A Method for the Calculation of Traffics Offered to an Alternative Routing Arrangement

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ABSTRACT
In planning multi-exchange networks the traffic distribution in the network must be calculated for several years ahead. For this calculation we need the existing traffic distribution in the network.

In the Swedish network all routes are measured two times each year. These measurements will give carried traffic per route in erlang. One important task is to calculate the part of the total offered traffic to each route which constitutes direct offered traffic, as the offered traffics to the routes are a mixture of both direct offered traffic and overflow traffic from other routes.

The method in this report determines the different offered traffics to an alternative routing arrangement in such a way that the sum of squared relative differences between calculated and observed carried traffic per route is minimized.

Carried traffics are calculated using the Wilkinson equivalent random method. The method is extended to accept offered smooth traffics. The calculations are performed using a standard iterative routine for minimizing a sum of squares. The initial values of the offered traffics are obtained by a "backwards" application of the Wilkinson method.

A computer program has been written for all these calculations and has been tested on several different routing arrangements with satisfying result.

1. INTRODUCTION
The normal traffic measuring equipment in the Swedish network can only measure carried traffic per route. For the planning and dimension of the network however we need to know the direct offered traffic to each existing route. As the network is an alternating routing network it is rather complicated to calculate these unknown traffic values.

Based on the Wilkinson equivalent random method a new routine has been designed which will give a better result than the old manually methods. The routine works in two steps.

First we assume that all routes have direct offered traffic. We calculate such values of these direct offered traffic that the calculated carried traffics for each route exactly corresponds with the measured carried traffics for the routes.

Mostly some routes shall have no direct offered traffic, only overflow traffic from other routes. Our calculation will probably give some small direct offered traffic, positive or negative even for these routes, due to measurement errors in the measured carried traffics on the routes.

Another cause for getting these small direct offered traffics instead of zero could be that the measured carried traffics are a mean value over several busy hours. To make the calculations more general we allow each direct offered traffic to have its own known variance to mean ratio. For normal Erlang distributed traffic we have this ratio \( Q = 1 \).

In the second step of the calculation we try to find the direct offered traffics that minimize the sum of squared relative differences between calculated and measured carried traffic per route. The calculated traffics in the first step are used as starting values for this second step.

Both steps are performed with iterative calculations and a computer program is written in the FORTRAN language.

2. METHOD FOR CALCULATION
Before we start the calculation we must know the following parameters.

a) The number of routes
b) The number of circuits for each route
c) The measured carried traffic for each route
d) The quotient variance/mean value for each route
e) The structure of the routing arrangement i.e. the direction of the blocked traffic from each route.

Fig 1 Example of a routing arrangement
The structure of a routing arrangement with m routes can be stored in two arrays

\[ M_j', (j=1, \ldots, L_{m+1} - 1); \]
\[ L_i', (i=1, \ldots, m+1); \]

In \( M_1', \ldots, M_{i+1} - 1 \) we store the identification numbers of those routes whose overflow traffic is offered to route number 1. In \( L_1' \) is stored a pointer to array \( M \) concerning route number 1.

The arrays \( M \) and \( L \) for the routing arrangement in fig 1.

\[ L = (1, 1, 1, 1, 1, 1, 4, 7, 9); \]
\[ M = (1, 2, 3, 4, 5, 6, 7, 8); \]

For each route \( i \) the following parameters will be of interest.

\[ A_i = \text{Direct offered traffic, mean value} \]
\[ W_i = \text{Direct offered traffic, variance} \]
\[ D_i = \text{Offered, overflow traffic from other routes, mean value} \]
\[ U_i = \text{Offered overflow traffic from other routes, variance} \]
\[ C_i = \text{Measured carried traffic on the route} \]
\[ N_i = \text{Number of circuits on the route} \]
\[ B_i = \text{Overflow traffic from the route, mean value} \]
\[ V_i = \text{Overflow traffic from the route, variance} \]

Fig 2 Parameters of interest for each route

\[ A_i, W_i = C_i, N_i \]
\[ D_i, U_i = B_i, V_i \]

For each route \( i \), we can calculate as the sum of the overflow traffics from other routes, in lower levels of the routing arrangement, that will be offered to route \( i \).

\[ b = L_i'; c = L_{i+1} - 1; \]

If \( c \geq b \):

\[ D_i' = \sum_{j=b}^{c} B_{M_j}; \]
\[ U_i' = \sum_{j=b}^{c} V_{M_j}; \]

If \( c < b \): \( D_i' = U_i' = 0; \)

We can now write the total offered traffic \((S_i, T_i)\) to the route \( i \) as

\[ S_i = A_i + D_i \]  \hspace{1cm} (3) \]
\[ T_i = W_i + U_i \]  \hspace{1cm} (4) \]

where \( S_i \) is the mean value and \( T_i \) the variance.

We can always find a number of full available circuits \((a_i)\) with a direct offered traffic \((R_i)\) that will give us the blocked traffic \((S_i, T_i)\) with the following equations.

\[ S_i = R_i \cdot E_{X_i}(R_i) \]  \hspace{1cm} (5) \]
\[ T_i = S_i \cdot (1 - S_i + R_i/(X_i+1 + S_i - R_i)) \]  \hspace{1cm} (6) \]

Eq (5) is the erlang loss formula and eq (6) is the Wilkinson formula for the variance of the blocked traffic.

If \( S_i < T_i \) we will get \( X_i > 0 \) and \( R_i > S_i \)

If \( S_i = T_i \) we will get \( X_i = 0 \) and \( R_i = S_i \)

If \( S_i > T_i \) we will get \( X_i < 0 \) and \( R_i < S_i \). See [1]

To solve eq (5) and (6) for all positive values of \( S_i \) and \( T_i \) we need a method to solve the erlang loss formula for all values of \( X_i \) \((0 < X_i < \infty)\). Such a method is presented in [2].

Fig 3

\[ X_i = X_i; \]
\[ R_i, R_i > 0; \ldots; S_i, T_i > 0; \ldots; B_i, V_i \]

To make the calculations more general we introduce the quotient

\[ Q_i = W_i/A_i; \]  \hspace{1cm} (7) \]

for all direct offered traffics. This makes it possible to have different kind of direct offered traffic to the routes. Normally we use the value \( Q_i = 1 \).

We can now write the following system of equations for each route in the routing arrangement.

\[ S_i = A_i + D_i \]  \hspace{1cm} (3) \]
\[ T_i = Q_i \cdot A_i + U_i \]  \hspace{1cm} (8) \]
\[ D_i = \sum_{j=b}^{c} B_{M_j}; \]  \hspace{1cm} (1) \]
\[ U_i = \sum_{j=b}^{c} V_{M_j}; \]  \hspace{1cm} (2) \]
\[ S_i = R_i \cdot E_{X_i}(R_i); \]  \hspace{1cm} (5) \]
\[ T_i = S_i \cdot (1 - S_i + R_i/(X_i+1 + S_i - R_i)); \]  \hspace{1cm} (6) \]
\[ B_i = R_i \cdot E_{X_i}(N_i)(R_i); \]  \hspace{1cm} (9) \]
\[ V_i = B_i \cdot (1 - B_i + R_i / (X_i + N_i + 1 + B_i - R_i)); \quad (10) \]

\[ S_i = C_i + B_i; \quad (11) \]

Eq (9) and (10) are the erlang loss formula and the Wilkinson formula on the route \( X_i + N_i \) (fig 3).

Eq (11) is deduced from the fact that offered traffic is equal to the sum of carried and blocked traffic. From this system of equations we can solve \( A_i, B_i \) and \( V_i \) if we know \( C_i, N_i, Q_i, M, L, D_i \) and \( U_i \).

\( S_i, T_i, R_i \) and \( X_i \) are only used to simplify the solution.

For routes in the lowest level we know that \( D_i = 0 \) and \( U_i = 0 \) because these routes will not be offered any blocked traffic from other routes.

For the set of equations (1, 2, 3, 5, 6, 8, 9, 10 and 11) we introduce the notation \( (A_i, B_i, V_i) = f(C_i, N_i, Q_i, (B M, V M, j=L_i, ..., L_i+1-1)); \) (12) for each route.

If we solve this set of equations route by route starting with routes in the lowest level we will always know the value of \( D_i \) and \( U_i \) for route \( i \) when we need it. The solution of the set of equations must be done by iterations (see section 3).

These calculations will give us values of all \( A_i, i=1, \ldots, m \). Mostly some routes have no direct offered traffic i.e. \( A_i=0 \). Our solution will probably give us \( A_i \neq 0 \) even for these routes.

The value of these differences show us how good the calculated direct offered traffics \( (A_i) \) satisfy the routing arrangement. Mostly the solution is not good enough due to several reasons e.g.

1. There are some faults in the measured carried traffic per route.
2. Some circuits have been out of order or continuously blocked.
3. The direct offered traffics have another variance than expected \( Q_i \neq 1 \).
4. Some routes are gradings and have not full availability.

We can now use the calculated direct offered traffics \( (A_i) \) as starting values in a minimization by iteration of the sum of the weight relative differences between calculated and measured carried traffic per route.

If we suspect case 1 or 2 we can minimize the differences by a method described in section 2.1. For the most suspect routes we can decrease the weight factor (see section 2.1) and we will get the largest correction on these routes.

If we suspect case 3 we can calculate new better fitting values of some \( Q_i \) parameters (see section 2.2).

For case 4 a possibility could be to replace the gradings with an equivalent full availability group.

2.1 SOLUTION WITH ALL Q-PARAMETERS KNOWN

If we assume that because of measurement errors we get \( A_i \neq 0 \) for some routes with no direct offered traffic, we can proceed as follows.

Replace the measured carried traffics \( C_i, i=1, \ldots, m \) in (12) by the unknown variables \( Y_i, i=1, \ldots, m \). In routes with no direct offered traffic we put \( A_i = 0 \). We can now denote these new systems of equations for each route as

\[ (Y_i, B_i, V_i) = g_1(A_i, N_i, Q_i, (B M, V M, j=L_i, ..., L_i+1-1)); \]

for routes with direct offered traffic and

\[ (Y_i, B_i, V_i) = g_2(N_i, Q_i, (B M, V M, j=L_i, ..., L_i+1-1)); \]

for routes without direct offered traffic.

We assume that \( n \) routes shall have direct offered traffic (\( n < m \)).

We now want to find the values of \( A_k, p=1, \ldots, n \) that minimize the sum of the weighted relative differences between calculated and measured carried traffic per route.

In \( K_p \) we store the indices of these \( n \) out of \( m \) routes which have direct offered traffic.

\[ h(A_k, p=1, \ldots, n) = \sum_{i=1}^{m} \left( Z_i, (Y_i - C_i) / C_i \right)^2; \]

where \( m > n \) and \( Z_i, i=1, \ldots, m \) are the weight factors. By changing the values of these factors we can show how much we trust the different measured carried traffics. The minimization is described in section 4.

2.2 SOLUTION WITH SOME Q-PARAMETERS UNKNOWN

If we assume that because of other variance than expected of the direct offered traffics (wrong values of the Q-parameters) we get \( A_i \neq 0 \) for some routes with no direct offered traffic, we can do as follows. Assume we have \( m \) routes but only \( n \) routes with direct offered traffic (\( m > n \)). We can now let some, say \( m_0 > n \), Q-parameters be unknown variables. We will get almost the same expression eq (13-15) as in section 2.1 but with the addition that also some \( Q_i \) parameters will be unknown variables. In the same way as in section 2.1 we now want to find the values of \( A_k, p=1, \ldots, n \) and \( Q_K, p=n+1, \ldots, m \) that minimize the sum of the weighted relative differences between measured and carried traffic per route.

In this case \( K_p \) also points out the number of the routes that have unknown Q-parameters.
To be able to solve this problem the $Q$-variables to be unknown must be selected with some care.

3. THE INITIAL SOLUTION

Before the minimization of (15) or (16) we need values for the direct offered traffic per route $A_{K,p}$, $p=1, \ldots, n$, to be used as an initial solution. These starting values we can get by solving (12).

Each of these functions is a system of equations (see section 2).

The calculation is done by iteration. (11) will give

$$G(A_i) = S_i - C_i - B_i;$$

This equation is solved by an iteration method described by Krautstengels [3].

In the evaluation of $G$ we will obtain $S_i$ and $T_i$ from eq (1, 2, 3, and 8). In order to calculate $B_i$ we use eq (5) and (6) to get

$$F(R_i, X_i) = R_i \cdot E(X_i) - S_i;$$

$$X_i = R_i / S_i - 1;$$

$$t = 1 - 1/(S_i \cdot T_i / S_i);$$

If $S_i \cdot T_i / S_i > 1$ we can calculate $R_i$ and $X_i$ by Newton-Raphson iteration.

As starting value of $R_i$ we choose $[4]$.

$$R_i = T_i + S_i \cdot T_i / S_i - 1;$$

The application of the Newton-Raphson formula

$$R_{i+1} = R_i \frac{F(R_i, X_i)}{F'(R_i, X_i)}$$

will require the calculation of $dE_r / dx$ for both negative and positive values of $x$. [2]

After calculating $A_i$ for all routes we also have the blocked traffic $(B_i, V_i)$ for all routes. For routes with no direct offered traffic we set $A_i = 0$. If the calculated absolute values of $A_i$ for these routes are large we adjust the remaining $A_i$ values. If we put

$$A_s = \sum_{i=1}^{m} A_i; \quad C_s = \sum_{i=1}^{m} C_i;$$

the final form for the starting values will be

$$A_i^* (C_s + B_s) / A_s;$$

where $B_s$ is the sum of the final blocked traffics in the highest level. $B_s$ is easily calculated as we know all $B_i$.

These values will now form the starting values for the minimization process (section 4).

4. METHOD FOR MINIMIZATION

The functions (14) and (16) in section 2.1 and 2.2 are non-linear and for the minimization we use "The damped Taylor's series method for minimizing a sum of squares and solving systems of non-linear equations" [5].

$$H(A_{K,p}, p=1, \ldots, n) = h(A_{K,p}, p=1, \ldots, n);$$

$$h(A_{K,p}, p=1, \ldots, n) = \sum_{i=1}^{m} \left( Z_i \cdot (Y_i - C_i) / C_i \right)^2;$$

$Z_i$ is a weight factor which indicates the estimated reliability of the measured carried traffic $C_i$. $Y_i$ is the calculated carried traffic on route $i$, $A_{K,p}$ is the transformed value of $A_{K,p}$.

$$a_{K,p} = \log(A_{K,p});$$

This transformation is done because the used method for minimization demands the unknown variables to be unrestricted but we have $A_{K,p} > 0$.

We also need the function for the calculation of carried traffic per route $Y_i$.

For routes with direct offered traffic;

$$(Y_i, B_i, V_i) = g_1(A_i, N_i, Q_i, \beta_n, V_n, j>1);$$

For routes without direct offered traffic

$$(Y_i, B_i, V_i) = g_2(N_i, Q_i, \beta_n, V_n, j>1);$$

The functions $g_1$ and $g_2$ we get from the system of equations described in section 2.

$$S_i = A_i + D_i$$

$$T_i = Q_i \cdot A_i + U_i$$
This system of equations can be solved in the same way as we solved the system of equations in section 2. We need to use iterations only to solve $R_i$ and $X_i$ out of eq (5) and (6) (see section 3), the rest is simple calculation.

We can now minimize the function $H(a_i,...,n)$ with the method described in [5].

If the measured carried traffics on the routes correspond badly with the erlang distribution the minimizing method will concentrate the changing to routes with high traffic per circuit. The reliability of the measured traffics probably stand in some proportion to the size of the traffic on the route. For this purpose we will use

$$Z_i = \sqrt{C_i + B_i},$$

as a normal weight factor for the routes, where $(B_i)$ is the blocked traffic we get when we calculate the startvalues $A_i$.

5. THE COMPUTER PROGRAM

A computer program is written in the FORTRAN-language. This program executes all calculations. The following information for each routing arrangement is stored on tape as input to the program.

1. The number of routes and levels in the routing arrangement.
2. The structure of the routing arrangement.
3. The number of circuits per route.
4. The measured carried traffic per route.
5. The weight factor per route.
6. Quotient variance/mean value for the direct offered traffic per route.

This information is stored in such a way that they are easy to change or correct year after year.

6. SOME EXAMPLES

The program has been tested with data from both simulations and traffic measurements. Below we show one example of these comparisons. One existing routing arrangement (fig 1) has been measured during the busy hour for five working days. With these measured traffics as input the program have calculated the initial direct offered traffics shown in fig 4. After minimizing the sum of the relative differences between measured and calculated traffic per route with the method described in section 2.1 (no unknown Q-parameters) we get a better result in fig 5. We can also minimize with some Q-parameters unknown (see section 2.2) and get a better result in fig 6. Which result we shall choose must be based on judgement.

In this example we have chosen the weight factors as shown in eq (26). If we suspect some particular measured traffic to be very uncertain we can decrease its weight factor and the program will allow a larger relative difference between calculated and measured carried traffic for this route.

<table>
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<th>Direct offered traffic</th>
<th>Carried traffic</th>
<th>Square of relative difference</th>
<th>Number of circuits</th>
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Total 61.16 61.10 61.1 0.0698 90

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REFERENCES


[2] Oppelstrup, J.: Calculation of the Erlang formula, $E_x(a)$ and its derivative $dE_x(a)/dx$ for $a>0$, $-\infty<x<\infty$. Institute of Applied Mathematics, report no. 36 (Stockholm 1976).

