The subject of the paper is the probability of blocking in non-hierarchical networks and related design and network management aspects. The examination is based on a method of calculating the probability of blocking in a non-hierarchical network; it is assumed that (a) trunk groups between switching centres in the network are unavailable with some probability, \( p \), and (b) the probability of a switching centre being unavailable is small and can be ignored. It is shown that the probability of blocking between any two switching centres can be expressed as a polynomial in \( p \), where \( p \) is the probability of a trunk group (link) being unavailable. The terms in this polynomial depend on the type of network, the relative location of the two nodes, and the number of route-classes allowed (the shortest routes comprise one route-class between those two nodes, the next-shortest routes comprise another route-class, and so on). The method is applied to a grid network and practical values for the probability of blocking between nodes are obtained. These are discussed in detail and on the basis of this discussion, practical considerations relating to the design and management of non-hierarchical networks are given.

1. **INTRODUCTION**

This paper examines the probability of blocking in a non-hierarchical network and relates the results to practical design considerations. The probability of blocking is assumed to depend on the traffic volume and the state of availability of each individual network element, viz., the nodes and the links. In practical terms, a node is a switching centre and a link is a circuit or a group of circuits interconnecting two switching centres. A link can fail due to traffic congestion, to circuit failure, or a combination of both these conditions. The probability that a link is unavailable is denoted by \( p \) and for reasons of simplicity it is assumed that \( p \) has the same value in all links and is independent of the availability state of any other link in the network. In contrast to links, nodes are considered to be elements that are completely reliable and cannot cause blocking. A route between two given nodes is defined as a sequence of links between those two nodes such that each intermediate node is common to only two of the links in the sequence. There may be several routes possible between the two given nodes, and blocking is considered to exist when no route is available between them as a consequence of traffic congestion or circuit failure. The situation in which a route exists between two nodes but the routing procedure employed by the network fails to discover it is not regarded as blocking but as routing inefficiency and is not discussed in this paper. Ref. 1 describes the development of two approaches to the determination of the probability of blocking between two nodes in a non-hierarchical network. This present paper briefly describes one of these approaches, the combinatorial approach, and derives numerical values for use in practical situations. The description of the approach is given in Appendix A and the calculations of the numerical values are given as Appendix B. The main text uses these values as a basis for discussing the practical considerations relating to the design and management of a non-hierarchical network.

2. **PROBABILITY OF BLOCKING**

In the sum of columns method (combinatorial approach), which is described in Appendix A, the probability of blocking, \( P \), between two nodes in a non-hierarchical network is expressed by a power series of the form

\[
P = k_1 p + k_2 p^2 + k_3 p^3 + k_4 p^4 + k_5 p^5 + \ldots + k_j p^j + \ldots (1)
\]

where, \( p \), the probability of a link being unavailable, is the same for all links; also, for small values of \( p \), the numerical values of successive terms decrease. The value of \( k \) is given by the expression

\[
k_j = \sum_{i=0}^{j} (-1)^{j-i} \binom{N-1}{j-i} C_i
\]

where \( N \) is the total number of links and \( C \) is the number of blocking possibilities, i.e., the number of different ways in which \( i \) links out of \( N \) can be unavailable and cause blocking between the two nodes under consideration. The probability that one particular link is unavailable is assumed to be independent of any other link being unavailable.

3. **ROUTE-CLASS**

A non-hierarchical network is characterized by having several outgoing link choices per switching centre and a traffic-routing procedure that permits each call to take one of several routes between its point of origin and its destination. The networks are non-hierarchical in the sense that all traffic except local traffic can, in principle, use any outgoing link from a switching centre; furthermore, no switching centre is superior to any other switching centre, and there is no key centre in any particular part of the network.

In a non-hierarchical network there is generally a very large number of routes between any two nodes \( O \) and \( D \). These routes can be classified \( A_1(O,D), A_2(O,D), A_3(O,D), \ldots, A_N(O,D) \), according to the number of links involved in the route. Class \( A_1(O,D) \) comprises all shortest routes between \( O \) and \( D \), where a shortest route is one that involves the minimum number of links in tandem between those two nodes; class \( A_2(O,D) \) comprises all next-shortest routes; class \( A_3(O,D) \ldots \) etc.

The number of permissible route-classes is a network design and management consideration. Generally, if \( p \) is small, the probability of blocking can be expected to decrease by allowing routes of many classes. Thus there are advantages in allowing many route-classes. However, since there is a delay and also a cost associated with each node in a route, it is desirable to keep routes as short as possible, i.e., to limit the number of route-classes allowed. Thus there are conflicting requirements, the impact of which are discussed in the following in relation to the grid-type network shown in Fig. 1.
4. THE GRID NETWORK

4.1 Sub-network associated with two given nodes and given route-classes

Fig. 2 shows a part of the grid network and the two nodes \(0(0,0)\) and \(D(x_d,y_d)\). The cartesian coordinate system is used to designate individual elements of the network so that each node is described by a discrete set of coordinates \((x,y)\) and each link is identified by the coordinates of the nodes which it interconnects, e.g. \((x_1,y_1)\) to \((x_2,y_2)\). Each link is regarded as being of unit length.

The part of the grid network that comprises all nodes and links which may be involved in any route of classes \(A_0\) and \(D(x_d,y_d)\) is regarded as being of unit length.

The grid network

![grid network diagram](image)

Fig. 2 The part of the grid network that comprises all nodes and links which may be involved in any route of classes \(A_0\) to \(A_n\) between nodes \(0(0,0)\) and \(D(x_d,y_d)\)

4.2 Probability of blocking between given nodes

The method which is described in Appendix A to this paper may be used to determine the probability of blocking between given nodes in the grid network. The two nodes are designated \(0(0,0)\) and \(D(x,y)\) and the probability of blocking between \(0(0,0)\) and \(D(x,y)\) is denoted by \(P(x,y,n)\), where \(x\) and \(y\) are the coordinates of node \(D\) and \(n\) indicates that routes of class \(A_0\) to \(A_n\) are allowed (see Fig. 2). The resulting expression for \(P(x,y,n)\) in Appendix B is used in Section 5 to derive specific conclusions.

5. CONCLUSIONS

Generally, the probability of blocking depends on the probability of a link being unavailable (all links having the same value of \(p\)) and on the number of route-classes allowed. If the probability of a link being unavailable is low, the probability of blocking between the two nodes is low and \(P(x,y,n) < p\). If the probability of blocking increases, and when \(p\) reaches a certain critical value, \(P(x,y,n)\) begins to exceed \(p\). This critical value of \(p\) depends on the values of \(x\), \(y\), and \(n\). The most critical situation is when \(x = 0\) and only route-class \(A_0\) is allowed \((n=0)\); then \(P(x,y,0) > p\) for all values of \(p\). By allowing also route-class \(A_1\), the critical value lies around \(p = 0.1\). For \(x \neq 0\) and \(y \neq 0\), the critical values are examined. By reason of symmetry, the same results apply with \(x\) and \(y\) interchanged. Large values of \(x\) and \(y\) are not examined, but Ref. (1) gives reason to believe that the results would be similar to those obtained for \(x + y \leq 6\).

The results suggest that network dimensioning should be done in such a way that sufficient circuits are provided in each link to ensure that the probability of link unavailability does not normally exceed the value of \(p = 0.1\).

The probability of blocking between two nodes is relatively high when only routes of class \(A_0\) (shortest routes) are available. In this case the leading term in the expression for the probability of blocking is:

\[
k_p^2 \text{ for } x = 0 \text{ or } y = 0 \text{ or } x + y = 6 \tag{B.1}
\]

and

\[
k_p \text{ for } x = 1 \text{ or } y = 1, \tag{B.17}
\]

(see expressions B.1, B.7, B.13 and B.17 in Appendix B)

\[
k_p^2 \text{ for } x = 1 \text{ or } y = 1, \tag{B.10}
\]

(see expressions B.10 and B.19 in Appendix B)

\[
2k_p^2 \text{ for } x > 1 \text{ and } y > 1. \tag{B.30}
\]

(see expressions B.30 and B.31 in Appendix B)

The probability of blocking is reduced by also allowing routes of class \(A_1\) (next-shortest routes) whereupon the leading term becomes \(k_p^2\) for \(x = 0 \text{ or } y = 0 \text{ or } x + y = 6\) and \(k_p\) for \(x = 1 \text{ or } y = 1\). Further reduction is obtained by also allowing routes of class \(A_2\), and in this case the leading term is always \(2k_p^2\) (the expression in this term is equal to the number of links per node, the network being regular; see Ref. 1), for all
values of \( p \) and \( n \). If routes of yet higher classes are allowed, the leading term in the expression for the probability of blocking is always \( p^2 n \) and the reduction in the probability of blocking is associated with the higher-order terms. The improvement is therefore small and of little practical interest. If routes of classes \( A_1 \) and \( A_2 \) are allowed, blocking between the two nodes \( 0(0,0) \) and \( D(x,y) \) is caused mainly by the outgoing links from \( 0(0,0) \) and the incoming links to \( D(x,y) \) being unavailable. This means that under these conditions the probability of blocking is nearly the same between all pairs of nodes, whatever their separation may be. It would be good engineering practice to allow routes of higher classes only in those situations where the leading term in the expression for the probability of blocking would be substantially reduced.

In the discussion up to now it has been assumed that the value of \( p \) remains constant and independent of the number of route-classes allowed. This would not be the case in a practical network.

The minimum number of circuits required to carry the traffic between two given nodes is defined as the number of links in the shortest route between those two nodes. If longer routes are used more circuits in tandem are involved in carrying the same traffic. Now, the more circuits that are used to carry a given amount of traffic, the higher is the traffic load on each individual link and the higher is the probability of a link being blocked with traffic. Therefore allowing routes of classes higher than \( A_0 \) causes the traffic on the links to increase and hence the probability of link unavailability to increase. If the probability of link unavailability increases, the probability of blocking between two given nodes increases. Thus the aim of reducing the probability of blocking by allowing more classes of routes might be defeated in a practical network by the resulting increase in the probability of link unavailability. This follows therefore that for any given pair of nodes \( 0(0,0) \) and \( D(x,y) \), the probability of blocking \( P(x,y,n) \) can be expected to exhibit minimum values for particular combinations of \( p \) and \( n \). This becomes an optimization problem which represents an extension of the determination of the probability of blocking between nodes in a non-hierarchical network and is not covered in this paper. However, the results in this paper suggest that the minimum value of \( P(x,y,n) \) for \( p \) close to unity, is obtained for \( n = 0 \) (or 1), and for small values of \( p \) the minimum value of \( P(x,y,n) \) would be obtained for somewhat larger values of \( n \).

### APPENDIX A

#### A1 INTRODUCTION

In this appendix an analytical expression is derived for the probability of blocking between two given nodes (switching centres) in a non-hierarchical network. For this purpose nodes are assumed to be perfectly reliable network elements which cannot cause blocking whereas links can be unavailable with some probability \( p \) where \( 0 < p < 1 \) and is independent of the availability of every other link in the network.

Blocking occurs between the two nodes \( 0 \) and \( D \) when all routes in the allowed route-classes \( A_0 \) (shortest routes) to \( A_2 \) are blocked. The probability of this occurring is denoted by \( P(0,D,n) \) and depends on the availability of the links in the sub-network comprising all nodes and links which may be involved in any route of class \( A_0 \) to \( A_2 \) between \( 0 \) and \( D \).

#### A2 COMBINATORIAL APPROACH

In the combinatorial approach to the problem of determining the probability of blocking in a non-hierarchical network, a link is a group of circuits interconnecting two nodes. A link can be unavailable due to traffic congestion, to circuit failure, or of course to a combination of both of these conditions.
\[ P_i(O,D,n) = C_i(O,D,n) p^i \sum_{k=0}^{N-i} (-1)^k \binom{N-i}{k} p^k. \]

Substituting \( j - i \) for \( k \) results in
\[ P_i(O,D,n) = C_i(O,D,n) \sum_{j=1}^{N-i} (-1)^{j-i} \binom{N-i}{j-i} p^j \]
(A.3)

Thus \( P_i(O,D,n) \) can be expressed as the polynomial in \( p \)
\[
\begin{align*}
&= a_{i0} + a_{i1} p + a_{i2} p^2 + a_{i3} p^3 + \ldots \\
&\quad + a_{iN} p^N.
\end{align*}
\]
(A.4)

where
\[ a_{ij} = 0, \quad \text{for } j < i \]
\[ a_{ij} = (-1)^{j-i} \binom{N-i}{j-i} C_i(O,D,n), \quad \text{for } j \geq i. \]

**A4 SUM OF COLUMNS METHOD**

The \( P_i(O,D,n) \) terms may be expressed in matrix form as
\[
\begin{bmatrix}
P_0(O,D,n) \\
P_1(O,D,n) \\
P_2(O,D,n) \\
\vdots \\
P_{N-1}(O,D,n) \\
P_N(O,D,n)
\end{bmatrix}
= \sum_{j=0}^{N} P_j(O,D,n) = \sum_{j=0}^{N} p^j \sum_{i=0}^{N} a_{ij}.
\]
(A.5)

By the use of (A.4) this may be written as
\[ P(O,D,n) = \sum_{j=0}^{N} P_j(O,D,n) = \sum_{j=0}^{N} p^j \sum_{i=0}^{N} a_{ij}. \]
(A.6)

Now let
\[ k_j(O,D,n) = \sum_{i=0}^{N-j} a_{ij} \]
\[ = \sum_{i=0}^{N-j} (-1)^{i-j} \binom{N-i}{i-j} C_i(O,D,n). \]
(A.8)

Then
\[ P(O,D,n) = \sum_{j=0}^{N} P_j(O,D,n) = \sum_{j=0}^{N} k_j(O,D,n) p^j. \]
(A.9)

Since \( P(O,D,n) = 1 \) for \( p = 1 \), we obtain from (A.9), by letting \( p = 1 \),
\[ P(O,D,n) = \sum_{j=0}^{N} k_j(O,D,n) = 1. \]
(A.10)

The assignment of values to the elements \( a_{ij} \) of the blocking matrix requires that the number of blocking situations \( C_i(O,D,n) \) be known for all values of \( i = 0, 1, 2, 3, \ldots, N \). As \( N \) increases, the time involved in testing all \( 2^N \) situations of available/unavailable links in the network increases exponentially, and beyond a certain value of \( N \) this time becomes prohibitive. If the network situations are tested in the order of decreasing contribution to the probability of blocking, the above power series can be truncated to give approximations to the probability of blocking. For small values of \( p \) approximations to \( P(O,D,n) \) can be derived from (A.7) above. For large values of \( p \) the corresponding expressions at (A.2a) to (A.7a) below are more suitable. Expression (A.2) can be written in the form
\[ P_{N-i}(O,D,n) = C_{N-i}(O,D,n) p^{N-i} (1 - p)^i. \]
(A.2a)

Substituting \( 1 - p \) for \( q \) gives
\[ P_{N-i}(O,D,n) = C_{N-i}(O,D,n) q^i \]
and expanding the last term by the binomial theorem gives
\[ P_{N-i}(O,D,n) = C_{N-i}(O,D,n) \sum_{j=0}^{N-i} (-1)^j \binom{N-i}{j} q^j \]

Finally, substituting \( j - i \) for \( k \) results in
\[ P_{N-i}(O,D,n) = C_{N-i}(O,D,n) \sum_{j=1}^{N-i} (-1)^{j-i} \binom{N-i}{j-i} q^j \]
(A.3a)

Thus \( P_{N-i}(O,D,n) \) can be expressed as the polynomial in \( q \)
\[
\begin{align*}
&= b_{i0} + b_{i1} q + b_{i2} q^2 + b_{i3} q^3 + \ldots \\
&\quad + b_{iN} q^N.
\end{align*}
\]
(A.4a)

where
\[ b_{ij} = 0, \quad \text{for } j < i \]
\[ b_{ij} = (-1)^{j-i} \binom{N-i}{j-i} C_{N-i}(O,D,n), \quad \text{for } j \geq i, \]
and where \( q = 1 - p \).

The \( P_{N-j}(O,D,n) \) terms can be expressed in matrix form as
\[
\begin{bmatrix}
P_0(O,D,n) \\
P_1(O,D,n) \\
P_2(O,D,n) \\
\vdots \\
P_{N-j}(O,D,n) \\
P_{N-j+1}(O,D,n)
\end{bmatrix}
= \sum_{j=0}^{N-j} P_j(O,D,n) = \sum_{j=0}^{N-j} q^j \sum_{i=0}^{N-j} a_{ij}.
\]
(A.5a)

From this matrix blocking, the final value of the probability of blocking can be found by the summation of rows or by the summation of columns. Notice that the sum of columns is
\[ P(O,D,n) = \sum_{j=0}^{N} P_j(O,D,n) = \sum_{j=0}^{N} q^j \sum_{i=0}^{N-j} a_{ij}. \]
(A.6a)

where \( q = 1 - p \).

**A5 SUMMARY**

To summarize this appendix, two expressions for the probability of blocking \( P(O,D,n) \) have been obtained:
- the sum of columns expression (A.6) for use when \( p \) is small
- the sum of columns expression (A.6a) for use when \( p \) is large.

The expressions are polynomials whose coefficients depend on \( C_i(O,D,n) \), the number of blocking situations.

A procedure for determining \( C_i(O,D,n) \) by computer has been developed and is described in Ref. (1). This method is very powerful but its application to large networks is limited by the computing time involved. This increases exponentially with the network size and is approximately \( 2^N \) milliseconds, where \( N \) is the number of links. Thus, for a network of 25 links about one hour of computing time is required whereas for a network of 35 links about one month is required.

In small networks with up to some 25 links the programs may be used to derive accurate expressions for the probability of blocking between two given nodes. In networks with some 25 to 60 links, approximate results can be obtained by taking into account those network situations which contribute significantly to the probability of
APPENDIX B

PRACTICAL VALUES FOR THE PROBABILITY OF BLOCKING

B1 INTRODUCTION

This appendix derives some practical values for the probability of blocking between nodes in the grid network (Fig. 1 of main text). The results were obtained by using the sum of columns method described in Appendix A in conjunction with the computer procedure described in Ref. 1. The two nodes 0(0,0) and D(x,y) shown in Fig. 2 of the main text are considered. For convenience these two nodes are referred to in the following as simply 0(0,0) and D(x,y). The two nodes are x+y links apart and different values for x and y are examined for which x+y = 1, 2, 3, 4, 6. The probability of blocking between 0(0,0) and D(x,y) is denoted by P(x,y,n), where x,y refer to the coordinates of node D(x,y) and n denotes that routes of class A0 to A2 are allowed (see Fig. 2) of the main text; A0 represents the class of shortest routes that involve x+y links.

Analytical expressions for P(x,y,n) are derived and used to compute numerical results for the probability of blocking as a function of p, the probability of unavailability of a link. A discussion is included on the significance of the results.

B2 ADJACENT NODES (x + y = 1)

For x = 1 and y = 0 (or y = 1 and x = 0) the two nodes are interconnected by a single link and the probability of blocking may be expressed as

\[ P(1,0,1) = 9p^3 + 18p^4 + 15p^5 - 6p^6 + p^7, \]  
\[ P(1,0,2) = 2p + 126p^6 - 12p^6 + 139p^7 - 18p^8 \]
\[ + 159p^9 - 106p^10 + 333p^11 - 72p^12 + 7p^{13}, \]
\[ P(1,0,3) = 2p^6 + ..., \]

where \( p \) is the probability of a link being unavailable.

Here (B.1) assumes that routes of class A0 are permitted (i.e., the single link interconnecting nodes 0(0,0) and D(1,0)), and (B.2), (B.3) and (B.4) assume that routes of respectively classes A0 and A1, A0 and A2, and A0 and A3 are permitted.

By using these expressions, the practical values shown in Fig. B1 for the probability of blocking were calculated; the curves (a), (b) and (c) correspond to \( n = 0, 1, 2 \) respectively and give the probability \( P(1,0,n) \) as a function of the probability \( p \); curve (d) is drawn as a solid line for \( p < 0.2 \) on basis of the result of using (B.4), and for \( p > 0.2 \) is extrapolated (dotted line) to approach curve (c).

The four curves (a), (b), (c) and (d) in Fig. B1 show that as the number of route-classes increases, there is a reduction in the probability of blocking between 0 and D (the lower the value of \( p \), the greater the improvement).

A lower bound on the probability of blocking between adjacent nodes is shown in Ref. (1) to be

\[ p(3^3 + (1-p^3)3) = 2p^4 - p^7 \]

Therefore:

\[ P(1,0,n) > 2p^4 - p^7, \]  

where \( n = 0, 1, 2, 3, 4, ... \)

* Note that in Appendix A which considers a non-hierarchical network in general, the notation corresponding to \( P(x,y,n) \) is \( P(0,D,m) \).

Fig. B1 The probability of blocking between adjacent nodes

In Fig. B1 the shaded area is limited by the curve associated with (B.5) and so any value of \( P(1,0,n) \) lies to the left of this area. Since curve (d) is close to the shaded area it can be deduced that the differences

\[ P(1,0,3) - P(1,0,n) \]

are small for all \( n \geq 4 \). Hence, there will be little improvement in the probability of blocking between adjacent nodes by allowing more than four route-classes.

In Fig. B1, the three dotted curves (e), (f) and (g) which are shown for comparison, represent a 3-times, 10-times and a 100-times improvement in the probability of blocking, (i.e., the expression \( P = \frac{1}{3}p \), \( P = \frac{1}{10}p \) and \( P = \frac{1}{100}p \)).

B3 NODES TWO LINKS APART (x + y = 2)

If nodes 0(0,0) and D(x,y) are two links apart, (x,y) = (2,0) or (1,1); the situation (x,y) = (0,2) is symmetrical to (x,y) = (2,0) and is ignored. For routes of class A0 alone, classes A0 and A1, and classes A0 to A2, being allowed, the following expressions for the probability of blocking are obtained

\[ P(2,0,0) = 1 - (1-p)^2 = 2p - p^2, \]  
\[ P(2,0,1) = 8p^3 + 16p^4 - 9p^5 + 116p^6 - 10p^7 - 11p^8 \]
\[ + 128p^9 - 67p^{10} + 18p^{11} - 2p^{12}, \]  
\[ P(2,0,2) = 2p^6 + 58p^7 + 18p^8 - 138p^9 + ..., \]  
\[ P(1,1,0) = 4p^3 - 4p^4 + p^5, \]  
\[ P(1,1,1) = 49p^4 - 168p^5 + 256p^6 - 220p^7 + 112p^8 \]
\[ - 32p^9 + 4p^{10}, \]  
\[ P(1,1,2) = 2p^6 + 572p^7 + ... \]

Practical values calculated from these expressions are shown as the six curves (c) to (f) in Fig. B2. The solid line of the two curves (c) and (f) are based on (B.8) and (B.12); the remainder of each curve is drawn to approach
the two curves (b) and (e). The shaded area represents a lower bound on the probability of blocking.

Curves (a) to (c) and (d) to (f) show that as routes of an increasing number of classes are allowed there is an improvement in the probability of blocking; and the lower the value of \( p \) the greater the improvement.

It follows from (B.7) that

\[
P(2,0,0) > P \quad \text{for all } p
\]

and

\[
P(3,0,0) > P(2,1,0)
\]

In Fig. B2 it can be seen that for small values of \( p \)

\[
P(2,0,0) > P(2,0,1)
\]

By allowing route-class \( \mathcal{A}_1 \), an improvement in the probability of blocking is obtained (B.8), but it may be noted from Fig. B2 that \( P(2,0,1) > P(3,0,0) \) only if \( p < 0.44 \). Similarly, \( P(1,1,0) < P(2,1,0) < 0.36 \), but \( P(1,1,1) < P \) only if \( p < 0.54 \). By comparing curves (a) and (d) it can be seen that for small values of \( p \)

\[
P(2,0,0) > P(1,1,0)
\]

By allowing route-class \( \mathcal{A}_1 \) - curves (b) and (e) - the difference between \( P(2,0,1) \) and \( P(1,1,1) \) is considerably reduced; by allowing route-class \( \mathcal{A}_2 \) - curves (c) and (f) - there is little difference between \( P(2,0,2) \) and \( P(1,1,2) \).

It may also be noted from Fig. B2 that curves (c) and (f) are close to the shaded area which means that little improvement can be obtained in the probability of blocking by allowing more than two detour links. If \( p < 0.24 \) then

\[
P(1,1,2) < P(2,0,2) < \frac{1}{5} p
\]

B.4 NODES THREE LINKS APART \((x+y=3)\)

The probability of blocking, \( P(x,y,n) \), can be expressed, for routes of class \( \mathcal{A}_0 \) alone, and classes \( \mathcal{A}_0 \) and \( \mathcal{A}_1 \), by the following expressions

\[
P(3,0,0) = 3p - 3p^2 + p^3 \quad \text{(B.13)}
\]
\[
P(3,0,1) = 9p^3 + 33p^4 - 135p^5 + 42p^6 + 26p^7 - 42p^8
\]
\[
- 298p^9 + 465p^{10} + 172p^{11} - 85p^{12} + 951p^{13}
\]
\[
- 55p^{14} + 188p^{15} - 36p^{16} + 3p^{17} \quad \text{(B.14)}
\]
\[
P(2,1,0) = 5p^2 + 2p^3 - 10p^4 + 13p^5 - 9p^6 + p^7 \quad \text{(B.15)}
\]
\[
P(2,1,1) = 29p^4 + 42p^5 + 62p^6 + 28p^7
\]
\[
- 184p^8 + 222p^9 - 85p^{10} + 65p^{11}
\]
\[
+ 1107p^{12} - 703p^{13} + 248p^{14} - 48p^{15} + 4p^{16} \quad \text{(B.16)}
\]

Practical values calculated from these expressions are shown in Fig. B3 as the solid curves (a) to (d). The shaded area represents a lower bound on the probability of blocking.

Comparing the two curves (a) and (c) with (b) and (d) in Fig. B3 shows that for given value of \( p \)

\[
P(3,0,0) > P(2,1,0)
\]

and

\[
P(3,0,1) > P(2,1,1)
\]

where

\[
P(3,0,1) < P(2,1,0) \quad \text{for } 0 < p < 0.48
\]

For large values of \( p \), the probability of blocking is higher than the probability of a link being unavailable. Thus,

\[
P(3,0,0) > P \quad \text{for all } p
\]
\[
P(2,1,0) > P \quad \text{for } p > 0.24
\]
\[
P(3,0,1) > P \quad \text{for } p > 0.34
\]
\[
P(2,1,1) > P \quad \text{for } p > 0.44
\]

To achieve a low probability of blocking between nodes
which are three links apart, route-class $A_1$ should be allowed, particularly where the two nodes are located on the same horizontal or vertical line (i.e., same x-coordinate or y-coordinate); the value of $p$ should not exceed 0.2 to 0.3, say.

B.5 NODES SIX LINKS APART ($x + y = 6$)

The probability of blocking, $P(x,y,0)$, on all shortest routes, and $P(6,0,1)$, can be expressed as

$$P(6,0,0) = 1 - (1-p)^6 = 6p - 15p^2 + 20p^3 - 15p^4$$

$$P(6,0,1) = 12p^3 + 24p^4 - 148p^5 + 3637p^6$$

$$P(5,1,0) = 17p^2 - 4np^3 + 38p^4 - 2p^5 - np^6 - 20p^7$$

$$P(4,2,0) = 2p^5 + 24p^6 - 50p^7 - 10p^8 + 40p^9 - 150p^{10}$$

$$P(3,3,0) = 2p^8 + 4p^9 + 37p^{10} - 56p^{11} - 24p^{12} - 80p^{13}$$

$$P(2,0,0) = 9p^{12} - 2922p^{13} + 2181p^{14} + 2456p^{15}$$

$$P(1,1,0) = 12p^{15} - 192p^{16} - 1562p^{17} + 10636p^{18}$$

$$P(0,0,0) = 0.0$$

$$(B.17)$$

$$(B.18)$$

$$(B.19)$$

$$(B.20)$$

$$(B.21)$$

Practical values calculated from these expressions are shown in Fig. B4. The results are similar to those in Fig. B3.

Practical values calculated from these expressions are shown in Fig. B4. The results are similar to those in Fig. B3.

[Diagram of blocking probabilities]

Fig. B4 The probability of blocking between nodes that are 6 links apart

Fig. B5 Comparison of blocking probabilities

When many routes are available between the two nodes, the most significant contribution to the probability of blocking is due to the outgoing links from the originating node or the incoming links to the destination node being unavailable. Thus, if route-class $A_1$ is allowed, there are several routes between the two nodes and the probabilities of blocking $P(6,0,1)$, $P(4,0,0)$, and $P(2,0,0)$ depend on the distance between the two nodes.

B.6 COMPARISON OF RESULTS

B.6.1 Relative locations of nodes

Fig. B5 shows the probabilities of blocking between nodes 0(0,0) and D(x,y), where $(x,y) = (6,0), (4,0), (2,0), (0,0), (1,0), (1,1), (0,1)$. When $x = 0$, there is only one shortest route available and it can be seen that the probabilities of blocking $P(6,0,0)$, $P(4,0,0)$ and $P(2,0,0)$ depend on the distance between the two nodes.

It can be concluded from the results that the probability of blocking between two nodes in the grid network is practically the same for all nodes provided route-class $A_1$ is allowed. Allowing also route-class $A_2$ leads to lower blocking probabilities and more uniformity in results, but allowing additional route-classes has little significance.

B.6.2 Relative values of $p$

If we denote by $Q$ the reduction in the probability of blocking between 0(0,0) and $D(x,y)$ in addition to that which would result if the two nodes were connected by a single link, we have

$$Q = \frac{p}{P(x,y,n)}, \text{ where } 0 < p < 1$$

$$(B.22)$$
and \( Q \) depends on \( x,y,n \) (see Figs. B1 to B5). For low values of \( p (< 0.01) \) and \( x > 0 \) and \( y > 0 \), the reduction is substantial and in general \( Q > 100 \) for \( n = 0 \). If \( x = 0 \) (or \( y = 0 \)), it is necessary to use \( n = 1 \) to ensure that \( Q > 100 \). For \( 0.01 < p < 0.1 \), depending on \( x,y,n \), the reduction \( Q \) is considerably less. For \( p > 0.1 \), the reduction is negligible and \( Q > 1 \) for all \( p > p_1 \) where \( p_1 \) depends on \( x,y,n \).

This means that there is a critical value of \( p \) above which the probability of blocking \( P(x,y,n) \) between two given nodes \( 0(0,0) \) and \( D(x,y) \) is high relative to \( p_1 \) below this value of \( p \), the probability of blocking \( P(x,y,n) \) is progressively reduced as \( p \) decreases. For \( n = 0 \) and \( x = 1, 2, 3, 4, 5, 6 \), where \( x + y \leq 6 \), the critical value is approximately \( p = 0.1 \). If \( x = 0 \) and \( y = 1, 2, \ldots, 6 \), it is necessary to use \( n = 1 \) to ensure that the critical value does not fall below \( p = 0.1 \). Symmetry dictates that the same results apply when \( x \) is replaced by \( y \) and \( y \) by \( x \).

**REFERENCE(S)**

(1) B.J.A. Vestmar. Design Considerations for the Grid Type of Communication Network. Doctoral thesis at the Technische Hogeschool, Delft, Department of Electrotechnology, 1975