THE CONFIGURATION THEORY
The Influence of Multi-Part Tariffs on Local Telephone Traffic

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ABSTRACT

Proper evaluation of usage sensitive pricing (USP) for telephone service requires knowledge of subscribers' behavior pattern when subjected to different kinds of measured tariffs. Total local revenue generated by a subscriber population is the aggregate of payments from each individual subscriber, which in turn is a function of his traffic and the tariff. This paper deals with the effect of the tariff configuration on individual subscribers' local traffic. The term tariff configuration refers to the geometrical pattern formed by tariff components when the tariff is presented in a linear price-usage diagram. The first part of the paper contains a graphical presentation of how a subscriber logically reacts to different tariff configurations. In this connection the concepts of equilibrium, diminishing relative savings and incentive are introduced. Later in the paper approximate mathematical functions for the relationship between traffic and tariff are developed; i.e., demand functions for local traffic.

1. THE MODEL

1.1. TARIFFS

The word tariff is used in the following narrow definition throughout the paper:

A local telephone tariff is composed of a fixed monthly rate (B) plus as the case may be a message unit rate (U), with or without a free message unit allowance (A); i.e., one-, two-, or three-part tariffs.

From these three components several types of tariffs may be composed as shown in Fig. 1.

![Fig. 1 Tariff Patterns](image)

FR, SNR and ANR are publicly offered; NR and AMR are not, but employees who are exempt from the fixed monthly rate enjoy them. As limit cases they are useful in the analysis.

The five depicted tariff types have different configuration. Varying values of tariff components within two- or three-part tariffs makes for further differentiation.

ANR is the prototype of the five tariffs. The other four are simply special cases of ANR, viz., SNR when A = 0; ANR when B = 0; NR when B = 0 and A = 0; and FR when U = 0.

1.1.1. TARIFF FACTOR

At ANR the price for a call in excess of the allowance is U and the price for a call within a fully used-up allowance is B/A. The ratio between them is called tariff factor and is denoted f.

\[ f = \frac{U}{B/A} = A \cdot \frac{U}{B} \]  

(1)

If a call within a fully used-up allowance were cheaper than a call outside the allowance, it would be cheaper for a large user to subscribe to two telephone lines and divide the traffic between them. Avoidance of such tariffs gives the restraint that

\[ f < 1 \]  

(2)

The significance of the tariff factor and empirical values will be treated in a separate paper by the author.

1.1.2. RATE RATIO

The ratio of message unit rate to fixed monthly rate is denoted s.

\[ s = \frac{U}{B} \]  

(3)

There is no such configurational restraint on the ratio s for SNR as there is on the tariff factor f for ANR. Nevertheless, there are upper and lower values of s, which if exceeded, make an undesirable tariff. The significance of the rate ratio with regard to usage repression, cost pricing, cost of traffic metering and inter-subscriber subsidisation, as well as empirical values of rate ratios, will be dealt with in a separate paper by the author.

1.2. CONCEPTS AND RESTRAINTS

The following concepts and restraints are included in the model to facilitate development of mathematical expressions of the relationship between traffic and configuration:

- The "individual guinea pig subscriber"
- The "common guinea pig subscriber"
- The "constant telephone bill restraint"
- The "constant revenue restraint"
- Equilibrium

A rigid "everything-else-being-equal" restraint should include the stipulation that the subscriber population must remain unchanged. In reality, the tariff configuration does affect the composition of the subscriber population. However, the problem may be eliminated by assuming that a telephone company would be allowed to subject one single subscriber, whom we shall call the individual guinea pig subscriber, to one experimental tariff after another — while
the tariff for all other subscribers remains unchanged. Together with the "constant telephone bill" concept (see below) this provides a model where neither the subscriber population, nor the company's revenue is affected by the experiment. Such discrimination is inexecutable in actual operation, but as a hypothesis it is very helpful in the analysis. To make this model even more confined, it is postulated that the individual guinea pig subscriber may have only one telephone line.

The constant telephone bill restraint limits the experimentation with the individual guinea pig subscriber to such tariffs which would cause his average total monthly bill for local service to remain unchanged — despite the usage-changing effect of different experimental tariffs.

Equilibrium denotes the situation where the tariff's repressing effect on the traffic results in an unchanged telephone bill.

Later in the analysis the restraint that the individual guinea pig subscriber is the only one subjected to tariff changes, as well as the restraint of his having only one line, will be dropped. We will still hypothesise about what effect various experimental tariffs have on our guinea pig subscriber's traffic when they apply equally to all subscribers; but the model now makes him a common guinea pig subscriber instead of an individual one.

In the model for the common guinea pig subscriber the restraint about his telephone bill being constant is superseded by the constant revenue restraint. This means that experimental tariffs would be confined to such configurations which would keep the average revenue per line for the whole subscriber population at a constant level. This model also has the simplifying restraint that the guinea pig subscriber's monthly traffic equals the mean traffic of the subscriber population and his bill equals the mean revenue per subscriber.

2. EQUILIBRIUM

Now equilibrium is established is shown in the following graphs. First, in Fig. 2, a switch from Flat Rate to Straight Measured Tariff and then in Fig. 3, from Flat Rate to No Rental Tariff. Both these graphs picture an individual guinea pig subscriber's reaction. Finally, in Fig. 4, is shown the additional effect on this subscriber's traffic when he becomes a common guinea pig.

![Fig. 2 Step by Step Establishing of Traffic Equilibrium at SMR for an Individual Guinea Pig Subscriber](image)

The Flat Rate is $ R per month and the individual guinea pig subscriber's monthly Flat Rate traffic is T. The first step is to try a straight measured tariff (SMR) with a fixed monthly rate of B and a message rate U₁ = (R - B)/T. If

the subscriber maintains his Flat Rate traffic, the amount of the bill would be R. But he can now realize some savings (which he couldn't under Flat Rate). So he reduces his traffic to T₂ which lowers his bill to B. If the company tries to compensate for this revenue loss by increasing the message rate to U₂ = (R - B)/T₂ he would reduce his traffic to T₃, which would prompt a new U₃ = (R - B)/T₃ — and so on. But for each new tariff adjustment he is less willing to make further reduction. Finally, the equal-revenue-traffic is established at Tₑq with a message rate of Uₑq = (R - B)/Tₑq. Of course, if one could know in advance exactly how the subscriber will react, it would be possible to select, right at the outset, the tariff with the precise value of U to equate the revenues. Since no subscriber's usage would be larger at SMR than at FR, an equilibrium may always be attained for the 'average' guinea pig subscriber. Obviously, in the reverse situation, when a company switches an average subscriber from SMR to a Flat Rate which equals his SMR-bill, his traffic will be T FR

What happens if B were reduced to zero? Obviously, the individual guinea pig subscriber would not reduce his traffic to zero. He would still generate some traffic, in spite of the sizeable message unit rate which will be necessary in order to make up the required revenue. How this No Rental traffic (TNR) is established is shown in Fig. 3.

![Fig. 3 Step by Step Establishing of Traffic Equilibrium at NR for an Individual Guinea Pig Subscriber](image)

In this case the first try is with U₁ = R/T NR. This causes the subscriber to reduce his traffic to T₂ — and so on. Equilibrium is established at the No Rental traffic. At this point the message unit rate is Uₙₗ = R/Tₙₗ. The point of departure in Fig. 4 is the individual guinea pig subscriber's No Rental traffic, Tₙₗ (same as in Fig. 3). It will be described in four steps what happens to this traffic when restraints are relaxed so that he becomes a common guinea pig subscriber.

First step. The restraint that the experimental tariffs apply only to the individual guinea pig subscriber is dropped, and precisely the NR-tariff which repressed his traffic to Tₙₗ now applies to everybody: (R = 0 and U = Tₙₗ).

Initially, however, two restraints will remain:

- There will be no change in number and composition of subscribers.
- No subscriber may subscribe to additional lines (over and above what he has at Flat Rate).
The individual guinea pig subscriber represents average customer and the mean traffic per line will thus shrink to about $\text{T}_{\text{NR}}$ when the tariff applies to everybody. But since mean revenue per line is to be maintained at the level of $R$, there is no need to increase the message unit rate further. Consequently the guinea pig subscriber's traffic is unaffected by the fact that he in this initial step the common guinea pig subscriber, and his traffic remains at $\text{T}_{\text{NR}}$.

**Second step.** The restraint that the subscriber population must remain unchanged is dropped. The absence of a fixed monthly charge in the No Rental tariff will cause a high influx of marginal subscribers, such as:

- Low-income customers who did not subscribe to FR, SMR, or AMR.
- Low-usage customers. This includes those who have a telephone primarily for the purpose of receiving calls (information services, ordering lines, message services, etc.). Some of these lines have no traffic at all; others only occasional calls.
- Such places as vacation homes, weekend homes, boats in marinas; any place can afford a telephone when there is no fixed charge.
- Husband, wife, children, relatives — any person sharing a home — can afford to have a telephone in his or her name and become a separate subscriber. Traffic on such lines will be a fraction of the whole family's.

Factors as those mentioned above will cause the mean traffic per subscriber line to shrink away below $\text{T}_{\text{NR}}$. Consequently the telephone company must increase the message unit rate further in order to maintain the mean revenue per line. A step by step increase along the principles in Fig. 2 and 3 (not shown in Fig. 4) would bring about a new equilibrium at a very low traffic. Since the increased rate per call applies also to the guinea pig subscriber, he will reduce his traffic to a new low, which is denoted $\text{T}_{\text{NR}\text{2}}$.

**Third step.** The restraint that a subscriber may have only one line (at one location) is relaxed for all subscribers, except our guinea pig subscriber. This brings about the following:

- Subscribers with heavy traffic would subscribe to one or more additional lines to avoid having to wait.
- Even small businesses would have "extra" lines, so that no business would be lost because of busy-line situations.
- Even a residence subscriber who is single could enjoy the convenience of having two lines, so that one would be free for incoming calls when the other is occupied. In case of trouble on one line, the other may be used to report it.

Because of all this traffic-splitting the mean traffic per line will be reduced substantially below $\text{T}_{\text{NR}}$. This requires additional increase in the rate per call in order to maintain constant mean revenue. Such increase will, in turn, have its effect on the guinea pig's traffic, which will settle at a new and lower equilibrium, $\text{T}_{\text{NR}\text{3}}$.

**Fourth step.** Up to this point the guinea pig subscriber has been charged a progressively higher rate per call, either because of primary reasons, such as his own reaction to a No Rental tariff, or secondary reasons, such as influx of low-users and traffic splitting by others. As a result, the only outgoing traffic he can afford is extremely important calls. Yet, at some point in the gradual shift from FR to NR he discovered the convenience of having two lines. Consequently his traffic per line is reduced to only one half of a traffic consisting of emergency and other important calls, or to point $\text{NR}^2$.

In summary: The common guinea pig subscriber's traffic per line at NR would be so small as to be regarded as zero in mathematical expressions of subscriber reaction.

The described reduction from $\text{NR}_2$ to $\text{NR}_3$ refers to mean traffic at No Rental tariffs. But it is clear that all factors mentioned above (influx of marginal subscribers, traffic splitting, etc.) are present all along, when a subscriber population is gradually shifted from Flat Rate to various Measured tariffs; and they progressively affect mean traffic as the rate ratio is increased. Thus one may say, that these factors have an accelerating effect on the repression of the guinea pig subscriber's traffic (over and above what happens when he is the only one subjected to such consecutive configuration changes). It is also clear, that empirical cross-section data, indicating relationship between traffic per line and configuration, encompasses the effect of subscriber influx, traffic splitting, etc.

When one realizes the effect of a substantially lowered fixed monthly rate on mean usage per subscriber line, it is easy to understand why No-Rental local tariffs are not offered publicly. The company would price itself out of the market long before this point is reached.

3. **DIMINISHING RELATIVE SAVINGS**

Before developing demand functions for multi-part tariffs, a brief description of what may be called the Law of Maintaining Relative Savings will be given. It states that:

The smaller the relative savings from a certain relative reduction in consumption, the lesser the consumer's incentive to effectuate that amount of reduction.

The difference between multi-part tariffs and a price, in this respect, is that:

- When a customer reduces his consumption of a common commodity by a certain percentage, his cost is reduced by the same percentage (disregarding quantity discounts).
- In that case the consumer enjoys commensurate relative savings.
- When a telephone subscriber with a publicly offered measured local tariff reduces his traffic, he can realize only diminishing relative savings.

The latter situation is illustrated in Fig. 5. The tariff is SMR with $\$30.00$ per month and $\$0.10$ per call. The subscriber's Flat Rate demand is 200 calls per month, which would make his bill $\$30.00$ if he maintained this traffic. If he reduces his traffic in steps of $50\%$ at a time, the corresponding reduction in his bill is $35.5\%$, $25\%$, $16.7\%$, $10\%$, and so on. Hence the expression diminishing relative savings.

Compare this with the fact, that had the commodity been cigarettes instead of telephone calls, any $x\%$ reduction in consumption would have rendered $x\%$ saving in the bill.
Incentive

4. INCENTIVE

In Fig. 6 are shown the same five types of tariffs as in Fig. 1. In addition to the commonly offered AMR, with a proper tariff factor \( f < 1 \), it also shows AMR₂, which is offered exceptionally, but is improper because \( f > 1 \).

The following postulations are made to facilitate algebraically simple observations:
- A subscriber's flat rate traffic is \( T_{FR} = 200 \) calls/month.
- The allowance for both AMR and AMR₂ is \( A = 100 \) calls/month.
- Measured tariffs have the same price/call \( U = \$ 0.05 \).
- \( f = 10.00, B₁ = \$ 7.00, B₂ = \$ 5.00, B₃ = \$ 3.00 \)

<table>
<thead>
<tr>
<th>Telephone Bill</th>
<th>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
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<td>8</td>
<td>10</td>
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<tr>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
</tr>
</tbody>
</table>

![Fig. 6 Effect on Bill of Cutting Traffic in Half.](image)

In order for a tariff to encourage the subscriber to save cost, it must offer him an incentive to economize. As an illustration of how various types of tariffs differ in this respect, we assume that a subscriber is consecutively subjected to all six tariffs. If, in each case, he tries to economize by cutting his traffic in half, the result would be:

### Table 1

<table>
<thead>
<tr>
<th>Tariff</th>
<th>Traffic Bill</th>
<th>Effective Price per Call</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reduction</td>
<td>Before After Relative</td>
</tr>
<tr>
<td></td>
<td>Relative</td>
<td>Reduo. Reduo. Change</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$ $ $ $</td>
</tr>
<tr>
<td>FR</td>
<td>50 %</td>
<td>0 % 0.05 0.10 Doubled</td>
</tr>
<tr>
<td>SNR</td>
<td>50 %</td>
<td>0.075 0.10 Increased 33 %</td>
</tr>
<tr>
<td>AMR₁</td>
<td>50 %</td>
<td>0.06 0.07 Increased 27 %</td>
</tr>
<tr>
<td>AMR₂</td>
<td>50 %</td>
<td>0.05 0.05 Unchanged</td>
</tr>
<tr>
<td>AMR₃</td>
<td>50 %</td>
<td>0.04 0.05 Reduced 25 %</td>
</tr>
<tr>
<td>AMR₄</td>
<td>50 %</td>
<td>0.025 0 Reduced 100 %</td>
</tr>
</tbody>
</table>

Incentive may be expressed in mathematical terms as follows: When switching from FR to Measured tariff, a small reduction in the traffic \( T_{FR} \) is denoted \( d(T_{FR}) \). Its effect on the total bill \( G \) is a reduction in the amount of \( G \). Comparing these ratio-wise to see what relative bill reduction results from a small relative traffic reduction, the ratio becomes:

\[
i = \frac{dG}{G} \quad \text{or} \quad i = \frac{dG}{G(T_{FR})} \frac{T_{FR}}{d(T_{FR})} = 1 (4)
\]

The ratio \( i \) expresses the incentive. Its definition is:

Incentive is the proportionate change in the bill divided by the proportionate change in demand.

Incentive should not be confused with elasticity. The latter is the proportionate change in the quantity in demand divided by the proportionate change in price.

The incentive concept will be applied to four cases of switch from Flat Rate to different types of Measured Tariffs.

**NR.** Pure price tariff. (No rental or allowance; only \( U \))

Bill at unmeasured traffic: \( G = Tₕ₁U \) for \( T_{FR} \) (7)

\[
i = \frac{dG}{G} = \frac{d(T_{FR})U}{T_{FR}} = \frac{1}{1 - \frac{A}{T_{FR}}} = 1 (6)
\]

Price-incentive = 1.

Relative savings increase as traffic is reduced.

**AMR.** Price tariff with allowance. (no rental; only \( U \) and \( A \))

Bill at unmeasured traffic: \( G = Tₕ₂U \) for \( T_{FR} \) (8)

\[
i = \frac{dG}{G} = \frac{d(T_{FR})U}{T_{FR}} = \frac{1}{1 - \frac{A}{T_{FR}}} = \frac{T_{FR}}{B + T_{FR}} (9)
\]

Rate-ratio-incentive \( > 1 \).

Relative savings diminish as traffic is reduced.

**SNR.** Two-part tariff. (no allowance; only \( B \) and \( U \))

Bill at unmeasured traffic: \( G = B + T_{FR}U \) (10)

\[
i = \frac{dG}{G} = \frac{d(T_{FR})U}{B + T_{FR}U} = \frac{T_{FR}}{B + T_{FR}} (11)
\]

Rate-ratio-incentive \( < 1 \).

Relative savings diminish as traffic is reduced.

**AMR₂.** Three-part tariff. (with \( B₁ \), \( U \) and \( A₁ \))

Bill at unmeasured traffic: \( G = B + (T_{FR} - A)U \) (12)

\[
i = \frac{dG}{G} = \frac{d(T_{FR})U}{B + (T_{FR} - A)U} = \frac{T_{FR}}{B + T_{FR}(T_{FR} - A)U}
\]

Relative savings diminish as traffic is reduced.
5. DEMAND FUNCTIONS FOR AN INDIVIDUAL GUINEA PIG SUBSCRIBER

The concept of incentive will now be used to set up mathematical expressions for an individual subscriber's demand for local calls under various tariff configurations.

NR. No Rental Tariff.

\[ T_{NR} = T(0) \]  

(13)

It is rather common to express a simple price-demand relationship by a logarithmic function of the type

\[ T_{NR} = C \cdot U^Y \]  

(14)

Such a formula implies that the subscriber's demand for telephone calls would be infinitely large if \( U \) were reduced to zero. In reality, the demand at this point has a finite value, namely \( T_{NR} \). The simplest way to reflect this restraint is to write the NR-demand as

\[ T_{NR} = T_{FR}(1 + CU)^Y \]  

(15)

Actually, \( C \) is a product of two factors. The motivation for the subscriber's reaction is that it would cost him to maintain his FR-traffic. Since this cost would have been \( T_{FR} \), and one may write \( C = T_{FR}/U \), we get

\[ T_{NR} = T_{FR}(1 + T_{FR}/U)^Y \]  

(16)

According to this formula, the subscriber would reduce his traffic considerably, even if a very small rate per call were applied. One cent per call would have quite an effect. Since the initial reaction is small, the formula should more realistically be written

\[ T_{NR} = T_{FR} \left[ 1 - \left(1 + \frac{1}{T_{FR}/U}\right)^Y \right] \]  

(17)

As may be seen in Fig. 7, this function produces an S-shaped demand curve.

Fig. 7 Demand Curve for an Individual Guinea Pig Subscriber at NR

At NR there is only one dependent variable, \( U \). Thus

\[ T_{NR} = f(U) \]

(18)

The traffic reduction at NR is \( T_{FR} - T_{NR} \) and at ANR it is \( T_{FR} - T_{ANR} \). One may assume, that the size of the traffic reduction in the two cases would have some kind of relationship to the maximum cost saving that could be accomplished in each case. In words

\[ \text{traffic reduction at ANR} = f \left( \text{max. possible ANR-bill reduction} \right) \]

\[ \text{traffic reduction at NR} = \text{max. possible NR-bill reduction} \]

A useful mathematical expression is

\[ \frac{T_{FR} - T_{ANR}}{T_{FR} - T_{NR}} = (1 - A/T_{FR})^X \]  

(19)

The function within the parenthesis is the same as the expression (9) for the allowance-ratio-incentive.

Insertion of formula (17) for \( T_{NR} \) into (18) gives

\[ T_{ANR} = \frac{T_{FR} - \left( T_{FR} - A \right)U}{T_{FR} - T_{NR}} \]  

(20)

\[ T_{ANR} = \frac{T_{FR}}{T_{FR} - T_{NR}} \left(1 - \frac{1}{A/T_{FR}}\right)^X \]

Traffic reduction from \( T_{FR} \) to \( T_{ANR} \), relative to the traffic reduction from \( T_{FR} \) to \( T_{NR} \) (for the same \( A \)) is assumed to be a logarithmic function of the relative maximum cost saving.
\[
\frac{t_{\text{FR}} - t_{\text{AMR}}}{t_{\text{FR}} - t_{\text{NR}}} = \left(\frac{(t_{\text{FR}} - A)U}{t_{\text{FR}} - A + B}\right)^x
\]  

(20)

Insert formula (19) for \( t_{\text{AMR}} \)

\[
t_{\text{AMR}} = t_{\text{FR}} - \left(\frac{1}{1 + \frac{1}{t_{\text{FR}} - t_{\text{NR}}}}\right)^y
\]

(21)

\[ y, z \text{ and } x \text{ indicate the subscriber's power of reaction to each type of incentive. One will find that } x = 1. \]

**SMR. Straight measured Tariff**

**Telephone Bill**

![Image of Graph](image)

**Traffic reduction from \( T_{\text{FR}} \) to \( T_{\text{AMR}} \) as related to reduction from \( T_{\text{FR}} \) to \( T_{\text{NR}} \) is assumed to be a logarithmic function of relative maximum cost savings, thus**

\[ \frac{T_{\text{FR}} - T_{\text{SMR}}}{T_{\text{FR}} - T_{\text{NR}}} = \left[\frac{T_{\text{SMR}}}{T_{\text{FR}} - T_{\text{NR}}}\right]^x \]  

(22)

Incorporate formula (17) for \( T_{\text{NR}} \)

\[ \frac{T_{\text{SMR}}}{T_{\text{FR}}} = 1 - \frac{T_{\text{PMR}}}{T_{\text{FR}}} \]  

(23)

**Demand function for SMR may also be obtained by giving \( A \) the value of zero in (21).**

Formulae (20) and (22) show that the incentive is actually the ratio between the usage-sensitive portion of the bill and the total bill.

Formula (21) for \( T_{\text{AMR}} \) may be used to build a three-dimensional demand model, as shown in Fig. 11. Traffic is measured on the vertical coordinate axis, and fixed monthly rate and message unit rate are measured on each of the two horizontal axes. Iso-revenue curves, (two of which appear in the figure) may be drawn from any point on the \( T_{\text{AMR}} \)-curve to the corresponding point on the \( T_{\text{FR}} \)-line. Together, all such iso-revenue curves form a demand surface for \( T_{\text{AMR}} \) for any configuration of Straight Measured Tariffs. Similarly, demand surfaces for AMR-tariffs may be obtained by drawing iso-revenue curves from points on the proper \( T_{\text{AMR}} \)-curve.

**Fig. 11 Demand Surface Model for Local Telephone Calls from an Individual Guinea Pig Subscriber**

The average power of reaction is, no doubt, stronger for residence than for business subscribers. One may also assume that subscribers within one class, but with different base demand for communication, may vary with respect to their power of reaction. Marginal customers are joining the system when the threshold (8) is lowered. On an average, such marginal subscribers would be expected to have lower usage but higher power of reaction. The reaction parameter might then be a function of the flat rate traffic. At present there is no data available to allow calibration of the demand function for a differentiated reaction parameter within one class of subscribers. Future before-and-after studies may supply the needed data.

6. **DEMAND FUNCTIONS FOR A COMMON GUINEA PIG SUBSCRIBER**

The difference between the demand function of an individual guinea pig subscriber and a common one is in the former case the No-Rental traffic is \( T_{\text{NH}} \), but in the latter case it becomes zero (to all intents and purposes). A fair approximation of a common guinea pig's demand function for calls may thus be obtained by using the value \( T_{\text{NH}} = 0 \) in equation (16) which then becomes (for \( x = 1 \))

\[ T_{\text{AMR}} = T_{\text{FR}} - T_{\text{FR}}(1 - A/T_{\text{FR}})^x \]  

(24)

Using this formula for \( T_{\text{AMR}} \) in equation (21) to

\[ T_{\text{AMR}} = T_{\text{FR}} - \left[ 1 - \frac{1}{T_{\text{FR}} - A} \right]^y \left( 1 - \frac{1}{t_{\text{FR}}} \right)^x \]  

(25)

This formula encompasses the earlier mentioned acceleration in regression. Consequently, \( x \) has a higher value in the demand function of a common guinea pig subscriber than of an
The formula for SNR is obtained by eliminating $A$ in (25)

$$T_{SNR} = T_{PR} - T_{FR} \left( \frac{T_{PR} U}{B + T_{PR} U} \right)^a$$

(26)

Formula (25) may be used to construct demand curves for $AMH_a$:
- in Fig. 12A as a function of the rate ratio ($s = U/B$), at various allowance ratios ($a = A/TPR$)
- in Fig. 12B as a function of $a$ at various values of $s$.

![Diagram](https://via.placeholder.com/150)

**Fig. 12 A**

![Diagram](https://via.placeholder.com/150)

**Fig. 12 B**

Demand Curves at AMH for a Common Gullage Pig Subscriber

From the graphs may be seen, that the rate-ratio-elasticity for telephone calls is negative and the allowance-ratio-elasticity is positive. Further analysis of formula (25) reveals that the demand for local calls is inelastic, and that the elasticity is rather small for commonly used values of the rate-ratio and allowance-ratio.

7. **AGGREGATION OF SUBSCRIBERS’ DEMAND FOR CALLS**

Because $TPR$ is an integral part of each subscriber’s demand function, it is necessary to know how the subscribers are distributed with regard to their Flat Rate traffic. The total traffic at Measured Tariffs is then obtained by integrating, over the whole traffic range, the product of the number of subscribers in each traffic interval and the average repressed traffic per subscriber in that interval.

$$TPR_{Max} = \sum_{0}^{TPR_{Max}} f(T_{PR}) T_{PR} \phi(T_{PR}) d(T_{PR})$$

(27)

where $f(T_{PR})$ = Subscribers’ Flat Rate distribution

$\phi(T_{PR})$ = Individual subscriber reaction

Example of curves depicting these two functions are shown in Fig. 13 and Fig. 14.

![Diagram](https://via.placeholder.com/150)

**Fig. 13** Subscriber Demand Distribution at $T_{PR}$

![Diagram](https://via.placeholder.com/150)

**Fig. 14** Subscriber Reaction

Associated with the aggregation process are such problems as:
- selecting standard Flat Rate subscriber distributions and their mathematical functions
- non-tariff-dependent fluctuations of a subscriber’s traffic
- part-time users
- zero-users
- over-reaction and other fringe phenomena
- duration of adjustment periods

These and other problems will be dealt with in a separate paper in connection with efforts to calibrate the demand functions on the basis of data which the author has accumulated during several decades from quite a few countries regarding:
- tariffs
- mean traffic per subscriber line
- subscriber distribution
- traffic before and after tariff changes

This material is presently in the process of being statistically analyzed.

Because of the complexity of the problem, and the many variables involved, it would be of great value if telephone administrations would contribute additional tariff- and traffic-data. A questionnaire is available for this purpose.
Addendum

8. INTENSIFIED REACTION AT AMR

Applying the test that $T_{AMR} = A$ for $U = \infty$ to formula (21) led to formula (25), which also may be written as:

$$ T_{AMR} = \frac{T_{FR} - A}{B + (T_{FR} - A)U} $$

(29)

The denominator indicates the degree of repression of the excess traffic $(T_{FR} - A)$. The expression within brackets spells out the incentive, if Flat Rate traffic were maintained, viz.

$$ \frac{(T_{FR} - A)U}{B + (T_{FR} - A)U} $$

(29)

One more factor has to be taken into account, however. Consider a household with a FR-demand of 100 calls/month. It is switched to AMR with $B = 10.00$ per month and $U = 0.08$ per call. The bill at maintained FR-traffic would be $18.00$, of which $8.00$ for usage. With some effort, the subscriber may reduce his traffic by 50 calls per month.

Later the tariff is changed to AMR with $A$ calls/month and $B$ and $U$ maintained at $10.00$ and $0.08$. By this time the household has a FR-demand of 200 calls/month; everything else being equal, including the household budget. Maintained AMR-traffic would also in this case produce a $18.00$ bill, of which $8.00$ for usage, since $(T_{FR} - A)$ is the same in both situations. This time, however, it takes less effort to reduce the traffic by 50 calls per month. This is, in fact, a way of saying that the marginal utility of the 50 relinquished calls is less in the second case than in the first.

Assuming that the utility of the most important call is the same in both cases, one may use a simple schematic to illustrate the difference in utility of the 50 least important calls.

In Fig. 15 there are $(T_{FR} - A) = 100$ excess calls in both diagrams, but the utility of the fifty-call reduction is different. This may be expressed as follows:

A given amount of excess traffic will — under otherwise like conditions — be more repressed if it lies 'on top of' a high allowance.

The fact that the incentive is increased under such conditions, may be given recognition in formula (25) by incorporating a factor that accelerates the repression:

$$ T_{AMR} = \frac{T_{FR} - A}{B + (T_{FR} - A)U/B} $$

Formula (30) may be expressed entirely in relative measures.

Let $u = T_{AMR}/T_{FR} \quad a_1 = A/T_{FR} \quad g = T_{FR}/B$

then

$$ u = 1 - (1 - a_1) \left[1 + (1 - a_1)^{-\frac{1}{(1-x)2k}} \right]^{-\frac{1}{x}} $$

(31)

where $0 < a_1 < 1$

$a_1$ denotes an individual subscriber's relative allowance. The above simple restraint does not apply to collective (aggregate) demand, because if the allowance were set higher than the mean traffic, there would still be some usage repression.

![Fig. 15. Intensified incentive at higher allowance.](image-url)