ABSTRACT

The purpose of this paper is to stress the importance of operational research techniques in the solution of problems encountered in teletraffic theory. Methods now in use in the operational research field are too numerous to permit their enumeration here, and therefore, this survey is confined to optimization problems.

In this review, some general optimization techniques are considered, together with areas in which these methods have been successfully applied. For continuous variables, various gradient based methods have been used to determine minimum cost configurations in multistage alternate routing networks. Penalty function methods (eg the inner method) have been successfully applied to the optimal design of computer communication networks. For discrete variables, integer programming has been used in the design of multidrop line networks connecting remote terminals to a central data processing centre. The principles of dynamic programming have been incorporated into models involving multistage decision processes.

The paper concludes with some comments on optimal control problems and identifies areas which may benefit from application of optimizing techniques.

1. INTRODUCTION

When the Program Committee invited me to present an address on this subject, it reminded me at first of Dr A. Jensen's paper "The applicability of decision theory in the planning and operation of telephone plant", presented at the first ITC in 1955(1). In his paper, Dr Jensen suggested the importance of interplay between telephony and other fields, referring to the theory of games, the decision function, and linear programming.

From the present trend, wherein mathematical modelling and optimization in communication system design are becoming increasingly common, in line with the rapid advances in computer performance, the need for better co-ordination between traffic engineering and operational research methods is becoming increasingly important. Methods now in use in the operational research field are too numerous to even allow enumeration of the large number of related contributions in the traffic engineering fields. Therefore, I would like to confine this survey to optimization problems.

2. MODELS FORMULATED AS CONTINUOUS NON-LINEAR PROGRAMS

I wish to begin with some formulation of continuous variable, non-linear programming problems. Analytic optimization methods, including the classical Lagrange multiplier method, have been widely used in traffic engineering. Some examples are: Moe's principle for multi-exchange area (2), optimal design of various link systems (3)...(8), and optimal capacity assignment in computer communication networks (9)...(14). Recently, however, formulation of mathematical programming problems and many heuristic algorithms have attracted attention. It is also noted that graph theory is becoming an indispensable tool for topological optimization of computer communication networks (15).

FIG. 1

OPTIMAL DESIGN METHODS FOR MULTI-STAGE ALTERNATE ROUTING NETWORKS

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>METHODS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of high usage circuits</td>
<td>Analytic (Marginal overflow,...)</td>
</tr>
<tr>
<td>Chain flow</td>
<td>Gradient (Caballero-Diaz)</td>
</tr>
</tbody>
</table>

One of the most important problems in telephone network design is the optimization of alternate routing networks. As the methods for triangular networks are well known I shall confine my remarks to multi-stage alternate routing networks. Optimization techniques now in use may be classified as shown in Fig. 1. Here, I would like to skip the explanation of analytic methods, although many excellent contributions have been made (16)...(19). The gradient method, one of the standard non-linear programming techniques, was originally applied to alternate routing networks with non-coincident busy hours (20).

FIG. 2

CHAIN FLOW MODEL

Given: Alternate routing pattern

Requirement matrix

Probability of loss \( B_k \) for OD pair \( k \)

Total network cost \( C(i) \rightarrow \min \) over global chain flow pattern \( \mathbf{b} \)

Constraints:

1. Chain flow: Multicommodity flow

2. Total chain flow for OD pair \( k \)

\[ = (1 - B_k) \times \text{Offered traffic for OD pair } k \]

Chain flow means the flow on a chain which consists of a series of links connecting an origin and destination pair (21), (22). The key of the chain flow model is that the variables used in the optimization are the chain flows of each OD pair, which are mutually independent, not the number of high usage circuits. Thus, the optimal routing problem reduces to a non-linear, constrained multicommodity flow problem. The trouble with standard gradient projection method lies in cumbersome matrix multiplication and inversion to obtain projection vector. However, for the chain flow model, very simple explicit formulae for this vector are now available (23).

FIG. 3

OPTIMAL FLOW ASSIGNMENT PROBLEM

Given: Network topology, Channel capacity \( (C_l) \),

Requirement matrix

Average delay \( T = \sum \frac{1}{r} \rightarrow \min \) over global flow \( \mathbf{f} \)

Constraints:

1. Multicommodity flow

2. \[ f_1 \leq C_l \]

\( C_l \): Capacity of link \( l \), \( f_1 \): flow on link \( l \).

\( r \): Total external traffic
As another example, I would like to take up one of the four optimal design problems related to computer communication networks (10), that is, the optimal flow assignment problem. The task is to minimize total average delay T per message with respect to traffic on each link. This reduces to a non-linear, unconstrained multicommodity flow problem, because objective function T becomes infinite when flow on a link approaches its capacity. In other words, T incorporates the second constraint as a penalty function as in SUMT.

FIG. 4

OPTIMAL FLOW ASSIGNMENT METHODS
Static routing
1. Flow deviation (FD) (Fratta, et al)
2. Gradient projection (GP) (Schwarz-Cheung)
3. Extremal flow (EF) (Cantor-Gerla)
   (Decomposition + GP)
4. Heuristics Cut-saturation routing
   (Chou-Frank)
Dynamic routing
Pontryagin's maximum principle
   (Segall)
For this problem, there exist at present five optimization methods. Of the methods available for static routing, flow deviation (24), extremal flow (26) and heuristics (27) have originally been developed for the flow assignment problem. This is because the standard, gradient projection method for non-linear programming (25) is not computationally efficient for reasonably large networks. Here, we can recognize the contribution of traffic engineering to operational research methods.

The key to the FD method is the association of a length, whose value is given by partial derivative of T with respect to flow Tij, with link i. These lengths may then be used as a factor in a shortest-route flow problem, with the resulting path to which some of the flow can be diverted.

The EF method is based on the decomposition principle, one of the standard optimization techniques for large systems. The application of Pontryagin's maximum principle for dynamic routing (28) is an example of interplay between system control theory and traffic engineering.

FIG. 5

FORMULATION TO MIXED INTEGER PROGRAMMING PROBLEM
Total cost

Constraints:
1. Conservation of flow
2. \( \sum_{j} Y_{ij} = 1 \)
3. \( Y_{ij} < C Y_{ij} \)

\( X_{ij} \): Flow from \( T_i \) to \( T_j \)

\( Y_{ij} = \begin{cases} 
1 & \text{if a link from } T_i \text{ to } T_j \text{ exists} \\
0 & \text{otherwise}
\end{cases} \)

\( d_{ij} \): Cost of a link from \( T_i \) to \( T_j \)

3. MODELS FORMULATED AS INTEGER PROGRAMS

Now, I would like to proceed to some formulation of integer programming problems. As the first example, I will take up topological optimization of teleprocessing networks. The problem is to obtain minimum cost, multifan line networks to connect remote terminals to a central data processing centre, subject to the constraints of line capacity and reliability.

If we neglect the reliability constraint, this problem reduces to a constrained minimum spanning tree problem, and can be formulated as a mixed integer programming problem, because variables \( x_{ij} \) are integer (30).

However, the standard techniques, such as the cutting plane or group-theoretic methods, are not applicable to a network of realistic size, because they require too much computer memory and running time.

FIG. 6

TREE NETWORK

1. Mixed integer programming
2. Unconstrained MST + Branch-and-bound
3. Matroid theory + Branch-and-bound
4. Heuristic algorithms

FIG. 7

EXAMPLES OF DEVELOPMENT STUDIES ON TELEPHONE NETWORKS

1. Optimum long-term plan for growing urban area (Jessop)
2. Optimum network modernization plan with CCIS (Balanian-Woodside)
3. Optimum extension plan for conduits and main cable (Rapp)

Optimization: Dynamic Programming

4. DYNAMIC PROGRAMMING MODELS

Now, I would like to proceed to some formulation to Bellman's dynamic programming problems. The problem of telephone network planning may be divided into two fields, namely, cross-sectional studies and development studies. The latter obtains the size and the point of time for future extensions, taking into account the time-value of money invested. Figure 7 shows some examples of development studies (43) . . . (47).

In long term plans for growing urban areas, for example, we must endeavour to determine the optimal strategy for when and what kind of new exchanges are to be added, and how exchange boundaries are to be altered, based on the present value of total investment over a given period. All of the problems shown in Figure 7 can be solved by dynamic programming as multi-stage decision problems.
I will refer to one more example. In developing a new switching system, such as a space-division electronic switching system, the optimum traffic capacity series is the first problem. This has also been solved by dynamic programming (48).

FIG. 8
MAIN CATEGORIES OF QUEUEING PROBLEMS

Behavioural problems

Operational problems

- Static model
- Optimal control policy
- Optimal queue discipline
- Optimal scheduling rule

5. DECISION PROCESSES

Finally, I would like to touch on some formulations of decision process problems. Problems in queueing theory may be put into two main categories, namely, behavioural and operational. Although studies on the former have so far occupied the main part of queueing theory, studies on optimal operation and control, which consider various kinds of costs and rewards, are gradually increasing. In these studies, Markov and semi-Markov decision processes theories are playing a leading role (49)...(51).

An example is the study on dynamic scheduling of multiclass message flow through a packet-switching node. This is attractive as a flow control policy in heavy traffic, because optimal policy implies that a server should deny service of some lower priority classes, even if a server is idle (52)...(56).

In relation to optimal control problems, Dr Kostén's pioneering work (53) presented at the third ITC on the subject "Operational Research Problems in Congestion Theory" is memorable.

It may be work adding that studies on approximate and numerical solutions for behavioural problems are also desired, in order to make it possible to apply standard optimization techniques to operational problems.

Since my survey was not as thorough as I would have liked, I was unable to explain thoroughly the interplay between traffic engineering and operational research methods.

REFERENCES


4. ..... and K. Iida : Optimum design of four-stage switching system, Rev. of ECL, 12, 3-4, p. 174 (1964).


