Hypothesis of a Toll Network with Separate Routes for First-Choice and Overflow Traffic

G. Diano and P. Pallotta
SIP — Società Italiana per l'Esercizio Telefonico, Roma, Italy

G. Masetti
SIP — Società Italiana per l'Esercizio Telefonico, Bologna, Italy

ABSTRACT

The paper develops the subject, dealt with previously, of finding a hierarchical toll structure which would not only be the economic optimum but would also operate well in overload conditions.

The structure envisaged, called book-network because of the configuration of its routing chart, has the characteristic of always keeping-choice and overflow traffic separate by splitting up the trunk groups and the transit exchanges.

Having defined the calculation methodology and established the design of this new network, it is compared with the traditional one from the point of view of costs and of efficiency, both in design and overload conditions; the latter are taken, for the sake of simplicity, as shortage of circuits on high-usage or final trunk groups.

The comparison shows that the book-network is more capable of guaranteeing, in overload conditions, the handling of the traffic of the small relations but is more expensive than the traditional network; however, when it becomes necessary to split up transit exchanges in operation the difference in cost becomes negligible.

1. INTRODUCTION

The design criterion necessary for the achievement of smaller streams of traffic in the event of accidental or permanent overloading, already dealt with in a paper presented to the Stockholm Teletraffic Congress (see Bibl. 1), are taken up and more thoroughly examined in this document.

Whereas the first phase of the study dealt with the subject in a general way — examination of several hierarchical networks, comparison criteria, overloading assumptions — the present work concentrates on a specific examination of a particular hierarchical network structure, which we shall call "book" type, where first-choice and overflow traffic are never mixed.

Firstly, the design parameter of this network are found and a calculation is then made to compare its costs and efficiency with the traditional hierarchical network, taking a real complex system, such as the SIP direct-dial network. This study is particularly relevant since in the Italian network (and presumably not only in it) the transit exchanges have, over the years, expanded at the same rate as the increase in the network (an average annual rate of over 10%), often causing saturation in the buildings where they originate. It soon becomes necessary, therefore, to split up the exchanges and this provides an opportunity to choose new network structures, taking "protection" against overloading of all the traffic streams into account in the economic design criterion (hierarchical network with alternate routings).

2. THE SIP DIRECT-DIAL NETWORK

As an essential reference, Fig. 1 gives the diagram of traffic routings in the present SIP direct-dial network, of the hierarchical type with three levels and with high-usage direct connections. Local areas with univocal numbering, for which the interurban code must be used for outgoing communications, are not included in the diagram.

![Diagram of the SIP inter-district Direct-Dial network](image)

Fig. 1 - Diagram of the SIP inter-district Direct-Dial network

A concise description of the network is given in Table 1, which shows the composition of the elementary traffic relations; in Table 2, regarding the composition of the traffic routing; and in Table 3 which gives the number and types of circuits composing the network in the year 1976**.

The data given in the tables show that the larger streams of traffic represent a very limited number of relations, while most of the relations, on the other hand, have very little traffic. This network guarantees a progressively better service to the larger streams, as a higher number of routing possibilities are provided for it.

** The data contained in the latter table do not correspond exactly to the real network (the plan of which includes appropriate redundancies and which is also affected by certain local plant constraints) but to its optimal design subject to the same constraints as the book network which will be dealt with next; this is the only way in which an economic and behavioural comparison can be made between the present network, taken as a reference, and the book network under consideration.
or with a group, originating from previous-choice groups, on which the traffic of as time passes, with the increase in volumes of traffic, affecting programmes etc.) the larger traffic relations a maximum overflow percentage, establishing, for many
work structure with first-choice and overflow traffic on the transit routes
or the said book network is adopted for comparison with the
structure, considerably affects the quality of service as well as of the basic elements needed for the traffic forecasts on which the network design is based.

### 3. BOOK-NETWORK

### 3.1 STRUCTURE AND CHARACTERISTICS

The basic characteristic of the book-network is that it provides for complete separation of first-choice from overflow traffic, both in the trunk-groups and in the CC transit exchanges. There are two choices for all the streams of traffic outgoing from each District Centre.

First-choice traffic routing can be:

- a) - with direct DC-DC trunk-groups for larger relations (on an economic basis);
- b) - or with direct groups originating DC - destination CC linked there to the first-choice transit section (to which only groups for first-choice traffic are linked towards all the dependent DC's), this is also done on the basis of economic considerations;
- c) - or with a group, originating DC - originating CC, towards the transit section reserved for first-choice traffic and from there routed towards the destination DC by a high-usage group, if this is more economical, or else via destination CC. The trunk-groups linking the first-choice transit sections of the CC with each other and with the dependent DC always exist for the routing of all the traffic relations which have not got a) or b) groups.

In this network measurement of traffic is also simpler and it is therefore possible to have a more exact idea of the quality of service as well as of the basic elements needed for the traffic forecasts on which the network design is based.

### Table 1 - Composition of DC-DC Traffic

<table>
<thead>
<tr>
<th>Group</th>
<th>Circuits No.</th>
<th>Trunk-groups No.</th>
<th>% of No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-DC</td>
<td>43,010</td>
<td>48,6</td>
<td>62,9</td>
</tr>
<tr>
<td>DC-CC high-usage</td>
<td>4,152</td>
<td>4,7</td>
<td>10,3</td>
</tr>
<tr>
<td>DC-CC final</td>
<td>14,416</td>
<td>18,3</td>
<td>5,1</td>
</tr>
<tr>
<td>CC-DC high-usage</td>
<td>3,552</td>
<td>4,0</td>
<td>9,2</td>
</tr>
<tr>
<td>CC-CC final</td>
<td>15,025</td>
<td>17,0</td>
<td>51</td>
</tr>
<tr>
<td>CC-DC</td>
<td>8,316</td>
<td>9,4</td>
<td>7,4</td>
</tr>
<tr>
<td>TOTAL</td>
<td>88,451</td>
<td>100,0</td>
<td>100,0</td>
</tr>
</tbody>
</table>

### Table 2 - Amount and composition of the SIP inter-district network

<table>
<thead>
<tr>
<th>With direct trunk-group</th>
<th>With 1 transit</th>
<th>With 2 transits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circuits No.</td>
<td>5,5</td>
<td>32,0</td>
</tr>
<tr>
<td>% of No.</td>
<td>87,5</td>
<td>68,5</td>
</tr>
<tr>
<td>Traffic offered</td>
<td>76,6</td>
<td>17,0</td>
</tr>
</tbody>
</table>

### Table 3 - Traffic routing

The smaller streams are routed via transit, where this first-choice traffic is routed along with the overflow from previous-choice groups, on which the traffic of the larger relations is routed. In the event of accidental phenomena (traffic overloads, breakdown, delay in effecting programmes etc.) the larger traffic relations are therefore less vulnerable, whether the relation itself is directly affected or whether it is a consequence of phenomena caused by other relations. Furthermore, as time passes, with the increase in volumes of traffic, direct trunk-groups also increase (in number and size) and anomalies which can affect the last-choice routings, where the remaining small traffic relations are routed, also increase in quantity and volume. This phenomenon, connected with the traditional network structure, considerably affects the quality of service given to the various traffic relations.

In view of the fact that no network structure can provide - within reasonable cost limits - the same level of service for smaller relations as it can for larger ones, many corrections have already been made to the present network plan, on a purely economic basis, to obviate - at least partly - the disadvantage mentioned (e.g. fixing a maximum overflow percentage, establishing, for many relations, direct trunk-groups not always economically justifiable etc.) nevertheless, there is still a mixing of first-choice and overflow traffic on the transit routes with the result that, in certain situations, many smaller relations may have a very unsatisfactory grade of service. The solution therefore seems to be a different network structure with first-choice and overflow traffic completely separate, as is the case with the book network.

Routing of second-choice traffic is done on a network which - when it is in its complete form - is similar to the traditional one, in which the criteria and order of the traffic routing remain unchanged. However, no first-choice traffic stream is routed on this network but only overflow traffic from the previous network; the entire layout could therefore be redundant. A simplified version of such a layout is shown in Fig. 2 where, up to CC level, the second-choice network consists of only hierarchical trunk-groups (DC-CC, CC-CC, CC-DC) : in practice this involves the abolition of intermediate overflows with the result that only two routing choices exist for each rate of traffic. Such a layout does not achieve the maximum possible performance of the complete network, however, in view of the basic information sought and moreover because it simplifies the optimization calculation, this simplified book network is adopted for comparison with the traditional one.
3.2 BOOK NETWORK CALCULATION

In traditional hierarchical networks mixing of first-choice traffic with overflow on final routings makes it necessary for such groups to be designed with a low loss, so as to ensure that the first-choice traffic along them will not have more than the maximum permissible degree of loss. This also ensures that the rest of the traffic on the toll network will be within the same limits. However, in the book-network under consideration, since each rate of traffic always has two alternative routings available and the last-choice routing only carry overflow traffic with a low loss design no longer necessary for the latter, to ensure the global grade of service required from the network. This does not mean that this criterion cannot be maintained but to do so another condition must be introduced: setting of the overflow degree of the first-choice groups parallel to the last-choice ones (DC-CC, CC-CC, CC-DC) which would not be defined in a pure and simple economic calculation (since the ratio of cost between direct and alternative routes is very close to 1). Instead of designing the last-choice routes in the "book-network" with a low loss, the more general criterion of ensuring that the global degree of loss of each traffic relation does not exceed a predetermined value, whatever the type of routing, can be adopted (this value can, if necessary, vary according to the type of routing)*. Not only the loss degree of last-choice routes but also the maximum values for the overflow from first-choice groups can be determined with this criterions: these values are related to each other by a series of equations which express the conditions fixed for the different types of traffic routing. These conditions and the consequent equations are given in detail in Appendix 1.

In the optimization calculation for the book-network a choice must first be made between the possible routings, since they exclude each other, before the various traffic streams are assigned to the trunk-groups. A calculation on an economic basis (see Bibliography 2) as for the traditional network, can only be made subsequently when the groups so determined are being designed (any high-usage groups found to be uneconomical by the iteration of the calculation are eliminated). The preliminary choice of the route is made by determining, for an incremental traffic, a minimal cost for the various alternative routings. This cost is obtained as a weighted average of the cost of the first-choice and final routes taking as weights the percentages of the streams of traffic handled by the various routes. More details on this subject are given in Appendix 2.

3.3 CALCULATION BY COMPUTER

In the same way as for the traditional network - for which SIP has had for some time a program which makes an optimal design by an iterative method based on Pratt's formulae (see Bibliography 3) - a procedure has been established for the optimal design of the book network. The procedure develops an iterative process which, taking as a starting point a predetermined marginal capacity arrangement of the final groups (H or EE), calculates the marginal occupancy values for high-usage groups, according to the formulae given in Appendix 2. On this basis, the procedure assigns the most economical first-choice routing to each DC-DC traffic relation, comparing the $K_1$ to $K_4$ cost indicated therein. At this point the traffic offered to each group can be determined and the number of circuits can then be calculated. The marginal capacity is consequently determined for the last-choice groups and the entire calculation repeated with these new values. The process works out in practice after 4 or 5 iterations when the cost variation between any iteration and the one before it becomes negligible.

The procedure was developed for an IBM 370/158 type computer on which it occupies 800 Kbytes of central memory, reducing processing times to the minimum; for an optimization calculation obtained with 5 iterations they are in the order of 10 minutes of CPU.

4. COMPARISON OF TRADITIONAL NETWORK WITH BOOK NETWORK

4.1 ELEMENTARY NETWORK CELLS

To facilitate the study of the behaviour of the network, without using a small-scale model, it was decided to subdivide the system into a limited number of elementary networks (called cells) each of which is the standard of a known number of relations. The standard consists of the routing which mainly and directly contribute to the determination of the overall grade of service. The elementary cells thus obtained are very few and are shown in Fig. 3.

<table>
<thead>
<tr>
<th>TRADITIONAL NETWORK</th>
<th>BOOK NETWORK</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="Fig.3.png" alt="Diagram" /></td>
<td><img src="Fig.3.png" alt="Diagram" /></td>
</tr>
</tbody>
</table>

*When this criterion is adopted, special attention must be paid to international traffic which enters the national network at the CC level, where it is routed either to or from the dependent DC's. The routing plan of the book-network allows connections for international traffic to be linked to the transit exchanges in such a way that groups of both first and last choice are always available to them between the CC and the DC.
works to be representative, they should be determined statistically, classifying all the traffic relations of the national network on the basis of the elementary routings. In this way, the appropriate traffic and circuit values can be assigned to the trunk-groups comprising each "cell" and the weight of each cell in the national network can be ascertained.

4.2 OVERLOAD CONDITIONS

The need for a comparison in overload conditions arises because in design conditions the behaviour of the various structures, even though not identical, is always satisfactory as it corresponds to the design criteria. The phenomenon of overload on the network can be caused either by an actual overload of traffic with respect to the values forecast, or by shortage of circuits because of breakdowns or of delays in plant implementation schedule and can affect quite large portions of the network in various ways.

In view of the difficulty of fully representing this phenomenon it was decided that, for the purposes of this study, it was significant enough to take shortage of circuits as the overload condition. This condition is easily expressed and moreover corresponds to one of the most common difficulties actually encountered. Shortage was considered separately for high-usage and for final trunk-groups, as the behaviour of the two networks can be influenced differently by it.

Maintaining the volume of traffic offered to the network unaltered, a simultaneous reduction equal to 15% of the project value on all the trunk-groups of the same type was chosen, which is approximately equivalent to a normal implementation.

For a uniform comparison with a traditional network in overload condition, the shortages of circuits on first-choice groups of the hierarchical routings in the book network were considered together with those of last-choice instead of with high-usage groups.

4.3 METHODS OF COMPARISON

To compare the two types of network it was necessary, first of all, to make the optimal calculation on the basis of constraints common to both:

- same traffic offered;
- same total traffic loss in design conditions (\( \leq 0.7\% \));
- existence of direct DC-DC groups for every traffic relation \( \geq 3 \) Erlang;
- mesh network between the CC's with a degree of loss of 1\% (to allow for the higher level network).

On the basis of the traffic data summarized in Table 1, designs were made as shown in Tables 2 and 3 on page 2 for the traditional network and in Tables 4 and 5 for the book network. Networks thus designed can be compared from the point of view of cost and behaviour, the latter being found from the loss degree in design and overload conditions.

For the evaluation of the behaviour in overload condition a new calculation was made for both networks, on the basis of the same design traffic but with the circuits reduced as indicated in paragraph 4.2; the new characteristic values for the elementary circuits were consequently obtained.

The loss degrees are calculated according to the classic congestion formulae, even though in reality the behaviour of subscribers in overload conditions differs from that in the Erlang hypothesis.

### Table 4 - Amount and composition of the book-network

<table>
<thead>
<tr>
<th>Group</th>
<th>Circuits</th>
<th>Truck - groups</th>
<th>C/TG</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC-DC</td>
<td>41,784</td>
<td>2,573</td>
<td>0.025</td>
</tr>
<tr>
<td>DC-CC</td>
<td>3,151</td>
<td>7,6</td>
<td>0.011</td>
</tr>
<tr>
<td>DC-CC high-usage</td>
<td>3,595</td>
<td>10,1</td>
<td>0.029</td>
</tr>
<tr>
<td>DC-CC final</td>
<td>6,965</td>
<td>4,8</td>
<td>0.041</td>
</tr>
<tr>
<td>DC-CC final</td>
<td>6,965</td>
<td>4,8</td>
<td>0.041</td>
</tr>
<tr>
<td>CC-CC</td>
<td>4,188</td>
<td>6,6</td>
<td>0.137</td>
</tr>
<tr>
<td>CC-CC final</td>
<td>7,169</td>
<td>6,7</td>
<td>0.210</td>
</tr>
<tr>
<td>CC-CC final</td>
<td>7,169</td>
<td>6,7</td>
<td>0.210</td>
</tr>
<tr>
<td>CC-CC final</td>
<td>7,169</td>
<td>6,7</td>
<td>0.210</td>
</tr>
<tr>
<td>CC-CC final</td>
<td>7,169</td>
<td>6,7</td>
<td>0.210</td>
</tr>
<tr>
<td>TOTAL</td>
<td>95,254</td>
<td>100,0</td>
<td>0.016</td>
</tr>
</tbody>
</table>

### Table 5 - Traffic routing

Indications of the quality of service in overload conditions more close to reality were sought by simulations, possible because of the subdivision of the network into elementary cells. The results thus obtained, which are not included here, confirmed the validity of the data obtained by calculation.

Starting with the value of the loss degree associated with each type of relation, a comparison was made between the overall efficiency of the two networks, obtaining the average loss degree of the whole network (B) and the average of all the relations' losses (Bm) regardless of their volume of traffic. In fact, B expresses the traffic loss quantitatively while Bm gives an indication of quality as regards the disparity in treatment between the various relations.

5. RESULTS OF THE COMPARISON

5.1 ECONOMIC ASPECTS

The book-network, designed in the way described in Chapter 3 and with the constraints mentioned in paragraph 4.3, costs about 7% more than the traditional network with the same constraints. An explanation for this higher cost can be found by comparing the data given in Tables 2 and 4 from which it can be seen that the book-network differs from the traditional one in the following way:

- a greater total number of circuits (+ 7.8%) and a greater overall number of trunk-groups, mainly because of the splitting up of hierarchical routings;
- less direct routing of traffic from the DC's (-12% for groups, -3.5% for circuits), due both to the different incidence of the efficiency and of the cost ratio of the
alternative routings on the two networks and, for DC-CC groups, to the fact that they handle only first-choice traffic;

- consequently, a higher number of circuits (+18.8%) on hierarchical routings, even though subdivided in two parallel groups.

These differences are mainly related to the different layout of the book network which tends to reduce high-usage groups because of the elimination of the mixing of first-choice and overflow traffic; these differences can, however, be reduced in quantity by implementing the second-choice network in its fullest configuration (i.e. also with high-usage DC-CC's and CC-DC's for overflow traffic). Another aspect to be taken into consideration regarding costs is the necessity, in the medium term, to split up the automatic transit exchanges of most of the CC's for plant and space reasons. This will involve a configuration with many split up groups for the traditional network also. A study was made on this point and it was found that the costs for the traditional network dealt with in this paper would be increased from a minimum of 3.5% to a maximum of 6%, depending on the way the two networks linked to the two automatic transit exchanges of each CC are interconnected. In this case the difference in cost compared to the book-network become very small and a comparison between the two networks based on their respective efficiency thus becomes acceptable.

5.2 EFFICIENCY

The comparison between the behaviour of the two networks is illustrated in Tables 6 and 7 and in the histograms of Figures 4, 5 and 6.

<table>
<thead>
<tr>
<th>CELLS</th>
<th>Network composition</th>
<th>B - Average loss per relation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% relations</td>
<td>% traffic</td>
</tr>
<tr>
<td>1</td>
<td>64.5</td>
<td>8.6</td>
</tr>
<tr>
<td>2</td>
<td>2.4</td>
<td>22.9</td>
</tr>
<tr>
<td>3</td>
<td>11.0</td>
<td>5.6</td>
</tr>
<tr>
<td>4</td>
<td>15.2</td>
<td>6.7</td>
</tr>
<tr>
<td>5</td>
<td>4.4</td>
<td>4.7</td>
</tr>
<tr>
<td>6</td>
<td>2.5</td>
<td>51.9</td>
</tr>
</tbody>
</table>

Average loss of the traffic (B) = 0.70
Average loss of the relation (Bm) = 0.89

Table 7 - Synthesis of book-network

The tables give the loss values, in design and overload conditions, for each type of elementary cell, separately for the two networks, together with the relative percentage incidence in terms of relations and of traffic. The B and Bm values for the whole network are also given.
handles traffic in a decidedly more satisfactory way than the traditional network. In fact, the small traffic streams are no longer sacrificed to the economy of the system, but are sure of acceptable treatment, even in overload conditions, with only a slight deterioration in the grade of service of the larger streams. This is even more important than may appear from this paper, since the Erlang hypotheses lose their validity in the presence of high congestion and the phenomenon of repeated calls is accentuated. Another advantage of the book-network is that it limits overload effects to the relations directly concerned and therefore reduces phenomena caused on others. From the cost point of view in the case of a traditional network with one automatic transit exchange per CC the alternative of a book-network would be too expensive as the high investment involved makes the higher cost, estimated at about 7%, very burdensome. However, when - as is the case in the Italian network - it is in any case necessary to split up the automatic transit exchanges of many CC's, the book-network is an acceptable alternative for achieving a better network structure, both from the points of view of traffic handling and of reliability of service and from the economic viewpoint.

In fact, the book-network provides a structure in which both transit exchanges of each CC "see" the whole national network and can therefore come to each other's assistance in case of breakdown.

7. ACKNOWLEDGEMENTS

Authors thank Ing. M. Gavassuti and Ing. U. Trimarco for their contribution in drafting of calculation programs.

8. REFERENCES

(1) G. Miranda, P. Pallotta: On design criteria for a telephone toll network with respect to overload protection (7th ITC, Stockholm 1973)
(2) C. W. Pratt: The concept of marginal overflow in alternate routing (5th ITC, New York 1967)
(4) G. Masetti, G. Miranda: Computer aided forecasting for point-to-point traffic offered to an automatic toll network (7th ITC, Stockholm 1973)
(7) C. Asgersen: A traffic routing strategy designed for overload protection downwards the hierarchy (6th ITC, Munich 1970).
DETERMINATION OF THE MAXIMUM LOSS DEGREE AND OVERFLOW VALUES OF TRUNK-GROUPS IN THE BOOK-NETWORK

Four equations can be formulated for national traffic expressing the condition where the loss degree must be lower than a predetermined overall value for first-choice traffic routed:
1) on group DC-DC
2) on group DC-CC of destination (in the transit exchange for first-choice traffic)
3) on group originating CC (for first-choice) - destination DC
4) through both the first-choice traffic transit exchanges, of originating and destination CC's.

For international traffic the two equations (5 and 6) can be formulated expressing the conditions in which - to comply with CCITT recommendations - the combination of the overflows of the hierarchical first-choice and final groups determines an overall loss degree \( B \leq 1\% \) for outgoing and incoming traffic.

With regard to Figure 2, indicating as \( B_1 \) to \( B_{11} \) the maximum overflow or loss degree for the different types of group envisaged on the book-network (excluding groups linked to National Centres) and the maximum losses fixed for the different types of routing of the national traffic as \( P_1 \) to \( P_4 \), the equations are:

1) \[ B_1 \left[ B_7 + (1-B_7) B_8 + (1-B_9) B_9 \right] = P_1 \]
2) \[ B_2 \left[ B_7 + (1-B_7) B_8 + (1-B_9) B_9 \right] + (1-B_2) B_6 \left[ B_{11} + (1-B_{11}) B_9 \right] = P_2 \]
3) \[ B_3 \left[ B_7 + (1-B_7) B_8 + (1-B_9) B_9 \right] + (1-B_3) B_4 \left[ B_{10} + (1-B_{10}) B_5 + (1-B_{10})(1-B_8) B_9 \right] = P_3 \]
4) \[ B_4 \left[ B_7 + (1-B_7) B_8 + (1-B_9) B_9 \right] + (1-B_3) B_5 \left[ B_{10} + (1-B_{10}) B_5 + (1-B_{10})(1-B_8) B_9 \right] + (1-B_3)(1-B_4) B_6 \left[ B_{11} + (1-B_{11}) B_9 \right] = P_4 \]
5) \[ B_3 (1-B_3) B_{10} = 0.01 \]
6) \[ B_6 \left[ B_{11} + (1-B_{11}) B_9 \right] = 0.01 \]

It can be seen from Fig. 2 that, in practice, the 11 variables indicated are reduced to 8, because:
- in the book-network, as in the traditional one, the higher level between CC and NC is only designed to constitute a safety valve in case of breakdown on lower level groups; it therefore guarantees, in design conditions, a degree of loss among the CC's of less than 1%; this means that it can be assumed that \( B_8 = B_9 = 0.01 \).
- the connections between the two transit exchanges situated in the same CC can be considered as internal exchange connections and designed at a negligible loss for this operation; it can therefore be assumed that \( B_{10} = B_{11} = 0 \).

To sum up, therefore, it is possible to study the book-network considering only trunk-groups up to CC-CC level; their maximum degree of loss and overflow values are connected with a system of 6 equations in 8 unknowns, which therefore allows twice as much liberty in seeking possible solutions. To arrive at a unique solution, other conditions must therefore be introduced which can express constraints of economic type, or related to traffic routing, or of other types.

As far as the study of the network dealt with here is concerned, the following conditions were imposed:
- predetermine the maximum overflow level for the DC-DC groups (\( B_1 = 20\% \));
- ensure that the last-choice hierarchical group outgoing from DC's has a degree of loss higher than the incoming last-choice hierarchical one, according to a predetermined ratio \( B_7/B_9 = 2 \). This means that traffic is preferably limited at departure, when it has not yet occupied the network, offering instead maximum probability of success when the network has already been occupied.

In the book-network designed for comparison with the traditional network the following maximum losses were adopted: \( P_1 = P_2 = P_3 = P_4 = 1.5\% \)

The maximum loss and overflow degrees of the trunk-groups are therefore:

\[
\begin{align*}
B_1 & = B_2 = B_4 = 20\% \\
B_3 & = B_5 = B_6 = 10\% \\
B_7 & = 4\% \\
B_8 & = 1\% \\
B_9 & = 2\% \\
B_{10} & = B_{11} = 0
\end{align*}
\]
BOOK-NETWORK OPTIMIZATION

For the calculation of the routing costs of groups designed according to the Economic Erlangs, the following parameters are necessary:

- the marginal capacity at \( H = \text{const.} \); \( \phi = \frac{\Delta A}{\Delta N} \mid H = \text{const.} \)

which expresses the increase in \( \Delta N \) circuits necessary for an increase \( \Delta A \) in traffic offered to the group so that the traffic offered to the last circuit of the group is equal to or less than the value \( H \) of the economic Erlangs;

- the overflow grade \( B = B(A, N) \mid H = \text{const.} \)

It can be shown that these two parameters, functions of traffic \( A \) and of the Economic Erlangs \( H \), do not vary much for sufficiently small \( \Delta A \) values (\( \geq 3 \) Erlang): the cost calculation is therefore made, determining the values of these parameters for the particular \( A \) and \( H \) values of the group under consideration and ignoring variations caused by the variation \( \Delta A \) of traffic offered to the group.

With these assumptions, and defining the costs per circuit of the various groups as \( C_1 \) to \( C_{11} \) and the switching costs per Erlang of the automatic transit exchanges as \( M_1 \) to \( M_4 \), and the marginal capacities of the groups with constant overflow or loss as

\[ \phi = \Delta A \]

the cost expressions for the various alternative routes are as follows:

a) cost of direct routing (DC-DC)

\[ K_1 = \Delta A \left\{ C_{11} + B_1 \left\{ C_7 + (1 - B_1) \left\{ M_3 + C_9 \left\{ M_4 + C_9 \right\} \right\} \right\} \right\} \]

b) cost of routing via destination CC (DC-CC)

\[ K_2 = \Delta A \left\{ C_2 + (1 - B_2) \left\{ M_3 + C_6 \left\{ B_2 + C_{11} \right\} + C_9 + M_4 \right\} \right\} + B_2 \]

c) cost of routing via originating CC (CC-CC)

\[ K_3 = \Delta A \left\{ C_3 + (1 - B_3) \left\{ M_3 + C_4 \left\{ M_4 + C_9 \right\} \right\} \right\} + B_3 \left\{ C_7 + (1 - B_7) \left\{ M_3 + C_9 \left\{ M_4 + C_9 \right\} \right\} \right\} \]

d) cost of routing via both CC's (CC-CC)

\[ K_4 = \Delta A \left\{ C_4 + (1 - B_4) \left\{ M_3 + C_5 \left\{ B_4 + C_{10} \right\} + M_3 + C_8 \right\} \right\} + B_4 \left\{ C_7 + (1 - B_7) \left\{ M_3 + C_9 \left\{ M_4 + C_9 \right\} \right\} \right\} \]

By a comparison of these four cost expressions, in which the parameters related to the various first-choice groups appear when necessary while those of the common last-choice routes appear systematically - the most economical routing and therefore the first-choice trunk-group can be determined for each traffic relation.

The only relations not subjected to this preliminary comparison are the DC-DC relations with more than 3 Erlangs: in order to fulfill the condition of comparing costs for sufficiently low \( \Delta A \) values it is necessary that such traffic relations be served by direct DC-DC groups - as is also done in traditional network.

To evaluate the marginal occupancy of each group, the formulae and parameters defined in Pratt's article already referred to (see Bibl. 2.) are used: this in particularly easy for the book-network since, in each case, the costs of only two routes have to be compared.

The marginal occupancy of the various types of group are given by the following expressions:

a) DC-DC group

\[ C_1 = C_7 + (1 - B_1) \left\{ C_9 \left\{ M_4 + C_9 \right\} \right\} \]

b) DC-CC group

\[ C_2 = C_6 + C_{11} \left\{ M_4 + C_9 \right\} + B_2 \left\{ \frac{C_7}{B_7} \right\} \]

c) CC-DC group

\[ C_4 = \frac{C_9}{B_7} + M_4 \left\{ M_4 + C_9 \right\} \]