Some Practical Problems of the Traffic Engineering of Overloaded Telephone Networks

Géza Honi
Hungarian Post Office, Budapest, Hungary

Géza Gosztory
BHG Telecommunication Works, Budapest, Hungary

ABSTRACT

In overloaded networks it is difficult to determine the basic data of traffic engineering. Some measurements were made to assess the interrelation between the traffic situation and the relative weights of the reasons of failure. A mathematical model of approximate nature allows to decide whether a traditional or a "repeated attempts" model should be adopted in dimensioning parts of a connection path. The data required for this are only the carried traffic, the number of call attempts and the holding times. The mathematical model presented for "repeated attempts" purposes assumes that the investigated group has in spite of repetitions an Erlang or Engset character with some fictitious offered traffic. Other parts of the networks and the called subscribers are also considered. This model neglects reattempt times but can take several failure type dependent perseverance functions into account. Simulation tests have shown that calculation results may be used in a wide range of practical cases.

INTRODUCTION

The problems of overloaded telephone networks have been discussed in many earlier and recent publications all over the world. Many suggestions have also been made concerning conceptions about the way of taking repeated calls into account in traffic engineering practice. According to experience an appropriate method that should sufficiently support the design engineers working in a country of moderate technical level still does not exist. In most Administrations the present day traffic measurement technique is not sufficient to determine the parameters required by the existing traffic engineering methods. Studying the data given for design purposes it is often impossible to find out whether they are data of an overloaded network or not.

In order to solve this problem an intensive research work took place in the last years. The three main research fields were:

- investigation of characteristics of overloaded networks on the basis of the interpretation of traffic measurement results;
- reliable determination of traffic intent as dimensioning starting data from traffic measurement results, choice between the traditional and the "repeated call" traffic engineering methods;
- elaboration of a traffic engineering method taking into account repeated call attempts and interaction of network and subscriber.

The most important results of this work are summarized in the following.

1. INVESTIGATION OF CHARACTERISTICS OF OVERLOADED NETWORKS

1.1. TRAFFIC OBSERVATIONS

Traffic measurements were carried out on the input and output trunks of a terminal exchange in an overloaded network. [1] The observation has been extended over a period of 4 days, altogether 22 hours. The most detailed observation has taken place in the case of long distance calls at the investigation. Altogether 2950 incoming calls and 4800 outgoing calls have been evaluated; they represent 87 erlh and 93 erlh traffic, respectively.

Observing the call attempts it was possible to determine whether a conversation has taken place or not and also the reason of failure could be given. The same investigation has taken place both for incoming and outgoing calls. The following reasons of failure were identified /the designation of the number of unsuccessful cell attempts because of the given failure is also indicated below and will be used in the following/:

- failures caused by congestion /these are functions of traffic dimensioning/, o₁;
- failures because of early disconnections caused by the calling subscriber and the network /most abandoned calls have taken place because the calling subscriber found it tiring to wait and interrupted the call/, o₂;
- failures caused by the called subscriber /who was busy or did not answer/, o₃;
- unsuccessful cell attempts caused by technical faults in the equipment /e.g. fault in signalling, interruption of the conversation, etc./, o₄.

1.2. MEASUREMENT RESULTS

Considering the naturally smaller failure rate of incoming calls the results given by outgoing calls will be evaluated. Thus the results will be of the same nature but the picture will be more demonstrative. Table 1 shows the proportion of the number of outgoing calls and the traffic given by them according to the reasons of failure. Almost every reason of failure - with the exception of technical faults - has a considerable weight in the columns for numbers of cell attempts and traffic. In the case of this magnitude of failure rates repeated call attempts effect not only the characteristics of common /marker, register, etc./ circuits but also the traffic characteristics of the speech path and in this way also the dimensioning.

The effect of different reasons of failure on traffic engineering is determined not only by the above given concrete particular proportions, but also by the steadiness of these and by their being a function of input parameters. Considering these facts the relation of the number of unsuccessful call attempts because of a given failure to all call attempts has been determined for a given period of time. The 22 hours investigation period has been devited into 15 minutes sections, and the relation of the number of calls attacked in each of these sections was determined by using regression analysis. The results of linear re-
gression are shown in Figure 1. Let us note that the relationship between \( c_f/c \), \( c_f/c \) and \( c \) can be better approximated by a parabol. Furthermore, curve \( c_f/c \) as a function of \( c \) will be rather a hyperbol even by interpreting the set of starting points by eye. However, the difference is not important and the picture given by linear regression is more demonstrative, \( c_f/c \) and \( c_f/c \) being a function of \( c \) can be approximated by a straight line.

**Table 1.** Distribution of the number of outgoing calls /first column/ and their traffic /second column/ between successful calls and cells unsuccessful because of different reasons.

<table>
<thead>
<tr>
<th>Rate of cells</th>
<th>Outgoing calls</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number of cell attempts</td>
<td>Traffic</td>
<td></td>
</tr>
<tr>
<td></td>
<td>%</td>
<td>%</td>
<td></td>
</tr>
<tr>
<td>successful</td>
<td>26.4</td>
<td>61.0</td>
<td></td>
</tr>
<tr>
<td>congested</td>
<td>27.5</td>
<td>11.1</td>
<td></td>
</tr>
<tr>
<td>engaged + no answer</td>
<td>23.7</td>
<td>18.4</td>
<td></td>
</tr>
<tr>
<td>premature abandonment</td>
<td>16.1</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>technical fault</td>
<td>2.3</td>
<td>2.2</td>
<td></td>
</tr>
</tbody>
</table>

The character of their relation is shown by the slope of the straight line in the figure. It is remarkable that \( c_f/c \) is almost parallel to axis \( c \). The situation is the same in the case of incoming calls. It means that there is a growing rate of \( c_f/c \) in the calls reaching the subscriber's line. However it must be taken into account that the number of call intents increases in the busy hour and the increase of the number of unsuccessful attempts because of the called subscriber can be taken as for quite normal in this period of time. This allows to look upon the called subscriber as a constant "reason of failure" in the calculations.

**Fig. 1.** Relative weights of successful and unsuccessful outgoing call attempts versus number of all call attempts /regression lines/.

The rate of unsuccessful call attempts taking place because of technical faults is relatively independent of \( c \) but the frequency of other reasons of failure definitely depends on it. This shows the constant need of examining the traffic measurement results from the aspect of distortion by overloads and from the point of view of applicability for traffic engineering purposes.

2. **DETERMINATION OF BASIC DESIGN DATA**

Traffic engineering of total telephone networks may nowadays rely on very well elaborated methods. These methods take also economic aspects into account. Their use is supported also by tables, graphs, etc. [2]

These methods rely on the traditional mathematical models and neglect the phenomenon of repeated cells. In the case of a network of good quality - if the overall failure rate is small - this neglect may be reasonable. However, in the case of an overloaded network, when enlarging it, the actual situation must be taken into consideration by traffic engineering as a radical change can not be expected taking place from one day to the other. Therefore though a partial network is of good quality it may belong to an overloaded network for a long time.

The task is twofold: /1/ the subscriber's actual traffic intent should be determined and /2/ it should be decided whether the traditional or the "repeated call" mathematical model should be used in the given situation.

In order to solve these problems the usual traffic measurement data as: carried traffic, number of incoming calls, holding time of call attempts are available.

It can be shown that in an overloaded network the offered traffic i.e. the starting data of the traditional mathematical model may obtain different values according to its being calculated from the carried traffic or from the number of offered calls. However, these offered traffics may differ considerably from the actual traffic intent. The degree of difference is affected by cell repetitions and failures. On the basis of investigation of differences the suitable mathematical model may be chosen for traffic engineering. The following chain of ideas involves approximations but these do not affect the qualitative statements of the final results.

2.1. **OFFERED TRAFFIC AS THE FUNCTION OF THE PROBABILITIES OF FAILURE** [3]

Let us examine a connection path consisting of \( r \) stages /Fig. 2/. These stages may be parts of a switching network or may be total exchanges, it is only supposed that the stages are controlled one after the other. The probabilities of failure valid for the individual stages /\( p_k/\) are available; these summarize the possible reasons of failure including the called subscriber at the end of the path if it is necessary. Furthermore, the average call set up time for the different stages /\( t_k/\), the number of call attempts directed towards the last stage at the beginning of the path /\( c_f/\) and the number of successful call attempts /\( C_f/\) are also known.

It is valid for every stage that:

\[
c_{f-1} = c_f + c_{uf} = c_{f-1}(1 - B_f) + c_{f-1}B_f \quad (1)
\]

where \( c_f \) is the number of cell attempts successful, i.e. getting through stage \( f \).

\( c_{uf} \) - is the number of cell attempts unsuccessful at the same stage.

**Fig. 2.** The investigated connection path

The actual traffic intent is:

\[
A_{ot} = c_0 + t_{ot} \quad (2)
\]

where \( c_0 \) is the number of cell intents; supposing a small probability of failure this number...
is almost the same for all stages. /Call intent is the first attempt in the series of repeated call attempts reaching the above named stage and being directed towards a given subscriber. A\(e_{c}\) is the average holding time of successful calls measured at stage \(\ell\). /It is supposed that both the average call set up time for every stage and the average time of conversation are independent of the probability of failure.

The offered traffic calculated from the number of all calls /c/ and from the average holding time of successful call attempts is:

\[
A_{hc} = c \cdot t_{sc}
\]

(3)

On the basis of the general theory of repeated calls and introducing the call repetition factor /\(\beta/\) the following formula will be approximately valid [4], [5]:

\[
\frac{A_{tc}}{A_{hc}} = \frac{c_{o} \cdot t_{sc}}{c} = \frac{1}{\beta} = 1 - PH,
\]

(4)

where

\[
P = 1 - \prod_{i=1}^{r+1} (1 - B_{i})
\]

is the overall probability of failure, if the failure probability \(B_{i}\) belongs to stage \(i\).

\(H\) is the average perseverance expressing the probability that subscribers will repeat calls unsuccessful for any reason; its average value according to measurements lies between 0,5 and 0,9 [6].

The offered traffic calculated from carried traffic may be expressed by using \(c_{o}\), the \(B_{s}\) and the holding times /see [3]/:

\[
A_{tc} = c_{o} \cdot \sum_{i=1}^{r+1} B_{i} \cdot t_{uc_{i}j} \cdot \prod_{i=1}^{j-1} (1 - B_{i}) + t_{sc} \sum_{i=1}^{r+1} (1 - B_{i})
\]

(5)

\[
1 - B_{c}
\]

where \(t_{uc_{i}j}\) is the holding time belonging to unsuccessful call attempts measured in stage \(\ell\) if the call attempt becomes unsuccessful at stage \(j\). It is valid that

\[
t_{uc_{i}j} = \sum_{i=\ell}^{j-1} t_{ki}, \quad \ell < j.
\]

It is obvious that first member of \(A_{tc}\) expresses the carried traffic originated by unsuccessful call attempts and its second member contains the carried traffic originated by successful call attempts. In both cases \(A_{hc} \geq A_{tc}\) and \(A_{hc} \geq A_{tc}\) but \(A_{tc} \geq A_{tc}\) may be alike valid.

Taking /5/ into account the ratio of the two different offered traffic will be:

\[
A_{tc} = \frac{\sum_{i=1}^{r+1} B_{i} \cdot t_{uc_{i}j} \cdot \prod_{i=1}^{j-1} (1 - B_{i}) + t_{sc} \sum_{i=1}^{r+1} (1 - B_{i})}{1 - B_{c}}
\]

(6)

where

\[
\Theta_{c_{ij}} = \frac{t_{uc_{i}j}}{t_{sc}}
\]

is the so called time factor. Comparing expressions /4/ and /6/ the possible extreme values of the \(A_{tc}/A_{hc}\) relation can be approximately determined for any cross section of the traffic path. If the probabilities of failure and the time factor are the same then these extreme values are determined by the possible extreme values of

H. Probabilities of failure and time factors can be determined by measurements and applying \(0,5\) or \(1\) as extreme values of \(H\), the limit values can be obtained.

/It should be remarked that if the above mentioned path consisting of \(r\) stages would be loaded only by the traffic given by the \(c\) call attempts on the input, the traffic measuring task would be much more simple. In this case:

\[
A_{tc} = Y_{c} (1 - B)^{r} = c_{c-1} t_{c},
\]

where

\[
t_{c} = \frac{Y_{c}}{c_{c}}
\]

is the average holding time of call attempts getting into stage \(c\). In order to determine the \(t_{c}\) holding time values, no measurements of time are needed and formula /5/ can be transformed to contain \(t_{c}\) times. /

It will be shown later that as a first approach it is enough to decide whether \(A_{tc} \geq A_{tc}\) takes place in the given traffic situation of not or rather what is the possible maximum value for the /\(A_{hc} - A_{tc}\)/ and /\(A_{tc} - A_{tc}\)/ differences. If one of these differences gives a suitable result, \(A_{tc}\) or \(A_{tc}\) can be adopted in traffic engineering. For the purpose of further investigations and to demonstrate the method, the following simplifications are introduced:

- the connection path includes three sources of failure \(/B_{1}, B_{2}, B_{1+1}/\)
- time factors are approximately \(0\), with the exception of \(\Theta_{c_{c-1}}\).

Equations /4/ and /6/ can be written in the following simple form:

\[
\frac{A_{tc}}{A_{hc}} = \frac{(1 - B_{1})[1 - (1 - B_{1} + (1 - \Theta_{c_{1}}))]}{(1 - B_{c})}
\]

(7)

\[
\frac{A_{tc}}{A_{hc}} = 1 - H(1 - (1 - B_{1})(1 - B_{2}) (1 - B_{c+1}))
\]

(8)

Applying formulas /7/ and /8/ simple nomograms can be constructed. With the help of these nomograms it is possible to determine the relation of \(A_{tc}/A_{hc}\) and \(A_{tc}/A_{hc}\) at different probabilities of failure and values of \(H/\) Figure 3. /
In Fig. 3, the related values of $A_1$ or $A_{ot}$ and $A_{of}$ or $A_{et}$ appear in the five marked ranges. For these ranges the parameters $B_1$, $B_2$, $B_3$, $A_1$, and $A_{et}$ don’t exceed the value of 0,01; 0,2; 0,3; 0,4 and 0,5 respectively and $H = 0,5 - 1$ were supposed.

This figure can also be adopted to determine the relative differences between offered traffics.

The example above proves that calculated traffics strongly depend on failure rate and on the amount of repetitions. The traffic engineering should consider this dependence.

2.2. TRAFFIC RANGES FOR ECONOMIC USE OF THE TRADITIONAL DIMENSIONING PRINCIPLES

To decide whether $A_{of}$ or $A_{et}$ is the starting design data it is necessary to determine the deviation of the results given by them from the optimum obtained by $A_{of}$.

One has to estimate the difference in the number of required switching elements (links, trunks etc.) gained by the different traffic values.

In practical cases the traffic may arrive to a stage of the investigated path from different directions. In this case traffics $A_{of}$, $A_{et}$ and $A_1$ may be interpreted as total traffics and different ranges should be determined for them, similarly to the ranges shown in Fig. 3. In this case the following part of the chain of ideas is also valid without any change. The above mentioned differences which are the measure of uncertainty of dimensioning may be characterized e.g. by the difference in the number of lines $\Delta N$ determined by the actual offered traffic and by that derived from the measured data or by the relative difference $\Delta N/N_1$, where $N_1$ is the number of lines in the bigger group.

From the data of the investigated system and from Figure 3 it is possible to determine the relation of the difference between traffic intent and offered traffic $\Delta A = A_1 - A_1$ or $\Delta A/A_1$, and the difference in the number of lines $\Delta N/N_1$, mentioned before. /Here $A_1$ denotes the smaller, and $A_1$ the bigger traffic, respectively./

In Figure 4 the $N_1 = \text{const.}$ and $N_2 = \text{const.}$ lines limiting $\Delta N = \text{const.}$ fields were drawn, into an $A_1$, $\Delta A/A_1$ coordinate system.

In order to demonstrate the method the number of required lines has been calculated, for the sake of simplicity, on the basis of Erlang’s loss model.

If the values of $A_1$ or $\Delta A/A_1$ change in a way that the point determining pairs of values remains within such a field in the coordinate system, the calculated difference in the number of lines will not change; e.g. if $A_1 = 2,2$ erl and $\Delta A/A_1 = 20%$ or $30\%$ then $\Delta N$ will be equal to unity in both cases. If one of the offered traffics $A_{of}$ or $A_{et}$ derived from measurements, leads to $\Delta N = 0$ then — independently of a possible big $\Delta A$ — it can be used for design purposes. /Further if $\Delta N = 1$ and $A_{of}$ or $A_{et}$ both $A_{of}$ and $A_{et}$ can be used./

According to the considerations above a limiting curve for offered traffics has been constructed. Under this curve traditional traffic engineering methods are acceptable (Figure 3).

If the traffic values exceed the limiting curve $A_1$ has to be exactly determined by adopting a traffic model taking also repeated calls into account.

Naturally in the above given method the acceptable $\Delta N$ depends on the traffic range used and the limiting curve shown in Figure 3 may suffer according changes.

![Traffic models with repeated calls](image)

**Fig. 5. Domains for the use of different traffic models**

3. MATHEMATICAL MODEL FOR REPEATED TELEPHONE CALLS

In order to describe the effect of repeated telephone calls many mathematical models have been introduced during the last years.

However, these models either represent difficulties of calculation (e.g. systems of equations of state) or their generalization seems difficult because of their empirical nature. What makes more difficult to solve the problem is that the repeated call attempts effect the total network and the analytical mathematical models introduced until now deal only with the most simple arrangements.

Therefore it seemed reasonable to elaborate a calculation method which was easy to handle, could easily take into consideration measurement results and at the same time the possibility of its generalization was not excluded. In order to make the verification easier, the idea has been elaborated for a simple system. On the basis of the encouraging results we can say that the mathematical model in most cases gives results which may be used in practice without correction.

**5.1. BASIC IDEA**

Call repetitions arrive to the input of the investigated system as a feed-back. This will change the character of the input process.

The feed-back characteristics will be determined
by the perseverance function valid for the given system and by the distribution of reattempt intervals. The perseverance function— as it is known—gives the probability that the subscriber will repeat the call after a certain number of unsuccessful call attempts. The new call attempt will take place after a reattempt interval.

The perseverance function determines the amount of unsuccessful call attempts remaining in the system for repetition. The distortion in the basic character of the input process is mainly determined by the reattempt interval /these distortions take place because of repetition/. Generally the shorter is this interval the more probable is that repeated call attempts will have an important role in the busy hour.

According to careful and detailed traffic measurements the incoming call attempts constitute in spite of repetitions a Poisson process [7], [8].

It seems that the distortion of the input process is rather of quantitative nature. This explains the basic idea of the following mathematical model according to which the changed input process may be described by a fictitious but random traffic. The volume of this traffic can be determined by the joint effect of an optional perseverance function and the reasons of failure perceived in the system. For a fully available group of lines, the congestion caused by this fictitious traffic can be given by Erlang's formula and in this way further simplifications may take place [4], [9].

Both the perseverance and the reattempt interval depend on the reason of failure of call attempts. The average perseverance function and the average distribution of reattempt intervals are valid for a given situation, for the given rates of the reasons of failure. Therefore it is reasonable if the mathematical model is able to take into account different perseverance functions and distributions of reattempt intervals depending on the reasons of failure. According to the above given basic idea it is enough if it is possible to apply different perseverance functions simultaneously in the model. This generalization and the variant of the model referring to traffic sources of finite number will be given in the following.

### 3.2. THE INVESTIGATED SYSTEM

The scheme of the system is shown in Figure 6. The arrangement corresponds to the route shown in Figure 2. At present this route consists of two stages. We suppose that the first stage is a fully available group consisting of N lines and the only reason of failure is group congestion. The second stage takes the remaining part of the network including subscribers in consideration. Here failure is represented by the constant probability p. The intensity of cell intents is \( C_0 \) and the input process arriving to the input of the system is of intensity. The distribution of the call attempts in the system takes place according to the Figure. The H-s drawn in the branchings mean that the unsuccessful call attempts will decide according to the perseverance function, whether more repetition should take place or not.

The average values of call set up times and conversation time have been indicated in the appropriate stages. In the second stage it has been taken into account that the times belonging to successful and unsuccessful call attempts are different. According to this the call set up times of the successful and unsuccessful call attempts and the total holding time of the successful call attempts are as follows:

\[
t_u = t_{k1} + t_{k21}
\]

\[
t_{ke} = t_{k1} + t_{k22}
\]

\[
t_s = t_{ke} + t_c
\]

The following formula gives the average holding time of the call attempts:

\[
t = t_s [1 - p (1 - \beta)]
\]

where

\[
\theta = \frac{t_u}{t_{ke} + t_c}
\]

is the time factor. It can be easily observed that t is always smaller than the total holding time of the successful call attempts and the degree of decrease is determined by p and \( \theta \).

![Fig.6. The investigated system with the flow of call attempts](image)

Before introducing the mathematical models we'd like to summarize the suppositions concerning the system:

- the offered traffic arriving to the input of the first stage is the product of the number of cell attempts and the average holding time:

\[
A^* = C_0 \beta t
\]

It is possible to include the \( A_0 \) traffic intent into the formula given above; this will be the traffic subscribers really would like to put through the investigated group. As \( A_0 = C_0 \cdot t_c \) is

\[
A^* = A_0 [1 - p (1 - \beta)] \beta
\]

The congestion caused by the group of lines is determined by the offered traffic \( A^* \); but this can not be expressed directly from formulas.
\[
\beta = \beta \left( A^s, p, H(u) \right).
\]

On the basis of the general theory of repeated call attempts it is known that

\[
\beta = 1 + \sum_{j=1}^{\infty} \frac{i}{1} \left[ \left( P_{(i)} H(u) \right) \right] (13)
\]

where \( P_{(i)} \) is the probability of failure in the case of the \( i \)th repetition. According to our supposition \( P_{(i)} = B = 1 - B \) \( P \) and it is valid that \( B = E_p/A^s \), thus

\[
\beta = 1 + \sum_{j=1}^{\infty} \frac{i}{1} \left[ \left( E_p(A^s) + [1-E_p(A^s)]p \right) H(u) \right] (14)
\]

Inserting the value of \( \beta \) into /12/ we get an implicit equation; this equation can be solved by iteration. As \( A^s \) is known the data belonging to the system shown in Figure 6 and necessary for traffic engineering can be determined. For example:

- carried traffic
  \( Y_4 = A^s \left[ 1 - E_p(A^s) \right] \)
- traffic of successful call attempts
  \( Y_5 = \frac{Y_4 \left( 1 - p \right)}{1 - p (1 - \Theta)} \)
- speech traffic
  \( Y_c = \frac{t_c}{t_k + t_c} \)
- ineffective traffic
  \( Y = Y_4 - Y_5 \)

Let us mention that the chain of ideas of the method described in [10] is similar to the preceding one with the important difference of \( H^1/ = 1 \).

3.3.2 ERLANG'S GROUP WITH H^1/ FUNCTIONS DEPENDING ON THE REASONS OF FAILURE

Formula /14/ may be simply modified in case of introducing \( H_j^1/ \), \( j = 1, 2, \ldots, n \), functions depending on failure reason where \( j \) denotes the reason of failure. We suppose that the reason of failure is independent of the previous reason of failure and the repetition always takes place according to the perseverance function belonging to the just observed reason. Let \( H_j^1/ \) denote the perseverance function belonging to congestion and \( E_p/H^1/ \) the perseverance function belonging to failures taking place because of the second stage. Then

\[
\beta = 1 + \sum_{j=1}^{\infty} \frac{i}{1} \left[ \left( E_p(A^s)H_j(u) + [1-E_p(A^s)]pH_2(u) \right) \right] (15)
\]

Thus for solving equation /12/ \( \beta \) given by /15/ should be applied. Other details of 3.3.1 are valid without any change.

3.3.3 ENGSET'S GROUP, AVERAGE H^1/ FUNCTION

If the traffic arriving to the system is originated by a finite number of \( S \) traffic sources then the call intensity is \( S \cdot \beta_0 \), where \( \beta_0 \) is the average call intensity of one traffic source. Formula /11/ will take the following form:

\[
\sum_{k=0}^{S} \alpha^s (5-k) \beta_4 \beta_5 = S \cdot \beta_0 \beta_1 \]

Also in this case one can find the offered traffic on the left hand side of the formula. The following notations were used:

- \( \alpha^s = \beta_0 \)
- \( \alpha^t = \beta_1 \)
- \( \beta^*, \beta_0^* = 1 - \alpha^s \left( 1 - EN(A^s) \right) \)
- \( \beta_0^* \) = the call intensity of free traffic sources,
- \( EN(A^s) \) = Engset call congestion.

On the basis of these the repetition coefficient will be:

\[
\beta = 1 + \sum_{j=1}^{\infty} \frac{i}{1} \left[ \left( EN(A^s) + [1-EN(A^s)]p \right) H(u) \right] (17)
\]

Substituting /17/ into /16/ one may obtain the well known implicit formula.

Comparing the preceding formulas with /16/ and /17/ the difference is that in the case of an Engset group the fictitious traffic is introduced for the average offered traffic of the individual traffic sources.

3.4. VERIFICATION OF THE MODEL

The simulation check of the exactness of the method can be found in [11], we would not go into details.

2.2 simulation with 95% confidence limits
2.2 calculation according to section 332

![Fig.7](attachment:image.png)

Fig.7. Comparison of results obtained from simulation and calculation
into details here. The investigations carried out refer to two variants of the Erlang group model; no comparison results exist up to now for the Engset group model.

As a conclusion one can say that the approximation improves if the number of lines in the group increases, the average perseverance decreases.

To demonstrate results in Figure 7 the repetition factor obtained by simulation and calculated on the basis of part 3.3.2 is given. On the horizontal axis the speech traffic intent can be seen, the 95% confidence limits of simulation are also indicated. In Table 2 one can find the average perseverance probabilities for some points of curve I with great perseverance and of curve II with small perseverance.

<table>
<thead>
<tr>
<th>Speech traffic intent /erl/</th>
<th>Curve</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I.</td>
</tr>
<tr>
<td>2</td>
<td>702</td>
</tr>
<tr>
<td>2,6</td>
<td>778</td>
</tr>
<tr>
<td>3,2</td>
<td>855</td>
</tr>
<tr>
<td>3,8</td>
<td>894</td>
</tr>
</tbody>
</table>

Table 2. Average perseverance values for the curves of Fig.7, derived from simulation tests

3.5. APPLICATION IN PRACTICE

The implicit system of equations determining fictitious traffic may be solved by iteration. In the case of $\Phi$ according to $/14$/ the conditions of iteration convergence have been analyzed in details by [9]. In all other cases similar considerations are valid. On the basis of equations $/14/$ tables were made for the practical work. Two different series of the $H/1/$ perseverance function have been applied, [7], [12].

Parameters $p$ and $\Theta$ have taken the values 0,1, 0,2 and 0,3; the range of $N$ and $A_0$ was $N = 2 + 20$ and $A_0 = 1 + 10$, respectively. The program of calculation has been written in FOCAL language for a PDP8/E computer.

The results may be useful from two points of view. On the one hand traffic engineering can be carried out if the parameters are known and if the actual $H/1/$ function is near to the supposed one. On the other hand one may get a general picture about the nature of the phenomenon and about the effect of parameter changes. Figures 8, 9 show general results of this kind.

$A^*$ is shown in Figure 8 as a function of $A_0$, two different perseverance functions were applied. Perseverance function according to $/12/$ /curve K./ results in larger $A^*$ than perseverance function according to $/7/$ /curve M./. In the Figure the $A^* - A_0$ relation can be observed too. However, if the probability of failure is small, it may happen that repetitions do not equalize the traffic decreasing effect of the abandoning calls. This phenomenon can be observed better in the case of smaller perseverance.

In Figure 9 the quantity $M = \beta - 1$ is shown as a function of $N$ in case of different $A_0$ and $p$ values. It can be easily observed that if $N$ is small then the rate of repetition will rather be determined by the congestion; but if the congestion is small than $p$ will dominate.

It should be emphasized that the model is able to apply perseverance functions depending on the reason of failure. This is very important, namely it can be supposed that $H_2/1/$ functions are stable in a given period of the day while the average $H/1/$ function evidently depends on the situation $/11/$.

CONCLUSIONS

The demonstrated mathematical model was elaborated for practical purposes i.e. to be suitable means for traffic engineering.

Tables and according graphs assure that the model can easily be adapted. The starting assumptions of the model are not contradictory to the
results of measurements. The model can be used widely because the measurement results can be taken into account in a simple and direct way. In possession of the required measurement data the calculation can be improved by considering different reasons of failure and according perseverance functions instead of average values. Simulation tests showed that calculations using this model will be correct enough for practice if the perseverance is not too high. For very high perseverance corrections will be derived. In this way the determination of traffic demand can be achieved reliably and so one can find the starting point of traffic engineering.

The measurement of the traffic characteristics of repeated calls is tiring and requires expensive measuring equipments. But if the congestion is low it is not necessary at all to use models considering repetitions, traditional method will also be adequate. The scope of application of traditional methods is determined by the difference of design results obtained by using these methods and the results arrived using models considering the real traffic intent. With the help of the method described one can determine the parameters of well established and simple measurements. On the basis of the degree of deviation it can be decided whether to adopt traditional methods or methods with repeated call attempts. An example of this kind of decision was shown in Fig. 5.

According to traffic measurement results shown, it is advisable to carry out the described decision not only in the case of devices with short holding times but also for devices busy during conversation. If this investigation shows that the traditional method will not give a reliable result, it will only then be necessary to carry out a more detailed series of measurements to determine the data required for a model with repeated call attempts.

ACKNOWLEDGEMENTS

The work, account of which was given, forms a part of the activity of the Research Institute of the Hungarian PTT and is to some extent shared with BHG Telecommunication Works.

The authors wish to express their thanks to Dr. G. Lajtha /PTT/ Deputy Manager and to Dr. P. Holnar /BHG/ Head of Development for their advices and support.

The authors are indebted to their colleagues especially to Miss R. Nagy /BHG/ and Mr. Z. Szentirmai /PTT/ for their help during the work. Thanks are due also to Miss M. Agostházi /BHG/ for her advices during the preparation of the manuscript.

REFERENCES