)^2}{H} C_d
\]

Where \( C_d \) is the cost of the direct circuit.

This cost represents the dimensioning cost penalty in not being able to estimate the offered traffic accurately.

As a matter of convenience we shall introduce two definitions:

\[
\text{Cost Penalty} \quad P_c = \frac{\Delta U}{H} C_d
\]

\[
\text{Penalty Function} \quad P = \frac{\Delta U}{H}
\]

For a given \( A \) and \( H \), the optimum number of direct circuits, \( N \) can be determined, and hence \( \Delta U \), provided that \( S \), the sample standard deviation is also known. For a given measurement technique, \( S \) is directly related to the measurement duration, \( t_m \).

Clearly, for a given \( A \) and \( H \), the penalty function is related to \( t_m \).

4. SIMPLIFIED RELATIONSHIP BETWEEN \( S \) AND \( t_m \)

For a scanning method for a route of infinite size, Hayward (Ref.3) showed that the variance of \( n \) as an estimator of \( Y \) is given by:

\[
V(n;Y) = \frac{N}{T} \left( \frac{r \cotanh \left( \frac{r}{2} \right) - 2} {360} + \frac{2}{6} \right)
\]

where \( r = \frac{m}{h} \) ratio of scan interval to holding time

\( T = \) length of observation period

\( M = \) the unbiased estimate of \( Y \), the carried traffic.

\[
\cotanh \left( \frac{r}{2} \right) = \frac{1-e^{-r}}{1+e^{-r}}
\]

\[
r \cotanh \left( \frac{r}{2} \right) = 2 + \frac{2}{6} - \frac{4}{360} + \frac{6}{15120}
\]

For practical cases, \( e^{-r} \rightarrow 0 \), and \( N \) and \( Y \) are nearly equal, and further assume that the offered traffic \( A \) is very close to \( Y \).

Hence \( V(n;A) \neq \frac{A}{T} \left( 2 - \frac{2}{6} + \frac{6}{r} \right) \)

Now

\[
S^2 = V(n;A)
\]

and if \( r = 1 \) (eg. 3 min. scan interval, 3 min. holding time)

then

\[
T = \frac{7A}{6S^2} + \sqrt{1.36A^2 - \frac{2}{S^2}} \quad (13)
\]

Normally, \( S = \frac{A}{10} \)

i.e. specifying that no more than 10% error in \( A \)

Then \( T = \frac{2.33A}{S^2} \quad (14) \)

For simplicity \( T = \frac{2A}{S^2} \quad (15) \)

Despite the simplifications and the assumptions made in Hayward's paper, this simple formula is sufficient for practical uses.

Given the holding time \( h \), the observation period in real time is given by

\[
t_m = \frac{2Ah}{S^2} \quad (16)
\]

For our purposes we shall take \( h \) as 3 minutes and \( A \) as the average time consistent busy hour traffic. A common practice used in determining this traffic is to perform the measurement in units of days, and hence select the time consistent busy hour from the samples. Clearly, unless there is some other means of obtaining prior information as regards the busy hour, this approach of data collection means that every 60 minutes of \( t_m \) in fact represent a day of observation.

5. RELATION BETWEEN THE PENALTY FUNCTION AND MEASUREMENT TIME

For a given traffic, cost factor and standard deviation, it is possible to work out the Penalty Function \( P \), and the length of the observation time \( t_m \).

Numerical analysis reveals that, in general an equation of the type

\[
P = \frac{1}{a+Bt_m} \quad (17)
\]

consistently gives a remarkably good fit to the data generated from the above system of equations. Table 1 is a list of the coefficients \( a, B \) for some typical \( H \) and \( A \) values.

<table>
<thead>
<tr>
<th>Traffic, ( A ) in Erlangs</th>
<th>( a )</th>
<th>( B )</th>
<th>Cost Factor ( H )</th>
<th>Optimum Circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.0763</td>
<td>7.50</td>
<td>0.4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>-0.3132</td>
<td>22.73</td>
<td>0.4</td>
<td>2</td>
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<td>-1.8</td>
<td>81.00</td>
<td>0.6</td>
<td>1</td>
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<td>27</td>
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<td>23</td>
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<td>20</td>
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<td>116</td>
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<td></td>
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<td>100</td>
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<td>1.0526</td>
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</tr>
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<td></td>
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<td>0.4</td>
<td>212</td>
</tr>
<tr>
<td></td>
<td>0.0036</td>
<td>3.6410</td>
<td>0.6</td>
<td>201</td>
</tr>
</tbody>
</table>

Table 1: VALUES OF \( a, B \) FOR TYPICAL VALUES OF TRAFFIC AND COST FACTOR

\( a \) may be ignored, and a simple relation between the Penalty Function and measurement time (in days) results.

\[
P = \frac{1}{Bt_m} \quad (18)
\]
It follows that the Dimensioning Cost Penalty is:

$$ P_c = \frac{C_d}{Bt_m} \tag{19} $$

6. MEASUREMENT COSTS

For a given $t$, equation (19) provides a convenient means of estimating the dimensioning cost penalty due to insufficient information regarding the true traffic. In principle, it is also possible to estimate the cost of acquiring this information.

In general, measurement cost is a complicated function. For most modern mechanised measurement systems, the following costs should be taken into account:

- Development of measurement equipment
- Capital investment in the measurement equipment
- Development and maintenance of processing software
- Measurement set up and check out
- Supervision of measurement
- Analysis of reports

As an approximation, assume that a linear relationship exists between the cost of measurement, $M_c$, and the time spent, $t_m$.

$$ M_c = C_m t_m \tag{20} $$

where $C_m$ = cost of measurement/group/day.

7. OPTIMUM MEASUREMENT DURATION $t_{\text{opt}}$

The total cost $D$, is a sum of the Dimensioning Cost Penalty $P_c$ and the Measurement Costs $M_c$ (see Fig.2):

$$ D = (P_c + M_c) \frac{Cd}{Bt_m} \tag{21} $$

Now

$$ \frac{dD}{dt_m} = -\frac{Cd}{Bt_m^2} \tag{22} $$

At

$$ \frac{dD}{dt_m} = 0 $$

$$ C_d = C_m \frac{Cd}{Bt_m^2} \tag{23} $$

Hence, the optimum duration of an occupancy traffic study is:

$$ t_{\text{opt}} = \frac{1}{\sqrt{BM}} \tag{24} $$

where

$$ M = \frac{C_m}{Cd} \tag{25} $$

In principle, $M$ can be estimated providing the cost of a direct circuit $C_d$, and the cost of measurement/group/day, $C_m$, is known. For a fair comparison, $C_d$ and $C_m$ must be at the same scale. If $C_d$ represents the "annual" charges, $C_m$ must also be annual charges.

8. ESTIMATING THE B CONSTANT

The optimum duration of a traffic study $t_{\text{opt}}$ is completely defined by $B$ and $M$. With $M$ estimated, the next task is to find the $B$ constant.

Now

$$ B = f(N,A,N) $$

Numerical analysis and a graphical plot of $B$ versus $H$ (Fig.3) reveals that in general, for a particular $N$, (uniquely determined by $A$ and a range of $H$) $B$ is a linear function of $H$ for ranges of $H$.

$$ B = \mu H \tag{26} $$

The simplicity of equation 26 is useful for practical uses. If $\mu$ is also known, then the problem of estimating $t_{\text{opt}}$ becomes easy.

From Fig.3, one is led to suspect that $\mu$, the gradient is some function of $N$ (and indirectly, related to $A$ and $H$).

Unfortunately, further numerical analysis of $\mu$ variation on $A$ or $H$ did not reveal any simple relation.

Fig.4 is a graphical representation of $\mu$ versus $N$ for different values of traffic. Clearly, as will be shown by an example, this graph can be used to estimate the optimum measurement time period, provided other parameters are available.
ESTIMATING THE OPTIMUM MEASUREMENT TIME: AN EXAMPLE

Suppose we wish to determine the optimum observation time for a direct circuit, given the following information.

(a) Cost of direct circuit (per annum) \( C_d = \$1500 \)
(b) Cost of measurement/group/day (per annum) \( C_m = \$5 \)
(c) Estimated traffic expected on route \( A = 10 \text{ Erlang} \)
(d) Cost factor to be used in dimensioning \( H = 0.6 \)

For \( A = 10 \text{ Erlang} \) and at \( H = 0.6 \), the optimum direct route size given by equation (5) or from standard graphs (Refs.4, 6) is \( N = 9 \).

From Fig.4, for \( N = 9 \) and \( A = 10, \mu = 35 \).

The \( B \) constant as given by \( B = \mu H \) is \( B = 35 \times 0.6 = 21 \).

The measurement cost/circuit cost ratio \( M = \left( \frac{C_m}{C_d} \right) = 0.0033 \).

Hence the optimum study duration, \( t_{\text{opt}} \) is \( \frac{1}{\sqrt{MB}} = 4 \text{ days} \).

Clearly, for a 5-day business cycle type of route, a minimum of 5 days will be spent on the actual measurement.

SENSITIVITY OF \( M \) AND \( B \) TO ERROR

The key equation in estimating the optimum measurement time is

\[
t_{\text{opt}} = \frac{1}{\sqrt{MB}}
\]

For a fixed \( M \) value, it is interesting to find the variation of \( t_{\text{opt}} \) with respect to \( B \) and \( M \) (Table 2).

<table>
<thead>
<tr>
<th>Traffic, ( A ) (Erlang)</th>
<th>Optimum ( C_{\text{ct}},N ) (( H=0.6 ))</th>
<th>( \mu )</th>
<th>( B=\mu H )</th>
<th>( t_{\text{opt}} = \frac{1}{\sqrt{MB}} ) (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
<td>62</td>
<td>37.2</td>
<td>2.85</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>46</td>
<td>27.6</td>
<td>3.11</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>38</td>
<td>22.8</td>
<td>3.65</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>33</td>
<td>19.8</td>
<td>3.91</td>
</tr>
<tr>
<td>12</td>
<td>11</td>
<td>30</td>
<td>18.0</td>
<td>4.10</td>
</tr>
<tr>
<td>14</td>
<td>13</td>
<td>27</td>
<td>16.2</td>
<td>4.32</td>
</tr>
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<td>16</td>
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<td>4.49</td>
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</tr>
<tr>
<td>20</td>
<td>20</td>
<td>16</td>
<td>9.6</td>
<td>5.62</td>
</tr>
</tbody>
</table>

TABLE 2: VARIATION OF \( t_{\text{opt}} \) w.r.t. \( B,N \).

The restriction placed on using this method is that an initial guess of the traffic must be available (possibly from past measurements). Table 2 shows the error made in estimating \( t_{\text{opt}} \) is marginal. On the other hand, errors in estimating \( M \) will have a marked effect on \( t_{\text{opt}} \). Clearly for practical usage, \( M \) should be estimated more precisely.

CONCLUSIONS

This study has shown that from some route parameters and an initial estimate of the traffic, it is possible to estimate the optimum time for a traffic measurement.

Although the study has been confined to the case of a full availability first choice route in a simple alternative plan, this principle can be extended to other high usage routes of more complex overflow patterns. Tests (not listed in this paper) indicate that the concept can indeed be applied to first choice routes with link access and limited availability.

The application of the method requires some prior knowledge of the traffic, so that the optimum number of direct circuits, and the parameter \( B \) can be found, and the optimum study period \( t_{\text{opt}} \) estimated.

Inspection of the key equation \( t_{\text{opt}} = \frac{1}{\sqrt{MB}} \) and the associated graphs show that in practice, \( t_{\text{opt}} \) is more sensitive to \( M \) than \( A \). Hence, even if the initial traffic estimate is not precise (making an error in \( B \)) the error in \( t_{\text{opt}} \) is relatively small.

Effort should be spent in getting a more precise estimate of the measurement-circuit cost ratio, \( M \). As in all engineering economics, the estimation of costs represent a real problem. In reality, \( M \) is a comparison between capital costs and labour costs. The effectiveness of the measurement-provisioning policy depends on how accurately one can estimate the ratio of manhour to equipment cost.

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the Staff of Traffic Engineering Section, New South Wales for the assistance.
If an element of A Erlang is estimated (measured) with a sampling error (standard deviation) of \( S \) Erlang, then the probability density function of the estimate is given by

\[
p(x) = \frac{f(x-A)}{S}
\]

(A-1)

normalised by letting

\[
B = \frac{(x-A)}{S}
\]

we have

\[
p(x) = \phi(B)
\]

(A-2)

Where \( \phi \) is the probability density function for a normal distribution, with mean = 0 and variance = 1.

Assume this traffic, \( A \) is offered to a route, and it is desired to estimate some quantity such as carried traffic, lost traffic, variance of lost traffic etc.

The estimate of this quantity is a function of the estimated traffic \( g(x) \), with an expected value of

\[
E(g) = \int p(x) \cdot g(x) \, dx
\]

(A-3)

Let \( G(B) = g(x) \),

and assume \( G(B) \) can be expanded as a series

\[
G(B) = a_0 + a_1B + a_2B^2 + \ldots
\]

(A-4)

Then

\[
E(g) = \int \phi(B) \cdot (a_0 + a_1B + a_2B^2 + \ldots) \, dB
\]

(A-5)

\[
= a_0 + a_1E(B) + a_2E(B^2) + \ldots
\]

(A-6)

(See Ref. 5 page No.147)

Now

\[
G(1) = a_0 + a_1 + a_2 + a_3 + \ldots
\]

(A-7)

\[
G(-1) = a_0 - a_1 + a_2 - a_3 + \ldots
\]

Hence

\[
\frac{G(1) + G(-1)}{2} = a_0 + a_2 + a_4 + \ldots
\]

(A-8)

If the 4th and higher powers are negligible, then

\[
E(g) = \frac{G(1) + G(-1)}{2}
\]

(A-9)

\[
= \frac{g(A+S) + g(A-S)}{2}
\]

(A-10)

This estimate differs from the value which would apply if the traffic was known precisely to be \( A \). The expected difference is:

\[
E(d) = \frac{g(A+S) + g(A-S) - 2g(A)}{2}
\]

(A-10)

If \( g(A) = A \cdot \text{En}(A) \), the overflow traffic, then

\[
E(d) = \Delta U \text{, the unpredicted overflow.}
\]

REFERENCES: