A Model Relating Measurement and Forecast Errors to the Provisioning of Direct Final Trunk Groups

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ABSTRACT

This paper describes a mathematical model of the provisioning of direct final trunk groups with forecasting and measurement errors. This model can be used to study the effects of applying standard trunking formulae to possibly inaccurate load forecasts.

An important consideration in the process is the degree to which the load forecast is actually followed. This so-called provisioning policy is modelled parametrically to allow consideration of a range of strategies, from following the forecast precisely to complete reluctance to remove trunks when indicated by the trunk forecast.

When load forecast errors are combined with a reluctance to remove trunks, there will be a net reserve capacity on the average, i.e., more trunks than would be needed if the loads were known exactly.

Using the mathematical model, a set of curves known as the Trunk Provisioning Operating Characteristics is calculated. These relate percentage of reserve capacity to service (as measured by the fraction of trunk groups with blocking exceeding 0.03). The accuracy of the estimate of the traffic load defines the curve on which one is constrained to operate. The degree of reluctance to remove trunks together with the traffic growth rate determines the operating point. Improved estimation accuracy corresponds to a more desirable operating characteristic.

The accuracy of the forecasted load estimate is influenced by many factors. Data base errors (e.g., measuring the wrong quantity due to wiring or other problems), recording errors (e.g., key punch errors) and projection ratio errors illustrate some of these factors. This type of modeling may be useful both in evaluating the potential effects of proposed improvements in measurement or forecasting accuracy, and in studying the effects of changes in provisioning policy.

1. INTRODUCTION

Traffic measurements play a major role in the Bell System. They are used as the basis of those efforts which are aimed at planning an efficient network by providing appropriate quantities of trunking and switching equipment. They also form the basis of many of the efforts which are aimed at efficiently administering the network as well as serving as primary inputs for purposes of evaluating network performance.

New traffic measurement systems typically bring with them a variety of benefits such as more accurate data, more detailed information and more automated and convenient collection and processing of the raw data together with possible new uses for the data which these improvements allow. Of course, to prove in economically, these advantages must offset the costs associated with installing and operating the system. Clerical savings associated with data collection and processing represent a good example of a relatively easily quantifiable advantage. Other advantages are not so simply equated to dollar savings.

In this paper we consider the effects of more accurate traffic measurements.

We describe a mathematical model of the provisioning of direct final trunk groups* with measurement errors. This model can be used to study the effects of applying standard trunking formulae to possibly inaccurate load forecasts. An important consideration in the process which ultimately determines the number of installed trunks is the degree to which the trunk forecast is actually followed. This so-called provisioning policy is modelled parametrically to allow consideration of a range of possibilities, from following the forecast precisely to complete reluctance to remove trunks when indicated by the trunk forecast. Errors in the load forecast will result in some trunk groups having more trunks than required while others will not meet the service criterion. When these errors are combined with a reluctance to remove trunks, there will be a net reserve capacity on the average, i.e., more trunks than would be needed if the loads were known exactly.

Using the mathematical model as a building block, a set of curves known as the Trunk Provisioning Operating Characteristics (TPOCs) is developed. These relate reserve capacity (i.e., the amount of trunks in service in excess of what would be required if the load were known perfectly) to service (as measured by the fraction of trunk groups with blocking >.03) in equilibrium. Figure 1 illustrates a typical set of TPOC curves. The accuracy of the estimate of the traffic load defines the curve on which one is constrained to operate. The reluctance to remove trunks together with the traffic growth rate determine the operating point. Improved estimation accuracy corresponds to a more desirable operating characteristic.

This type of modeling can be useful in evaluating the potential effects of proposed changes in measurement or forecasting accuracy as well as studying the effects of changes in provisioning policy. The accuracy of the forecasted load estimate is influenced by many factors. Sampling errors (e.g., switch count errors associated with TIS usage measurements), data base errors (e.g., measuring the wrong quantity due to wiring or other problems), recording errors (e.g., key punch errors) and projection ratio errors illustrate some of these factors. Since a number of uncertainties contribute to the overall error associated with the estimation of next day's traffic level, reducing a particular component (e.g., reducing data base errors by introducing an improved traffic measuring system) can be expected to affect the results significantly only on those groups where the error source is a major contributor to the overall error. Thus the potentially achievable improvements are limited by those error sources over which we have no control (e.g., switch count errors might represent one such source).

It should be pointed out that the results which are presented in this paper are based on a simplified model of the provisioning process for the case of direct final trunk groups only and are intended to serve primarily as a way of viewing the problem which puts into focus some of the important trade-offs which are present. The

* The model assumes that a sequence of statistically similar provisioning cycles characterizes each group.

† Traffic growth and reluctance may be related (e.g., high growth may cause high reluctance).

Also called full direct or nonalternate route groups.
emphasis is on methodology rather than on specific numerical results. Much work remains to be done to develop and more fully exploit these ideas.

The paper is broken down as follows: Section II contains a description of the trunk provisioning process considered. Section III describes the mathematical models which we use to characterize the trunk provisioning process and generate the TPOC curves. Basically, they consist of a procedure for estimating the traffic load for the next busy season coupled with an engineering rule describing the provisioning policy for converting this estimate into the number of trunks provided. The mathematical details associated with these models are presented in Appendix A.

Section IV considers the sensitivity of the results to a number of the important submodels and assumptions which are used in the development. The TPOC curves are shown to be quite insensitive to the particular model of provisioning policy used as well as to the traffic growth factor and the distributions which characterize the various measurement and forecasting errors (though both the reluctance and growth affect the operating point). The curves do depend somewhat on trunk group size (corresponding to true offered load), however. While specific results are not presented here, the curves also depend upon measurement or forecasting biases which may exist.

Section V summarizes and discusses further work which is needed.

II. A SIMPLIFIED MODEL OF THE TRUNK PROVISIONING PROCESS

The trunk provisioning process actually used in practice is very complex and varied. It is, in fact, complex based only on what is explicitly recommended and standardized. Overlaid on this are decisions made on a judgment or discretion basis which take into account a multitude of practical considerations.

We shall abstract some essentials of the trunk provisioning process to form the basis of our simplified model.

The following steps summarize the highly simplified version of a portion of the trunk provisioning process for direct finals which we will analyze.

1) From measured usage, peg count, and overflow (UFCO) (where the usage estimate U includes both traffic and maintenance usage) an estimate of present offered load, \( \bar{q}_i \), is obtained from

\[
\bar{q}_i = \frac{U_i}{1-O_i/PC_i} \tag{II-1}
\]

where \( O_i \) and \( PC_i \) are the overflow and peg count, respectively.

2) This estimate is the offered load, \( \bar{q}_i \), is projected ahead with projection ratio \( \bar{g}_i \) to yield a load \( g_i \bar{q}_i \) in a future period for which a trunk requirement is determined.

3) The estimated number of trunks required, \( \bar{N}_{i+1} \), to meet a grade-of-service objective is determined from

\[
B(\bar{N}_{i+1}, \bar{g}_i, \bar{q}_i) = 0.01 \tag{II-2}
\]

where \( B \) is the Erlang B function.*

4) If \( N_{i+1} \) exceeds the number of trunks currently in service, \( N_i \), the extra trunks indicated are added. If \( N_{i+1} \) is less than \( N_i \), a fraction \( \frac{N_{i+1}}{N_i} \) of those indicated to be removed are actually removed. The fraction is a number between 0 and 1, depending on the reluctance to remove trunks. More explicitly,

\[
N_{i+1} = \max\left(\bar{N}_{i+1} - \frac{N_i - N_{i+1}}{N_i}, 0\right) \tag{III-1}
\]

Reluctance refers to the following. If the calculated number of trunks required is less than the number installed, there is a reluctance to take trunks out unless the equipment is needed for other purposes. There are a number of factors which quantify reluctance. There are. There is yet another reluctance to remove trunks and that is the possibility of increasing TUR errors when trunks are moved around. Improved data could reduce the reluctance to service down.

In practice, the reluctance to remove trunks is quite complicated and appears to be based largely on judgment. For this reason, we have modeled it parametrically to get a feel for its effect.

We shall see that this model, simplified as it is, is still rich enough to provide insights into the trunk provisioning process. In the next section, we shall see how the errors in (i) and the uncertainty associated with (ii) combine with (iv) to result in reserve capacity.

III. A PROVISIONING MODEL FOR DIRECT FINAL TRUNK GROUPS

III.1 A Model of Provisioning Policy

The essence of the mathematical model for the simplified version of the provisioning process is included in eqs. (II-1), (II-2) and (II-3).

While the process proceeds from load measurements through traffic estimates to trunk requirements and the number of trunks provided, it is useful in discussing the model to begin with the implications of eq. (II-3). That equation models the provisioning policy which relates the current number of trunks, \( N_i \), and the estimated requirements for the next period, \( N_{i+1} \), to the number of trunks to be provided for the next period, \( N_{i+1} \).

Repeating it here,

\[
N_{i+1} = \max\left(N_{i+1} - \delta(N_i - N_{i+1}), 0 \right) \tag{III-1}
\]

where \( \delta \) is a parameter which we introduce to provide a measure of the reluctance to provision down. If \( \delta = 0 \), trunks will never be removed while, at the other extreme, \( \delta = 1 \) corresponds to the situation where exactly the estimated number of trunks is always provided, even if that requires removing a large number of the trunks currently installed. Intermediate values of \( \delta \) correspond to intermediate provisioning policies.

It is convenient to rewrite eq. (III-1) in normalized form by dividing through by \( M_i \), the number of trunks required to handle the true traffic load in year \( i+1 \).

This yields

\[
\frac{N_{i+1}}{M_i} = \max\left[\frac{N_{i+1}}{M_i} - \delta\left(1 - \frac{N_{i+1}}{M_i}\right), 0\right] \tag{III-2}
\]

where \( \delta \) represents the growth in trunks required to handle the true traffic load in going from year \( i \) to year \( i+1 \).† The equilibrium solution to eq. (III-2) has been determined both from (approximate) analytic techniques and from simulation. Specifically, important characteristics of the distribution of the quantity \( N_i/M_i \) have been modeled it parametrically to...
are found. The mean value of this variable is a direct measure of the reserve capacity of the trunk groups under consideration while the grade of service provided by a collection of trunk groups is related to the distribution of \((N-m)/M\) where \(m\) is the number of maintenance trunks (e.g., the fraction of groups with blocking \(> 0.03\) is given by \(\text{Prob}[N-m)/M < a]\) where \(a\) is a suitably chosen constant which depends upon group size). Fig. 2 qualitatively shows this relationship. In fact, the TPOC is a plot of \(B(N/M) - 1 + B(N/m)\), the reserve capacity, vs. \(\text{Prob}[N(N/m) < a]\).

Inspection of eq. (III-1) or (III-2) indicates that the various forecasting uncertainties which enter into the provisioning process are all summarized in the distribution of \(N_{i+1}\), which represents our estimate of the number of trunks which will be required in year \(i+1\).

III.2 A Model of Trunk Forecasting

Having discussed the model which converts the estimated trunk requirement, \(N_{i+1}\), into the number of trunks provided, \(N_{i+1}\), and a simplified discussion of the way it leads to the tradeoff curves, we now turn to the process of estimating the trunks required from the traffic measurements.

To identify the potential improvements associated with a new traffic measurement system, it is necessary to consider a specific estimation model. We have selected direct final trunk groups for this purpose. For convenience, we repeat here the relevant equations which were discussed in Section II.

\[
a_i = \frac{\bar{U}_i}{1 - B_i} \quad \text{(III-3)}
\]
\[
\hat{a}_{i+1} = \frac{\hat{B}_i}{\hat{a}_i} \quad \text{(III-4)}
\]
\[
B(N_{i+1}/N_{i+1}) = 0.01 \quad \text{(III-5)}
\]

In these equations, \(\bar{U}_i\) represents the measured carried (which typically consists of the sum of the measured traffic load plus the measured maintenance usage), \(\bar{B}_i\) is our blocking estimate (determined from measured peg count and overflow), \(\hat{a}_i\) is the estimate of offered load, \(\hat{B}_i\) is the projection ratio (which is simply an estimate of the traffic growth factor on the trunk group) and \(B(N/a)\) is the Erlang blocking formula. As can be seen from eq. (III-5), the distribution of the estimated number of required trunks is determined by the distribution of the estimated offered load, \(\hat{a}_{i+1}\). The (normalized) variance of the estimate of \(\hat{a}_{i+1}\) turns out to play a major role in results which we will soon describe. If we denote this normalized variance by \(c^2\), then the first order analysis of (III-3) and (III-4) given in Appendix A yields

\[
c^2 = \frac{c^2}{\hat{B}^2} + \frac{c^2}{\hat{a}^2} + \frac{c^2}{\hat{e}^2} \quad \text{(III-6)}
\]

where

\[
c^2 = \text{var} \left(\frac{N_{i+1}/N_{i+1}}{N_{i+1}/N_{i+1}}\right) = \text{normalized variance of error in forecast load},
\]
\[
c^2_{\hat{a}} = \left(\frac{\bar{U}_i - \bar{U}_i}{\hat{a}_{i+1}}\right) = \text{normalized variance of total carried usage error},
\]
\[
c^2_{\hat{B}} = \left(\frac{\hat{B}_i - \hat{B}_i}{1 - \hat{B}_i}\right) = \text{normalized variance of error in blocking estimate and}
\]
\[
c^2 = \left(\frac{\hat{e} - \hat{e}}{\hat{e}}\right) = \text{normalized variance of error in traffic growth factor}
\]

These terms are defined more fully in Appendix A.

To further facilitate the identification of the various error sources which contribute to our total estimation error, we may (approximately) decompose \(c^2\) into its component parts as follows:

\[
c^2_{\hat{a}} = c^2 + c^2_{\hat{a}} + c^2_{\hat{e}} \quad \text{(III-7)}
\]

In the above expression, \(c^2_{\hat{a}}\) represents the variance of the data base and data handling induced errors in usage, \(c^2_{\hat{a}}\) represents the statistical errors in usage and \(c^2_{\hat{e}}\) represents the variance of the fraction of trunks plugged busy for maintenance purposes. Each of these quantities is normalized to the group traffic usage. In the above expression, the data base and handling induced usage errors refer to errors in the load due to measurement system deficiencies such as T&B wiring errors or data handling or recording errors. The maintenance usage error term reflects the fact that, in general, one does not know how much of the total usage is attributable to trunks plugged busy for maintenance purposes and how much is traffic usage.

In order to compute the reserve capacity and the fraction of groups with blocking greater than \(0.03\), it is necessary to know the expected traffic growth, the number of trunks actually required, and the statistical behavior of the fraction of trunks plugged busy for maintenance. The form of the model output is shown in Figure 3. The model parameters used to generate the curve apply to trunk groups of nominal size \(25\), a traffic growth factor of 1.05 (i.e., a 5 percent annual increase), \(\alpha_{\text{om}} = 0.02\) and \(\alpha_{\text{om}} = 0.02\). Each point on the curve corresponds to choosing one value of \(\theta\) and then computing the corresponding \% reserve capacity for the groups and the fraction of groups which have blocking greater than 0.03. Several values of the provisioning parameter, \(\theta\), are indicated on the curve. Thus, this curve, which we shall refer to as the Trunk Provisioning Operating Characteristic (TPOC), indicates the tradeoff which exists between \% Reserve Trunk Capacity and fraction of groups with blocking exceeding 0.03.

Figure 4 shows the same tradeoff curve as Figure 3, but also includes the tradeoff curve for a system which has \(\alpha_{\text{om}} = 0.1\). The operating point on the tradeoff curve depends on the amount of reluctance to service down, but the tradeoff curve depends on the \(\alpha_{\text{om}}\) of the provisioning system. Improving the accuracy of the data gathering and processing for the trunk provisioning process causes a decrease in \(\alpha_{\text{om}}\). That is, it places the operating point on a more desirable tradeoff curve. The service improvement or trunk savings realized by such an improvement depends on the location of the operating point on the curve.

One additional point: a perfect measuring system corresponds to a nonzero \(\alpha_{\text{om}}\) due to nonmeasurement errors, such as statistical errors. This is indicated in Fig. 4 by the shaded area which contains TPOCs which cannot be reached by only decreasing measurement errors.

A number of studies were conducted to assess the sensitivities of the tradeoff curves to the model assumptions. These are discussed in the next section.

* The statistical error term represents the effects of basing an estimate on a fixed number of samples of the busy-idle state of each trunk over a finite time interval.

† Normality is assumed; sensitivities to distribution (and to other factors) are considered in the next section.

‡ Other factors, such as day-to-day variations, also contribute to a minimum \(\alpha_{\text{om}}\). At this point in time, insufficient information exists to adequately quantify the "background noise".
IV. SENSITIVITY RESULTS FOR TRADEOFF CURVES

The provisioning policy model used is an approximation to the actual provisioning decisions which occur in the provisioning process. In fact, there probably isn't any universally applicable model since many decisions are largely based on judgment. The important question is whether a different model for provisioning policy would significantly change the tradeoff curves. To investigate the sensitivity of the curves to the assumed model of provisioning policy, we considered a different model for the reluctance to remove trunks.

This model (the w-model) is given by

$$N_{w+1} = \max(N_{w+1}^0, N_w)$$

where w is between zero and one. When w is zero the w-model always provides the estimate of the trunks required, just as the 8-model, eq. (III-1), does when $\delta = 1$. Also, when w = 1 the w-model never removes trunks, just as with the 8-model when $\delta = 1$. Thus, the two models are identical when there is no reluctance to remove trunks, $w = 0$ and $\delta = 1$, and when there is complete reluctance, $w = 1$ and $\delta = 0$. However the models are quite different in the case of an intermediate amount of reluctance. The w-model will freely remove trunks down to $w_{NI}$, but won't remove any beyond that, while the 8-model removes a fixed fraction of any excess of $N_{w+1}$ over $N_8$. TPOCs for each of these models are shown in Figure 5 for nominal parameters of trunk group size (25), traffic growth factor (1.05, 5 percent growth rate), fraction of maintenance busy trunks (0.02) and values of $\delta$ of 0.1 and 0.3 (standard deviation of load forecast error). As expected, the end points of these plots are identical since for $w = 0$, $\delta = 1$ and $w = 1$, $\delta = 0$ the models are identical. Furthermore, the curves are seen to be close throughout the entire range. We thus observe that the two reluctance functions provide different mechanisms for tracing out approximately the same tradeoffs.

Figure 6 is a plot of the tradeoff curves for several different growth rates (2%, 5%, 8%). We note that the curves are quite insensitive to the particular traffic growth rate although the specific operating point, for a given reluctance, can be quite sensitive. Also, the extent of the curves are sensitive to growth. We noted earlier that, for a given growth, increasing reluctance causes one to ride up a tradeoff curve. This is illustrated by the $\delta = 0$ (complete reluctance) points indicated on the figure. We are thus observing, as expected, higher growth rates tending to reduce reserve capacity.

The results were also shown to be relatively insensitive to the distribution function of the forecast of trunk requirements and the level of maintenance busies, however, trunk group size and forecasting bias did influence the tradeoff curves.

V. CONCLUDING REMARKS AND FURTHER WORK

In this memorandum, we have developed a mathematical model of the trunk provisioning process which allows one to study the relationship of traffic measurement and forecasting errors and reluctance policy to reserve capacity and service. We emphasize that this is a first-cut model based on many simplifying assumptions.

Direct final groups were considered since they represented the simplest case. The results were displayed as tradeoff curves of percent reserve capacity vs. percent of groups with blocking > 3 percent. The curves were parameterized by $\sigma^2$, the normalized error is forecasted load.

These TPOC curves were found to be relatively insensitive to reluctance policy, probability distribution of errors, and growth. This robustness of the curves is useful not only from the point of view of convenience of presentation but also because of our uncertainty concerning these quantities. The basic TPOC concepts were validated using field data collected on approximately 250 direct final trunk groups. In the course of this data collection effort it became necessary to extend the basic model to include the effects of forecast bias since it became clear that proper interpretation of data required a decomposition of forecast error into a bias and a random component. In addition, to properly interpret the data, the TPOC curves were modified to reflect the fact that estimates of reserve capacity and service were themselves corrupted by various errors.

Starting with the conceptual framework provided by the TPOC curves, a number of important implications naturally follow. To begin with, since TPOC shows that the effectiveness of the provisioning process is significantly influenced by the accuracy of the forecast, improving that accuracy will pay clear benefits. Separate studies have strongly suggested that measurement errors can be a significant component of the forecast error. Thus reducing measurement errors, either by installing improved measurement systems or by providing improved maintenance has clear payoffs.

For a given measurement accuracy, the next natural area of concern deals with administration of the provisioning process in the presence of forecast errors. The key observation in this regard is that it is the effective forecast accuracy which ultimately controls the process. Thus any technique which improves that effective accuracy will yield benefits. For example, the use of independent short term and long term forecasts would be expected to result in an improved forecast accuracy as would the development of improved screening methods for identifying potentially poor data points.

The TPOC framework also offers a conceptually appealing approach to the monitoring of trunking performance since it quantifies the relationship between trunks and service in a way which highlights the dependence on key system parameters.

Much remains to be done to further develop and extend the concepts we have described. We indicate here only a few of the many areas worthy of further study. The results we have described assume an equilibrium condition corresponding to reluctance and true growth which are constant from year to year. These assumptions could be relaxed and transient solutions developed in more detail than we have done. Extension of the modeling approach to other probability engineered groups would seem like an obvious next step while the appropriate framework for viewing network clusters involving MU groups is less clear.

To capitalize on the ideas we have discussed, some of the obvious problem areas which present themselves involve the development of improved forecasting capabilities, disconnect policies (i.e. "reluctance") and data screening techniques.

Finally, we note that in order to draw conclusions about the value of specific proposed improvements (e.g. an improved measurement system), the overall forecast error must be decomposed into its component parts and each of these components must be quantified for each of the alternatives under consideration. In addition, it should be kept in mind that casual attempts to plot reported trunking data on existing TPOC curves can be misleading because of factors that might be influencing the data but left out of the curves (e.g., bias errors, trunks installed as a result of new machine cut-overs, etc.).

* It might be noted that they also represent a large function of the total Bell System trunks and groups.

* Where one operates on the curve depends on the reluctance policy and growth.

* Some of these extensions as well as results involving day-to-day variations were developed by E. J. Masserini.
We also noted earlier that reducing data base (or forecasting) errors by introducing improved measurement systems (or forecasting systems) can be expected to provide trunk savings only on those groups where the measurement (or forecasting) error is the major contributor to the overall error. Additional work (e.g., further modeling and statistical analysis of usage errors) is needed to more accurately quantify the effect of improvements in measurements (or forecasting) when one is dealing with a nonuniform collection of trunk groups.

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APPENDIX A

MODEL DEVELOPMENT

AII: Model and Definitions

Our model for the provisioning of direct final groups with no day-to-day load variation assumes that peg count, overflow and usage are measured during a base period, e.g., a busy season. The offered load is estimated according to eq. (III-3), and the next period's (e.g., next busy season's) offered load is found by multiplying by a projection ratio. The estimated number of trunks required for the next busy season is such that the projected offered load would cause a blocking probability of 0.01. Once the number of trunks required for next year, \( N_{i+1} \), has been estimated, the number actually provided, \( N_i \), is chosen using eq. (III-1), or eq. (IV-1), depending on the provisioning policy modeling. This model assumes trunks are provided only once each period with measurements from the last period.

In order to write the model equations we need some definitions:

\[
\begin{align*}
\bar{a}_i &= \text{Mean of the true traffic usage during study period } i. \\
\bar{u}_i &= \text{Measured traffic usage during study period } i. \\
m_i &= \text{Actual maintenance usage during study period } i. \\
m^{'}_i &= \text{Estimated maintenance usage during study period } i. \\
\bar{u}^{'}_i &= \text{Mean of the true total (maintenance plus traffic) usage during study period } i. \\
\bar{U}_i &= \text{Measured total (maintenance plus traffic) usage during study period } i. \\
\bar{e}_i &= \text{Usage error due to TUR wiring, etc. with separate traffic usage measurement.} \\
\bar{e}U_i &= \text{Usage error due to TUR wiring, etc. with joint usage measurement.} \\
\bar{a}_i &= \text{Mean of the true offered load during study period } i. \\
\hat{a}_i &= \text{Estimate of } \bar{a}_i \text{ based on measurements during study period } i. \\
\hat{a}_i &= \text{Estimate of } \bar{a}_i \text{ based on measurements during study period } i. \\
M_i &= \text{Number of traffic trunks required in study period } i. \quad (e.g., B(M_i, a_i) = .01). \\
\hat{M}_i &= \text{Estimate of } M_i \text{ based on measurements during study period } i. \\
\end{align*}
\]

*Nonuniformity can result from dealing with groups with different measurement devices (e.g., ESS and TUR's). Furthermore, even with the same measurement devices (e.g., all TUR measurements), one can expect differences in accuracies (e.g., well maintained TUR vs. poorly maintained TUR).

\[
\begin{align*}
N_i &= \text{Number of trunks in place in study period } i \quad \text{(includes maintenance)}. \\
\hat{N}_{i+1} &= \text{Number of trunks estimated as required for period } i+1. \\
e_i &= \frac{\hat{a}_i}{a_i}, \text{ the traffic growth.} \\
\hat{e}_i &= \text{The estimate of } e_i. \\
B_i &= \text{Measured average fraction overflowing during study period } i. \\
\hat{B}_i &= \text{Mean of the fraction overflowing during study period } i. \\
eB_i &= \hat{B}_i - B_i. \\
\hat{e}_i &= \hat{e}_i - e_i. \\
\end{align*}
\]

AII: Estimating \( N \) from Lumped Traffic and Maintenance Usage

Estimating \( N_i \) in terms of its mean and variance, requires estimating \( a_i \). The estimate of \( a_i \) depends on whether or not maintenance usage is measured separately. If it is not, then \( \bar{u}_i \) is measured and we estimate \( a_i \) by

\[
\hat{a}_i = \frac{\bar{u}_i}{\bar{u}_i - B_i} \quad \text{eq. (AII-1)}
\]

where

\[
\begin{align*}
\bar{u}_i &= \frac{m_i + eU_i}{u_i - eU_i + eU_i}. \\
\bar{m}_i &= \frac{m_i + eU_i}{u_i - eU_i + eU_i}. \\
\end{align*}
\]

Since \( B(\hat{a}_i, \hat{a}_i) = .01 \) is the design criterion, expanding and regrouping terms yields

\[
\frac{N_i}{N_{i+1}} = 1 + \left( \frac{B_i}{eU_i} \right) c = \frac{B_i}{eU_i} \quad \text{eq. (AII-4)}
\]

where

\[
B_i = \frac{2B_1}{3N_i} M_i^{1.84} \quad (B_1 = \frac{2B_1}{3N_i} M_i^{1.84}) \\
\]

\[
\text{By eq. (AII-6), } \frac{B_i}{eU_i} \text{ depends only on } M_i \text{. It is 0.647 for } M_i = 10 \text{ and increases toward 1 as } M_i \text{ increases.}
\]

To find the mean of \( N/M \) we assume that all measurements and estimates are unbiased. That, together with eqs. (AII-4), (AII-5) and the definitions, gives

\[
\frac{\hat{N}_{i+1}}{N_{i+1}} = 1 + c \frac{eU_i}{u_i} = 1 + d \frac{E[m_i]}{M_i} \\
\]

where

\[
\begin{align*}
\hat{N}_{i+1} &= \text{Estimate of } N_{i+1} \\
E[m_i] &= \text{Mean of the true total offered load during study period } i. \\
M_i &= \text{Estimated number of trunks required for next busy season, (e.g., } B(M_i, a_i) = .01). \\
\hat{N}_{i+1} &= \text{Estimated number of trunks required for next busy season.} \\
\end{align*}
\]

*Nonuniformity can result from dealing with groups with different measurement devices (e.g., ESS and TUR's). Furthermore, even with the same measurement devices (e.g., all TUR measurements), one can expect differences in accuracies (e.g., well maintained TUR vs. poorly maintained TUR).
To find the variance of $\hat{N}/M$ we assume that the terms in eq. (AII-4) are statistically independent. That gives

$$\text{Var} \left[ \frac{\hat{N}+1}{\hat{M}+1} \right] = c^2 \sigma^2_x$$

(AII-9)

where

$$\sigma^2_x = \sigma^2_{u_1} + \sigma^2_{B_1} + \sigma^2_{e_1}$$

$$\sigma^2_{u_1} = \text{Var} \left[ \frac{u_1}{u_1} \right]$$

$$\sigma^2_{B_1} = \text{Var} \left[ \frac{e_{u_1}}{u_1} \right]$$

$$\sigma^2_{e_1} = \text{Var} \left[ \frac{e_{B_1}}{B_1} \right]$$

(AII-10)

In the rest of the paper we will assume that

$$\text{Var} \left[ \frac{e_{u_1}}{u_1} \right] = \text{Var} \left[ \frac{e_{B_1}}{B_1} \right]$$

since neither contain variation due to maintenance usage and all the other errors have the same source.

A similar analysis is possible for the case when the traffic usage is measured separately from the maintenance usage.

AIII: Solution of Model Equations

The solution of eq. (III-2) for the distribution of $\hat{N}/M$ has been obtained in several ways. Analytic approximations and characterizations of properties of the solution have been developed by D. L. Jagerman while the actual TPOC curves presented in this paper were obtained by employing simulation techniques to solve eq. (III-2).
TRUNKS GROWTH FACTOR, 1.05

\[ \sigma_e = 0.20 \]
\[ \sigma_m = 0.02 \]

FIGURE 3: TYPICAL MODEL OUTPUT, TPOC TRADEOFF CURVE

FIGURE 5: SENSITIVITY TO RELUCTANCE MODEL

FIGURE 4: TYPICAL MODEL OUTPUT, TPOC TRADEOFF CURVES

FIGURE 6: SENSITIVITY TO GROWTH