Discussion Record

SESSION NO. 51 - NETWORK AND SYSTEM OPTIMISATION

Chairman - V.I. NEIMAN (U.S.S.R.)
Discussion Leader - M. ANDERBERG (Sweden)

Paper No. 511
Author: B. TSANKOV (Bulgaria)

C.W.A. JESSOP (Australia): Optimisation of a network in terms of standard transmission modules (eg 12 channel groups) is less useful if other uses than telephony are involved, eg telegraphs. How often, in your experience, do routes occur where only telephony is involved, to which your method of optimisation can be applied?

B. TSANKOV: If there are other channels in the multiplexing plan (eg telegraph channels or channels of private networks) the method considered is useful, but the considerations in Appendix [7] are not applicable. For example if there are 3 telephone channels multiplexed for telegraph transmission in a route, the telephone route group sizes, in the case of \( z = 12 \), have to be 9, 21, 33 ... instead of 12, 24, 36 ... .

J.P. FARR (Australia): With modular engineering, do you agree that the larger the module size the greater is the need to use both way rather than one way trunks? This is of particular importance in the case of PST-PCM systems where you suggest a module size of 30.

B. TSANKOV: Yes.

J.P. FARR: What is the variable on the vertical axis of figure 6 a ?

B. TSANKOV: \( D = \sigma^2 - R \).

Paper No. 512
Author: R.J. HARRIS (Australia)

C.W.A. JESSOP: The paper states that the cost of a final choice is less than expected in practice. Please explain how this circumstance eventuates?

R.J. HARRIS: In the first data set it has been found that some relay set costs have been omitted in some instances and also, allowance was not always made for the heavier gage cable required for final choice routes.

C.W.A. JESSOP: What method do you propose for manual adjustment of the optimised network, should this be necessary due to changes in traffic levels or network configurations? In particular ensuring that OD grades of service are maintained.

R.J. HARRIS: a. For changes in traffic levels I would suggest that the chain flows which are affected could be increased (or decreased) in proportion to the changes in these levels. Iterations could then be recommenced from this new "near optimal" point and terminated when a suitable accuracy had been obtained for the new minimum.

b. Several different types of changes in network structure are possible. Firstly, if the routing strategies for particular OD pairs are to be altered I would suggest that the old chain flows could be used as a starting solution in the new network arrangement (where possible).

Once again, iterations would need to be performed in order to re-optimize the network. Secondly a new exchange (or group of exchanges) could be added or deleted. In such a case, the extent to which this change affects the network would be a prime factor in determining the best way to proceed. If changes were expensive it may be more practical to restart the entire optimizing process from the beginning. For minor alterations it should generally be possible to retain old values of the chain flows and simply re-assign these values to the new routes and recommence iterations until a sufficiently accurate solution has been obtained.

C.W.A. JESSOP: In Section 4 paragraph 3, you state that user optimised solutions fail to recognize the advantages of bulking traffic on alternate routes. However this method rejects first alternate routes in favour of bulking traffic on the final routes. Could you explain this apparent contradiction please?

R.J. HARRIS: Both solutions recognize the advantages of bulking traffic in the alternate routes and the differences are largely differences in degree, ie. In the system optimised network, extensive use is made of the alternate routing section of the network; while in the user solution the majority of the traffic remains on the first choice routes and the remaining traffic flows onto the route with the most attractive cost per unit Erlang - which would appear to be the third choice chain in these particular instances.

Y.M. CHIN (Australia): How does the Chain Flow method compare with the traditional method (ie Pratt et al) in terms of computing time?

R.J. HARRIS: [Adelaide Network] At each iteration step of the system optimization process it is necessary to dimension the network from the current chain flow pattern, this process takes 0.2 seconds CPU time on a Cyber 74 computer. For the same computer, one dimensioning step in the conventional approach can take from 15 to 30 seconds to perform depending on the methods used to determine variances and equivalent random traffic parameters.

M. ANDERBERG: You state in Section 3.1 of your paper that the iterative process took approximately 15 minutes of CPU time, and then you arrived at a solution which was 1 - 2% from the true System optimal solution.

1. How is this true solution defined?
2. How long will it take to find the true solution for the studied network?


2. In general, a non linear programming algorithm will not necessarily converge to the exact minimum in a finite number of iterations although at each iteration on the new chain flow pattern will define a network of lower cost. It is possible to compute a lower bound on the value of the minimum cost from duality theory of non linear programming. In practice, it is necessary to terminate computations once the difference between the current cost and the estimate for the minimum is less than a prescribed figure.
W. LORCHER (Germany) : In your paper you compare 2 optimizing principles: System and user optimization. You compare these 2 principles by means of a real network and you obtain differences of about 15% with respect to the cost.

My question is: Have you compared these results also with results obtained by conventional methods?

R.J. HARRIS : I have compared my results with the conventional methods using data set 2 but the comparisons should be treated with caution for 3 reasons:
1. The 2 models have different grade of service criteria and it is difficult to assess the O-D grades of service from the conventional model.
2. The conventional methods minimize OD costs in a "competitive" manner which is related to the concept of Game Theoretic Optimization discussed in Ref. 1.

The results of this comparison show that the cost of a conventional network falls about midway between the System and the conventionally optimized networks. In particular, the numbers of direct route circuits were roughly comparable in the 2 networks, and hence the direct route costs are comparable but the cost of the alternate routing networks differed by about 8 - 10% and circuits were assigned to links in different ways. In some particular cases for Berry's model third choice routes had more circuits than the conventional approach - for reasons cited in Section 4.

A.J. FREEMAN (Australia) : In Australian metropolitan networks the use of routes X-B has generally been avoided because it makes optimisation using cost factors unstable. As your results show that A-Y routes are often not used in allowing X-B routes. What do you expect in terms of increased computer time and possible non uniqueness of solutions if this routing is permitted in your optimisation procedure?

R.J. HARRIS :
1. It is possible that X-B routes may be advantageous in this situation. The input data requires that the network routing pattern must be specified for the optimizing model. Thus we would need to rerun the System optimizing program to establish the new solution and determine whether such routes would be more attractive to flow than A-Y routes.
2. If we retain the requirement of 3 choices for traffic from a given OD pair computing times would not vary greatly from those which were obtained incorporating A-Y routes. If we permit all 4 possible routing strategies the computing times would probably increase by about 20% per iteration of the System optimizing algorithm.
3. One of the difficulties of a non linear programming formulation is that optimizing algorithms may not necessarily locate the global optimal solution if the objective function is non convex. As some of the link cost functions in the dimensioning model are not convex with respect to the chain flows it is not clear that the total network cost will necessarily be convex. Studies of the total network cost for networks with A-Y routing permitted have not indicated non convexity in the objective function when data set 1 was employed. We have not done any studies on the use of X-B routing and so we cannot comment upon the possible existence of multiple solutions due to non convexity of the objective function.

J.S. HARRINGTON (Australia) : I refer to Section 5 of the paper in which it is suggested that the network would stand up to overload conditions better under "User" optimization than it would under "System" optimization. I do not disagree with this but I suggest a more economic solution would be to build up the final routes, links 3, 4 and 5, of figure 1, of the "System" optimized solution. This might result in unequal but improved overall grade of service for most traffic parcels. This would not be undesirable because I suggest there is no real merit in designing for equal end to end grade of service for all traffic parcels as the customer is more likely to assess the service provided by the telephone network on the network's ability to handle, successfully, the first repeat attempt call after congestion was encountered, than be would be on knowing the network was designed for overall grades of service of 1 in 100 or 1 in 200, etc. Could you comment please?

R.J. HARRIS : I am afraid that I cannot agree with your statement concerning the "lack of merit" in using end to end grade of service for at least 2 reasons.
1. The conventional approach of specifying grades of service on backbone routes ensure that all OD pairs get service of better than (or equal to) that grade of service. In fact, if the majority of the traffic parcels are carried on the early choice routes, then there is almost no loss for such parcels. This must result in more circuits being provisioned overall than are necessary because of which dimensions for OD grades of service and thus I suggest there are economic advantages in applying the principle of end to end grade of service.
2. I also suggest that subscribers are unable to distinguish between grades of service of 0.01 or 0.005 [say] under normal conditions. Furthermore, Berry's model gives the planner the opportunity to specify end to end grade of service for particular OD pairs, thus it is possible to design a network where these OD pairs have different grades of service to meet particular requirements.

It is not clear from your comments how you propose to "build up" the backbone routes from the System optimised solution. Presumably this must be done in some regular and ordered way which avoids specifying circuit quantities which does not lead to large imbalances in OD grades of service - such as the present conventional scheme.

There are many possibilities which could be considered in order to reduce the sensitivity of the System optimized network to overload. One suggestion is to reformulate the constraints of the problem in such a way that overload is considered explicitly: eg A certain percentage increase in offered traffic load should not result in degradation in OD grade of service of more than a specified value, and then find a network optimal solution for this condition. An alternative suggestion involves the provision of service protection routes for those OD pairs which would normally be offered to the backbone routes only.

J.S. HARRINGTON : I refer to Section 4, paragraph 1 of your paper, and Section 4, paragraph 1 of your table 1.

R.J. HARRIS :
1. Are the costs given in table 1 for integer value of circuits?
2. Do the unrounded costs in Section 4, paragraph 1 refer to costs for non-integer circuit values?
3. What is the reason for the small cost difference, for data set 2, between the values given in Section 4, paragraph 1 and those given in table 6?

R.J. HARRIS :
1. The costs given in Table 1 refer to unrounded circuit values.
2. Yes, the unrounded costs in Section 4, paragraph 1 do refer to costs for non-integer circuit values.
3. A typographical error in Table 1 is responsible for the cost difference. Please note that the figures given in Table 6 are the correct figures.
J.S. HARRINGTON: Refer please to Section 4 paragraph 2. I would like to make a comment on the rejection of the second choice route - link 2 of figure 1, in preference to the third choice route - links 3 and 4 of figure 1. The probability of this occurring increases:

1. As the cost of the 2 paths tend towards one another,
2. If the own area tandem is an XY tandem as against an 'X' and a 'Y'. (ie) 1 inlet as against 2 plus a R/S,
3. As the efficiency of the final route increases (more offered traffic and full availability switching),
4. As the number of terminals served by a 'Y' tandem decreases, ie more 'Y' tandems.

R.J. HARRIS: In general I would agree with the various possibilities which you suggest, in fact all of them appear to be evident in the network which I have considered. Note however that 1. occurs as a consequence of possibility 2, in your list of suggestions.

J.G. VAN BOSSE: In section 2 d. of the paper I indicated that one of my starting assumptions is the assumption of "random selection". The results of my paper always depend on this type of selection; I have not been able to further generalize the results by dropping this assumption.

G. HARLAND (United Kingdom): Your paper assumes the same, though arbitrary, distribution on the links at a given stage of compared linking patterns. In practice, the distribution is affected by the linking pattern, as briefly discussed in my paper No. 246. I believe that, on links close to the centre of the channel graph, an improved linking pattern generally results in a smoother traffic distribution but that the converse may well be true on the first and final stage links. I doubt whether the effect would ever reverse the result of a comparison between 2 linking patterns but would appreciate your comments on this aspect.

J.G. VAN BOSSE: It is plausible that the re-arrangement of linking patterns also modifies the occupancy distribution on individual link groups. I doubt whether these modified distributions can be determined analytically. It would be interesting to study this problem further, say by simulation. However, I have no great hope that simulations of reasonable length will positively identify the superior graph in a group. I base this on the fact that the relative difference in congestion between the optimum graph and one of the near-optimum graphs is small.

K. TAKAGI (Japan): First, I would like to express my thanks to this wide generalization about the assumption of occupancy distribution. In my work, I prefer to simplify the calculation, taking into account the comprehensive treatment of switching networks. So, the blocking probability estimation of my method may have some defects in the accuracy. In this study, you have completely solved one of these defects. But, I think, also the dependence between stages must not be neglected to improve the accuracy. So, I would like to ask you, if the optimum graph holds good under such a more realized condition. (Of course, as your anticipation.)

J.G. VAN BOSSE: My paper intends only to generalize your results in one respect: removal of the Bernoulli-type distributions on some of the link groups. The ultimate generalization, is the removal of assumptions of "random selection" and "inter-stage independence" pose problems which so far have defied all my attempts at resolution.

Paper No. 514
Author: J.G. VAN BOSSE (U.S.A.)

J.G. KAPPEL (U.S.A.): The author is to be complimented on an interesting and informative paper. It is regrettable that there was no-one at the Congress to answer questions on this work.

My question is basically the same one I had on Reference 5, the author's 1974 paper. How can one be assured that an optimal link graph has been constructed, when the model does not take into account the order of path search within the network?

J.G. VAN BOSSE: In Section 2 d. of the paper I indicated that one of my starting assumptions is the assumption of "random selection". The results of my paper always depend on this type of selection; I have not been able to further generalize the results by dropping this assumption.

G. PARSONS (United Kingdom): Quite rightly the paper takes into account as a factor the selection of a tandem exchange. It is regrettable, however, that the paper should have omitted to include the cost of providing an additional tandem.

J.G. VAN BOSSE: It is plausible that the re-arrangement of linking patterns also modifies the occupancy distribution on individual link groups. I doubt whether these modified distributions can be determined analytically. It would be interesting to study this problem further, say by simulation. However, I have no great hope that simulations of reasonable length will positively identify the superior graph in a group. I base this on the fact that the relative difference in congestion between the optimum graph and one of the near-optimum graphs is small.

G.D. BOLAM (Australia): In your analysis you seem to ignore the cost of the tandem exchanges, for example you quote $100 for a direct circuit and $51 for an alternate circuit. The cost of an exchange building (and land) could contribute more than $1000 per alternate circuit: taking this into account would seriously alter your conclusions. The cost of switching equipment is additional. The cost of providing an additional tandem exchange in Melbourne would be quite prohibitive at the present time so an upper limit on alternative routes should appear in your calculations. Please comment.

L.T.M. BERRY: Please refer to the answer given to Mr R.B. Leigh's question. The above costs were arbitrarily chosen simply to illustrate that network cost is a function of the chain flow pattern. The actual costs used when the model was applied to the Adelaide telephone network were realistic costs - provided by the Traffic Engineering section of Telecom Australia. These costs included switching costs at tandems.

The model takes a fixed network topology, ie the routing pattern and tandem exchanges are fixed. We do not consider the provision of additional tandems.
R.S. KRISHNAN IYER (Australia): In your paper you have briefly described an approach to integer optimization. Another approach to integer (discrete) optimization is Dakin's Tree Search Algorithm (modified by Bandler and Chen) which begins by finding a continuous solution. If the solution is non integer, partitioning is introduced and corresponding constraints added. The process continues until an integer (discrete) solution within some limits is obtained. We have worked with this approach and have found that there is good convergence. Is it possible to apply such an approach to your model?

L.T.M. BERRY: No, for the following reasons. To apply either a tree search or branch and bound algorithm it would first be necessary to formulate the problem completely in terms of the numbers of circuits \( n_j \). However, no satisfactory model is known which gives the OD traffic congestions as a function of the vector of circuit numbers \( n \). Apart from this, there are the added difficulties that these constraints are non-linear and define a non-convex region of feasible solutions. The non-linear program is also very large.

The formulation considered has both continuous variables \( n_j \) related linearly to OD congestions and discrete variables \( n_j \). When the constraints involving \( n_j \) are incorporated in the objective function it is possible to solve very large minimum cost problems by either of the methods given in references [4] and [5].

J.P. FARR: In the results presented in your paper have you calculated the variance of traffic offered to the alternative routes before computing the circuits required on these routes? If not, the traffic congestion will not be identical for each origin - destination pair as you define. We have worked with this approach and limits is obtained. We have worked with this approach and have found that there is good convergence. Is it possible to apply such an approach to your model?


W. LORCHER: In your paper you give an example that an increase of 1 circuit on a link could result in an increase in probability of loss of 20% or more for certain OD-pairs. Please can you explain this effect?

L.T.M. BERRY:

![Origin Destination Diagram]

Traffic between (1) - (3) small (no direct link provided). The addition of 1 circuit on link T - (2) results in an increase in the carried traffic on link (1) - T destined for exchange (3) with a resultant decrease in the carried traffic on chain (1) - T - (3). This effect (viz. the increase in traffic congestion for traffic from exchange (1) to exchange (3) is just the contribution of an extra circuit on link T - (4). Similarly an increase on link (4) - T increases the chain flow between exchanges (4) and (3) but reduces the chain flow between exchanges (1) and (3).

Paper No. 515
Author: U. YECHIALI (Israel)

M. ANDERBERG: In section 3 you state that if the traffic is large enough it will be possible to say that the number of trunks are proportional to the offered traffic, and you use the proportional constant \( C \). In what regions of \( A \) would you find it possible to use the assumptions of the proportional constant \( C \)?

U. YECHIALI: There are 2 answers to this question. Firstly, the exact range of \( A \) for which one can use the same depends on the specific grading scheme that is being used. For example, in various sections of the Israeli network where O'دت's grading is used with availability of \( K = 20 \), \( A \) is approximately between 1.2 and 1.3 for \( A > 30 \) Erlangs and grade of service (i.e. loss probability) 0.02 \( \leq P \leq 0.05 \). Secondly, the use of linear relation between the offered traffic and the number of trunks allocated is not a necessary one. We may obtain exactly the same analytical results if the cost function \( C_2(\mu_i) = C_1(\mu_i/v_i) \) is a concave function. This is indeed the case, since for a specified grade of service, the number of trunks, \( N_{(v_i)} \), is a monotone increasing concave function of the load \( v_i \).

Paper No. 516
Authors: T. DONNS and R.S. KRISHNAN IYER (Australia)

M. ANDERBERG: Mr Iyer, in the presentation of your paper, you included some comments on the accuracy of the Rapp's approximation:

\[
A = V + 3M \left( \frac{1}{2} - 1 \right).
\]

(I should perhaps point out that this is the only approximation formula, the value of \( n \) is found exactly, when knowing \( A \).) Since it is an approximation and is a such very useful for a starting point in the iterative process of finding \( A \) and \( n \), I do not understand your comments on its inaccuracy.

R.S. KRISHNAN IYER: In answer to your question, we would like to make the following points.

We are quite aware of the procedures under which Rapp's approximation is usually used. If equivalent random theory (ERT) is used in the normal fashion, Rapp's approximation is used to determine approximate values of \( A \) and \( n \). These values are then adjusted to the correct values by means of an error minimization procedure. The correct values of \( A \) and \( n \) can then be used in a relation of the form

\[
M_n + 1 (A) = A_n M_n (A)
\]

\[
n + 1 = M_n (A)
\]

The above form is used C times, where \( C \) is the number of circuits from which one wishes to determine the overflow. This method is well-established, is widely used, and is well-known to give satisfactory results. It is another procedure which we seek to question, although, before proceeding to our main argument we would like to draw attention to one point of interest.

In the presentation of the paper, we showed that according to the ERT the variance of the overflow traffic is extremely sensitive to the number of trunks, \( n \). Indeed, as was demonstrated in the presentation, a change of less than 0.04% in \( n \) can lead to a change of 100% in \( V \). In normal practice this phenomenon does not cause problems, since small changes in \( n \) are accompanied by corresponding adjustments in \( M \). However, we feel that it is rather odd that the mathematical model should exhibit such extreme sensitivity, as measured by the first partial derivative.

We would now like to turn our attention to the application of ERT in optimization.

In the optimization process, one is attempting to determine the optimum number of trunks.

As we pointed out in the presentation, we may write:

\[
V = (1 - M + \frac{A}{1 + P})
\]

where \( D = M + n - A \).
Since \( M + n + 1 = A \) (as can be seen from Rapp's results), and since, in addition, the difference between \( M \) and \( \left[ 1 + \frac{1}{n} \right] \) is very small, \( V \) is evaluated by forming the difference between 2 numbers which are close together. Thus \( V \) is very sensitive to \( D \).

If this result is applied in a continuous optimization procedure, continuous values are obtained for the numbers of circuits. If these values are rounded off, the values of variance, as calculated by ERT, are likely to deviate quite considerably from the optimum on each link.

For example, assume one is trying to optimize two link variables \( X \) and \( Y \). Suppose, for the purposes of our argument, that the optimum numbers of trunks are 25.6 and 16.7. If these values are to be rounded off, one cannot be sure how the variance is going to vary from its value at the optimum.

It has been suggested that an integer optimization procedure be employed. It is not clear to us how such a procedure will overcome the sensitivity problem. For instance, if the integer solution obtained for the above example were 24 and 25, then, once again, there is no indication of how the variance has deviated from the optimum.

Since ERT is a method which takes the second moment into account, we feel that procedures which use ERT and which do not take proper account of the second moment require further investigation.

M. ANDERBERG: In your paper you introduce a system of state equations. Considering the amount of states that will occur, it would be of great interest to hear of the preliminary results, envisaged in your paper, of the programming of your formulae and the use on some networks.

T. DONNS: Unfortunately, our answer is not a very satisfactory one since we do not have any preliminary results. However, there are 2 major reasons why this is so. Firstly, as Mr Krishnan Iyer mentioned, we recently found that our model could be modified in order to eliminate special assumptions on traffic arrivals at tandem exchanges. Secondly, our interest has been recently directed toward the problem of formulation of switching strategies, which, in the simple form outlined by Mr Krishnan Iyer, we feel may constitute a more attractive starting point for the implementation of our model. We agree that there is a large number of network states but for the formulation of switching strategies it seems likely that only a small number of state probabilities need be considered. In addition, a close examination of the rather forbidding looking explicit solution reveals that a large amount of computation is common to a large number of states and the savings in computational effort which can be achieved by taking advantage of this fact remain to be seen.