Dimensioning of Traffic Routes According to the EERT-Method and Corresponding Methods

Kauko Rahko
Helsinki University of Technology, Helsinki, Finland

ABSTRACT

In this paper the ERT, EERT, NDM and WDM dimensioning methods are compared. It is proved that the NDM and (E)ERT are valid dimensioning methods based on different criteria. The NDM is based on the time congestion criteria. For EERT it is calculated values according to time congestion (EERT-T) and compared both methods in this case.

Methods based on time congestion improve the dimensioning of telephone networks on the basis of overall grade of service because then the total congestion can be determined as a sum of the time congestion in successive routes. Whether the NDM is better than the EERT can be proved only by measurements.

1. INTRODUCTION

The Equivalent Random Theory (ERT) has been widely used for dimensioning of traffic routes. Its limitation is that only such routes in which the ratio of the variance of offered traffic to the average is more than or equal to one (z>1) can be dimensioned by this theory /1/. Dr. G. Breschneider presented at ITC 7 in Stockholm an extension to the ERT (EERT) /2/ so that it also covers smooth traffic (in which 0<z<1). For EERT the author has calculated tables and made curves on the basis of call congestion /3/.

Reference /5/ presents a dimensioning method of traffic routes based on time congestion and on the use of a Gaussian distribution model for carried traffic (the Normal Distribution Method, NDM). This method has been tabulated /8/ for both time and tail congestion. From the table the number of lines needed can be read for different average and dispersion values of carried traffic. We have started the calculation of tables for the ERT and EERT on the basis of time congestion.

This paper compares NDM, ERT and EERT on the basis of different service criteria.

2. EXTENDED EQUIVALENT RANDOM THEORY

2.1 General

By the ERT such fully available overflow routes can be dimensioned on the basis of the average m and the variance $\sigma^2$ of offered traffic in which the peakedness factor, $z=\sigma^2/m$, is more than or equal to one. With the EERT systems can be dimensioned in which the peakedness factor is greater than zero.

There are three types of traffic, smooth traffic (0<z<1), pure chance traffic (PCT1 z=1) and peaked traffic (PT z>1). Traffic is smoother than pure chance traffic if a finite number of sources generates pure chance traffic (PCT2) or when traffic carried by a finite number of lines is offered from some node forward. When congested pure chance traffic generated by a finite number of sources of the first choice route is directed to an overflow route, its peakedness factor is less than, equal to or more than one depending on the value of the average /11/.

Figure 1: Described Network

PCT1 on High Usage Groups

PCT 1 or PCT 2

PCT 1 or PCT 2

PCT 2 on High Usage Groups

PCT 1 or PCT 2
By the EERT it is possible, for example, to dimension the route m-n (Fig. 1). Traffic offered to this route can be smoothed traffic (to nodes i PCT1 or PCT2 is offered) in routes i-m (i=1,2,...I). On the other hand, it can be overflow traffic of PCT1 coming from first choice routes m-k (k=1,2,...K), overflow traffic of PCT2 coming from first choice routes m-j (j=1,2,...J) or direct traffic PCT1 or PCT2. This dimensioning problem is dealt with in the following.

2.2 The Principle of the ERT and EERT

In ERT the real system (Fig. 1) is replaced by such an equivalent system of Leq servers that the offered equivalent Poisson traffic Aeq generates an overflow traffic with the same mean and variance as the real traffic. The mean M and variance V are (Fig. 2) 1,2:/

\[ M = \sum_{i=1}^{T} M_{si} + A + \sum_{k=1}^{P} M_{pk} \]  
\[ V = \sum_{i=1}^{T} V_{si} + A + \sum_{k=1}^{P} V_{pk} \]  

A is direct offered Poisson traffic to route m-n  
T \sum_{i=1}^{T} M_{si} is the sum of the means of smooth traffic offered to route m-n  
P \sum_{k=1}^{P} M_{pk} is the sum of the averages of peaked traffic offered to route m-n  
P \sum_{k=1}^{P} V_{pk} is the sum of the variances of peaked traffic offered to route m-n

The equivalent parameters Aeq and Leq can be determined by equations 2a and 2b.

\[ M = A_{eq} \cdot E_{1,Leq}(A_{eq}) \]  
\[ V = M(1-M + \frac{A_{eq}}{Leq+M+1-A}) \]  

Aeq and Leq are determined either by iteration or by approximate formulae. Aeq and Leq calculated from the average and the variance of offered traffic behave as follows (Figures 3 and 4) depending on the peakedness factor.

Figure 2: Traffic diagram of equivalent system

Figure 3: Equivalent traffic as a function of offered traffic, peakedness factor as a parameter

Figure 4: Number of equivalent lines as a function of offered traffic, peakedness factor as a parameter

When M, V, Aeq and Leq are given, the number h of lines in the route m-n is determined by iteration from call congestion.

\[ B = E_{1,Leq} + N(A_{eq}) \]  

\[ E_{1,Leq}(A) \] is Erlang's congestion.
2.3 Numerical Solution of the EERT

According to the equation (3) it is possible to calculate congestion with both positive and negative numbers of lines. In the calculation the integral form of Erlang's congestion formula can be used /2/.

\[ E_{1,L}(A) = \frac{A^{-1}}{e^{-A}A^{1+y}} dy \] (4)

The congestion behaves /9/ according to Figure 5 as a function of \( L \).

\[ \begin{array}{ccc}
A & B_{E}(A,-L) & B_{E}(A,L) \\
5 & 50 & 450 \\
10 & 20 & 400 \\
20 & 10 & 350 \\
100 & 5 & 300 \\
\end{array} \]

Figure 5: Congestion as a function of number of lines, traffic as a parameter

The equation (4) can be solved for \( L \) numerically. Suitable numerical approximations for congestion are continued fractions and Laguere's polynomials. Because it is difficult to determine the number of lines by iteration and to calculate the congestion numerically, the numbers of lines calculated by the EERT have been tabled as a function of offered traffic and peakedness factor with several congestions. The numbers of lines according to the EERT as a function of offered traffic \( M \), peakedness factor \( z \) and congestion \( B \) are shown in Figures 6...11.

\[ \begin{array}{ccc}
M & \text{B=1%} & \text{B=0.2%} \\
50 & z=0.5 & z=0.66 \\
100 & z=0.2 & z=0.66 \\
\end{array} \]

Figure 6: Number of lines calculated by the EERT as a function of offered traffic with 1 % congestion, peakedness factor as a parameter

\[ \begin{array}{ccc}
M & \text{B=1%} & \text{B=0.2%} \\
50 & z=0.5 & z=0.66 \\
100 & z=0.2 & z=0.66 \\
\end{array} \]

Figure 7: Number of lines calculated by the EERT as a function of peakedness factor with 1 % congestion, offered traffic as a parameter

\[ \begin{array}{ccc}
M & \text{B=1%} & \text{B=0.2%} \\
50 & z=0.5 & z=0.66 \\
100 & z=0.2 & z=0.66 \\
\end{array} \]

Figure 8: Number of lines calculated by the EERT as a function of congestion, peakedness factor as a parameter when \( M = 50 \) Er\( \text{l} \)}
3. METHODS BASED ON NORMAL DISTRIBUTION

3.1 Time Blocking based on the NDM

The methods which will be described here have a traffic model with normally distributed traffic intensity /5/. The mean \( m \) and the variance \( \sigma^2 \) of the normal distribution represent the average and the variance of traffic carried at low congestion in the route under study. The time congestion based on the NDM is calculated from formula (5).

\[
B = \frac{n-0.5}{n+0.5} \int f(x)dx
\]

where

\[
f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-m)^2}{2\sigma^2}}
\]

\( m \) = mean value of carried traffic
\( \sigma \) = dispersion of carried traffic

In Fig. 9 the number of lines \( n \) is shown as a function of the peakedness factor. The number of lines has been tabulated in Reference /8/. An example of the said table is shown in Table 1. When \( m \) and \( \sigma^2 \) have been determined, the number of lines corresponding to congestion \( B = 1 \% \) can be read from Table 1.

<table>
<thead>
<tr>
<th>( m^2 )</th>
<th>( B = 0.01 )</th>
<th>( 1 \ldots 36.66 \ldots 200 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>3.81 \ldots 12.84 \ldots 22.02</td>
<td></td>
</tr>
<tr>
<td>20.00</td>
<td>22.81 \ldots 31.84 \ldots 41.02</td>
<td></td>
</tr>
<tr>
<td>21.00</td>
<td>23.81 \ldots 32.84 \ldots 42.02</td>
<td></td>
</tr>
<tr>
<td>22.00</td>
<td>24.81 \ldots 33.84 \ldots 43.02</td>
<td></td>
</tr>
<tr>
<td>50.00</td>
<td>52.81 \ldots 61.84 \ldots 71.02</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Number of lines needed when time congestion \( B = 1 \% \), \( m = 1 \ldots 50 \) Erlang and \( \sigma^2 = 1 \ldots 200 \) Erlang².

3.2 Tail Congestion (TC)

The tail congestion for normally distributed traffic (TC) /8/ is calculated from formula (6).

\[
B = \frac{1}{n+0.5} \int f(x)dx
\]

where the notations are as in (5). The number of lines as a function of the peakedness factor is shown in Fig. 10. The number of lines for a
congestions underdimensions it and gives the most exact values with commonly used dimensioning congestions (curve d: exact values /10/). When offered traffic is PCT2, routes can be dimensioned approximately by the EERT by calculating the parameters of offered from the binomial distribution /2/ \( M = q a, Z = 1 - a \), where \( a \) is the average traffic of the source and \( q \) is the number of sources. In Figure 20 the results of the EERT are compared with values dimensioned by Engset’s call congestion /7/. It can be noted that the EERT dimensioning quite well approximates the dimensioning based on Engset’s call congestion.

![Diagram](image)

**Figure 18:** Studied system where smooth traffic is offered to node \( m \).

![Diagram](image)

**Figure 19:** Dimensioning of the route \( N_m \) in Figure 18 by different methods

4.3 Comparison on basis of time congestion

The NDM and the WDM are based on time congestion. Earlier researches prove that with smooth traffic time congestion is greater than call congestion /10/ and vice versa with overflow traffic /1/. The same conclusion can be made by studying the number of lines \( N \) dimensioned for 1 % congestion by the NDM and the EERT as a function of \( z \) (Figure 21). When the average traffic \( M \geq 20 \) Erl, the intersection of curves is in point \( z = 1 \) with a high accuracy. The Figure also shows the tail congestion method (TC). The number of lines dimensioned by the TC-Method is usually greater than by the NDM and depending on the values of \( M \) and \( z \) either greater or smaller than by the EERT.

With certain overflow traffic the number of lines by the NDM is smaller than that by the ERT based on time congestion (ERT-T) (Figure 22). The smooth traffic system in Figure 18 is dimensioned in Figure 23 by the NDM (curve a). The values of the NDM follow the number of lines calculated from exact time congestions /10/. From Figures 19 and 23 it seems that as well as the EERT overdimensions the number of lines based on call congestion so does the NDM overdimension the number of lines based on time congestion, but to a smaller degree.
Figure 21: Number of lines N dimensioned for 1\% congestion by (---) EERT, (--.--.) NDM and (--- ---) TC method as a function of z.

Figure 22: Average traffic according to ERT-T and NDM as function of peakedness factor.

Figure 23: Comparison of the number of lines in the system described in Figure 18 dimensioned by the NDM (curve a) to exact values (curve b) as a function of time congestion B.

5. CONCLUSIONS

Above we have presented an extension to the curves of Wilkinson's method in the smooth traffic area. These curves are useful in the dimensioning when using Wilkinson-Bretschneider's Method. Complete tables are available and will be sent at request. The results of the EERT have been compared to exact values with smoothed traffic and to values by Engset's call congestion formula with PCT2 traffic. In addition we have compared the results of the NDM to the results of the ERT-T with overflow traffic and to exact values with smoothed traffic.

Methods based on time congestion improve the dimensioning of telephone networks on the basis of overall grade of service because then the total congestion can be determined as a sum of the time congestions in successive routes. This is the basis from which also the CCITT's definition of grade of service should be determined and thus it could be avoided that the measured congestion is usually greater than the estimated congestion. The NDM and the EERT are different dimensioning methods and they are based on different assumptions. Whether the NDM is better than the EERT can be proved only by measurements.

6. ACKNOWLEDGEMENT

The author wishes to thank Mr E Kivelä for programming the numerical calculations /16/, Mr Palo for his assistance in the time congestion comparison /18/, Mr Hertzberg for assisting in the comparison of methods and Mr M Tossavainen for his help in finishing the manuscript.

This work has been sponsored by Finnish Academy.

REFERENCES:


3. Rahko K: Dimensioning tables for smooth, Poisson and peaked traffic based on EERT. Technical Research Centre of Finland.
given tail congestion has also been tabulated in Reference /8/. An example is shown in Table 2. For \( m = 21 \) Erlang, \( \sigma^2 = 36.66 \) Erlang and \( B = 1 \% \) we get \( n = 35.61 \) lines.

<table>
<thead>
<tr>
<th>( m^2 )</th>
<th>1 ... 36.66</th>
<th>200.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>3.83 ... 15.61</td>
<td>34.45</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>20.00</td>
<td>22.83 ... 34.61</td>
<td>53.45</td>
</tr>
<tr>
<td>21.00</td>
<td>23.83 ... 35.61</td>
<td>54.45</td>
</tr>
<tr>
<td>22.00</td>
<td>24.83 ... 36.61</td>
<td>55.45</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>50.00</td>
<td>52.83 ... 84.61</td>
<td>83.45</td>
</tr>
</tbody>
</table>

Table 2: Number of lines needed when tail congestion \( B = 1 \% \), \( m = 1 \ldots 50 \) Erlang and \( \sigma^2 = 1 \ldots 200 \) Erlang.

3.3 Karlsson's Method

S.A. Karlsson calculated the number of lines /6/ as follows:

\[
n = x_T + tz
\]

which is the so-called peak traffic dimensioning method. He showed that

\[
z = 2 \int (x-x_T)f(x)dx - \frac{2}{\sqrt{\pi}} \sigma z
\]

where \( f(x) \) is the frequency function of the continuous normal distribution (5) and \( x_T \) is the average value of the traffic distribution. For example, with \( B = 1 \% \), the value of \( t \) would be 3 /6/.

3.4 The Weibull Distribution Method (WDM)

The NDM is a dimensioning method with two parameters \( (m, \sigma) \) and it gives better results than the Poisson Distribution Method with one parameter \( (m = \sigma^2) \). Should more exact time congestion be required than in the NDM then could be used the dimensioning method with three parameters presented in reference /5/ and based on the Weibull fitting of carried traffic (three parameters \( \alpha, \beta \) and \( \gamma \)). These parameters are determined from carried traffic by numerical methods (with the help of a processing measuring device). The time congestion of the WDM can be calculated from the formula:

\[
B = \frac{\left(\frac{(n+0.5-\gamma)^\beta}{\alpha}\right)}{1-e^{-\frac{(n-0.5-\gamma)^\beta}{\alpha}}}
\]

where \( \alpha = \) scale parameter
\( \beta = \) shape parameter
\( \gamma = \) location parameter

Figure 12: Blocking calculation based on Weibull frequency distribution function.

4. COMPARISON

4.1 NDM and traffic measurement results

The two traffic parameters of the NDM, \( m \) and \( \sigma \), are determined by measuring carried traffic in the route under study during a busy hour, a busy period, a working period (6 h) or during several successive periods of 24 hours (e.g. 5 days).

Figures 13, 14 and 15 give some examples of measured traffic distributions. The measurements have been done in Finland on the routes Forssa-Helsinki-Kuopio (Figure 16) from 8 a.m. to 4 p.m. five consecutive days in 1967 /12/.

Figure 13: Traffic intensity distribution \( p(x) \) in a first choice route (route 1 in Fig. 16), 5 days, 8 a.m. to 4 p.m.
Figure 14: Traffic intensity distribution \( p(x) \) in a final choice route with first-routed traffic (route 2 in Fig. 16).
In all non-overflow cases the normal distribution fits well into the whole distribution, so the dimensioning can be based on it /5/. This is also valid for routes with high congestion and for a final choice route when the congestion of the preceding route is high /15/.

Into the normal distribution can be fitted measuring results on the basis of the average and the dispersion or with the help of a normal distribution paper. In the last-mentioned case the fitting can also be done for a certain part of the normal distribution. Then the parameters of the fitted distribution differ from the parameters of the measured distribution. Associated measuring techniques are treated with in a paper at this congress /13/.

With small values of \(m/\sigma \leq 1,5\) negative states of occupancy should be left out from the NDM. This means that the integral in the denominator of the time congestion formula (5) is calculated from \(-0.5\) to \(n+0.5\). The time congestion obtained in this way is denoted by \(B(n)\) in Figure 17, which shows how the value of NDM time congestion \(B\) changes as a function of traffic parameters when \(B(n)\) is 1 % /9/. In direct routes \(B\) usually equals \(B(n)\).

4.2 Comparison of results by the EERT on basis of call congestion

With overflow traffic the EERT gives the same results as the ERT based on call congestion. Some results by the EERT are compared with the results of other methods.

To the system in Figure 18 smoothed traffic is offered and the route \(N_m\) is dimensioned in three ways: a. according to Erlang's 1 congestion formula so that it is assumed that to the route \(N_m\) Poisson traffic \(A = \frac{1}{2} \cdot A_{in}\) is offered,

\[ B(n) = 1\% \]

b. according to Erlang's 1 congestion formula so that Poisson traffic offered to the route is assumed to be the sum of \(A\) and c. by the EERT. On the basis of the dimensioning results (Figure 19) it seems that the EERT with small congestions somewhat overdimensions the route requirement, with high congestion.

Figure 15: Traffic intensity distribution in a final choice route carrying only both-way overflow traffic (route 3 in Figure 16), 5 days, 8 a.m. to 4 p.m.

Figure 16: Traffic routes. F = terminal trunk exchange in Forssa, H = transit exchange in Helsinki, K = transit exchange in Kuopio. 1, 2 and 3 = routes studied. 1 = high-usage route, 2 = both-way low-loss route, 3 = both-way low-loss route.

4.2 Comparison of results by the EERT on basis of call congestion

With overflow traffic the EERT gives the same results as the ERT based on call congestion. Some results by the EERT are compared with the results of other methods.

To the system in Figure 18 smoothed traffic is offered and the route \(N_m\) is dimensioned in three ways: a. according to Erlang's 1 congestion formula so that it is assumed that to the route \(N_m\) Poisson traffic \(A = \frac{1}{2} \cdot A_{in}\) is offered,

\[ B(n) = 1\% \]

b. according to Erlang's 1 congestion formula so that Poisson traffic offered to the route is assumed to be the sum of \(A\) and c. by the EERT. On the basis of the dimensioning results (Figure 19) it seems that the EERT with small congestions somewhat overdimensions the route requirement, with high congestion.

Figure 17: NDM time congestion \(B\) as a function of average and dispersion of the traffic model, when corrected time congestion \(B(n)\) is 1 %.

With small values of \(m/\sigma \leq 1,5\) negative states of occupancy should be left out from the NDM. This means that the integral in the denominator of the time congestion formula (5) is calculated from \(-0.5\) to \(n+0.5\). The time congestion obtained in this way is denoted by \(B(n)\) in Figure 17, which shows how the value of NDM time congestion \(B\) changes as a function of traffic parameters when \(B(n)\) is 1 % /9/. In direct routes \(B\) usually equals \(B(n)\).


