RELIABILITY OF COMBINED MEASUREMENT OF MEAN HOLDING TIME AND TRAFFIC LOAD BASED ON COUNTING CALLS AND SAMPLING SEIZURE STATES

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ABSTRACT
This paper deals with a combined measurement of the traffic load and the mean holding time. The measuring facility entails:
- counting the seizures
- determining the number of trunks seized at equidistant points of time.

The fundamental correlations between actual and sampled holding times are investigated for the general case of an arbitrary arrival process and arbitrarily distributed holding times.

It is shown that the inaccuracy caused by the scanning procedure is unbiased and not correlated with the holding time.

Simple formulas for calculating confidence intervals for traffic load and mean holding time are derived for application to an undisturbed Poisson arrival process and negative exponentially distributed holding time.

1. INTRODUCTION
Computer-controlled switching systems provide a facility for straightforward combined measurement of traffic load and mean holding time. Measurement of these two variables for a particular trunk group (or other switching equipment) entails:
- counting the seizures
- determining the number of trunks seized at equidistant points in time and registering the accounted number on a second counter over a selected period of observation of sufficient duration. The final total of the two counters can then be used to calculate "measured values" for the two variables in the light of known correlations.

This technique thus corresponds to what is known as sampling method in terms of both traffic load and holding time. The reliability of such sampling methods has been treated in several articles in the past (e.g. /1/ ... /5/).

However, the questions arising with respect to the combined data acquisition suggested the investigations in this paper.

The analytical concept used corresponds basically with Iversen's idea of the traffic process being represented by two components which are corresponding structurally but stochastically independent: the arrival process and the holding time process.

However, it also is referring to Hayward /2/ in regarding the inaccuracy effected by the scanning procedure as to be an additive component of the total measuring inaccuracy, it is shown that this concept holds generally for arbitrary arrival and holding time processes.

Due to its objectives the investigation is restricted to the case of regular scan intervals. But, nevertheless, the methodical concept used may be applied to arbitrary scan intervals.

2. NOMENCLATURE
2.1 Probability Functions
Let X be a random variable. Expressed are then:
- the probability that "..." is true: P(\ldots)
- the density function of X: p(X=x)
- the distribution function of X: E[X^k]
- the variance of X: V[X] = E[X^2] - E[X]^2
- the standard deviation of X: \sigma[X] = V[X]
- the stochastic sum of N identically distributed random variables X_i (i=1,2,...,N)

2.2 Symbols
2.2.1 Fixed values
- the (duration of the) measuring period T
- the (duration of the) scan interval Q
- the arrival intensity \lambda
- the mean holding time \tau
- the traffic intensity (traffic load) A=\lambda T

2.2.2 Random Variables
- the actual duration of the holding time H
- the integer transform of the holding time H, \text{int}(H/Q)
- the modulo transform of the holding time H, R = H \text{ mod } Q (0 \leq R < Q)
- the location of the beginning of the holding time within the scan interval, defined as its distance from the last preceding scanning point (0 \leq U < Q)
- the location of the termination of the holding time within the scan interval, defined as its distance from the last preceding scanning point (0 \leq V < Q)
- the recorded duration of the holding time S=0,1,2,\ldots,
- the scanning error, defined by D=S-H (-Q < D < Q)
- the number of arrivals within the measuring period N
- the total count of occupations observed at scanning points within the measuring period Z

(see next page)
Due to Iversen's concept the traffic measured is composed of holding times (seizures) observed within a particular trunk group. The beginning and the termination of the holding times is depending on chance.

Because of symmetry this will not affect the variance of the durations of the holding times which be truncated. It is to say that the total of time covered by the measurement usually comprises several measuring periods, each of which is placed on a busy hour. However, since the traffic of particular measuring periods are to be supposed as to be mutually independent, the results apply analogously to the measurement in toto. The measuring procedure entails a combined recording of:

(a) The **arrival process**
   - constituted by the points in time on which a holding time is beginning. This points in time will be denominated as arrival points, and the time distance between two arrival points in sequence as inter-arrival time.
   - independent of the absolute time (which provides stationarity)
   - continuously distributed (which provides regularity). As shown in 9, this requirement guarantees the location of an arbitrary arrival being equally distributed within a fixed time interval.

(b) The **holding time process**
   - constituted by the durations of the holding times. We necessarily assume the holding times to be mutually independent.

The **traffic volume observed** is an approximate for the measured value representing an approximate for the mean holding time \( \bar{t} = Y/N \)

The measuring procedure entails a combined recording of:

(a) The number (N) of seizures (i.e. holding times) beginning within the measuring period (i.e. the number of arrivals)

(b) The total count (Z) of occupations observed at equidistant points in time, designated as "scanning points".

The beginning of the measuring period is placed exactly on a scanning point and its duration (Z) is a multiple of the scan interval. If \( \omega \) denotes the (length of the) scan interval, then

\[ Y = Z \omega \]

is the traffic volume observed. On base of \( Y \) there results

\[ t = Y/N \]

as an approximate for the mean holding time \( t \)

\[ A = Y/T \]

as an approximate for the traffic intensity \( A \).

6. **STOCHASTIC REPRESENTATION OF THE MEASURED VALUES**

6.1 **Interpretation of the Data Recorded**

Due to the fixed measuring period some of the holding times are truncated:

There may be "tails" of holding times which begin prior to the measuring period as well as "heads" of holding times which survive the measuring period.

The variance of the sum \( X + Y \) follows generally by


where \( K[X,Y] = \text{cov}(X,Y) \) the covariance of \( X \) and \( Y \).

For the sum holds:

- in any case

\[ E[XY] = E[X]E[Y] \]

and the variances of \( X \) and \( Y \) are additive. If \( X \) and \( Y \) are independent then they are uncorrelated, but the reverse is not necessarily true.

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The traffic measured is composed of holding times (seizures) observed within a particular trunk group. The beginning and the termination of the holding times is depending on chance.

Due to Iversen's concept this traffic can be represented by two processes which are structurally coordinated (with respect to the sequential number of seizures) but stochastically independent:

- independent of the absolute time (which provides stationarity)
- continuously distributed (which provides regularity). As shown in 9, this requirement guarantees the location of an arbitrary arrival being equally distributed within a fixed time interval.

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as an approximate for the mean holding time \( t \)

\[ A = Y/T \]

as an approximate for the traffic intensity \( A \).
Due to the combined measurement we have, however, to pay attention to an additional phenomenon:
The truncated "heads" will be counted as complete seizures (i.e. holding times) within N. The other hand the truncated "tails" will be recorded within the traffic volume Y, but not counted within N, i.e. the recordings on N and Y are somehow distorted (what has an effect to the correlation between N and Y).

Nevertheless, to simplify matters we suppose the measuring period as being chosen of sufficient duration (T>>t), and thus the effects of truncation and distortion as to be neglectable in practice. On this account we assume that each holding time counted within N may be exatly recorded within Y.

6.2 Fundamental Relations

Referring to 6.1 and section 7, we define the following random variables:

H: the actual duration of a (particular) holding time
S: the holding time as recorded by the scanning procedure
D: the scanning error, defined as D=S-H
N: the number of arrivals during the measuring period

Then (as shown in 7.3):

(1) \[ S = H + D \]
(2) \[ E[S] = E[H] \]
(3) \[ V[S] = V[H] + V[D] \]

The variance \( V[D] \), applying for (3) is obtained by applying the modulo transform on H (see 8.).

As traffic volume measured Y we define the sum of all recorded holding times beginning within a fixed time period of the length T.

In 3.3 then is obtained

(4) \[ Y = N \cdot S \]
(5) \[ E[Y] = E[N] \cdot E[S] \]
(6) \[ V[Y] = V[N] \cdot E[S] + V[S] \cdot E[N]^2 \]

In the case of a Poisson arrival process with an arrival intensity \( \lambda \), is \( V[N] = E[N] = \lambda T \), and (6) thus may be simplified to

(7) \[ V[Y] = \lambda T \cdot (V[S]+E[H]^2) \]

If the constants

(8) \[ \lambda = E[N]/T \quad \text{(arrival intensity)} \]
(9) \[ T = E[H] \quad \text{(mean holding time)} \]
(10) \[ A = \lambda T = E[N]E[H]/T \quad \text{(traffic intensity)} \]

are regarded as parameters of a stationary traffic process, then the random variables

(11) \[ A = Y/T = (N \cdot S)/T \]
(12) \[ Z = Y/n = (N \cdot S)/N \]

are measured values applying as approximates of A resp. \( T \) in practice.

Since T is a constant, for the measured value \( A \) follows by means of (5)(6)

(13) \[ E[A] = E[Y]/T = A \]
(14) \[ V[A] = V[Y]/T^2 = (\lambda T \cdot V[S] + \lambda^2 V[N]) / T^2 \]
(15) \[ V[A]^2 = (V[S] + \lambda^2) / T \]

Formula (15) applies in the case of a Poisson arrival process.

In contrast to \( A \) the measured value \( Y \) has a denominator, represented by the random variable N. Thus, under the condition \( N=n > 0 \), is obtained

(16) \[ E[Y|N=n] = (n \cdot E[S]) / n = E[S] \quad \text{(const)} \]
(17) \[ V[Y|N=n] = (n \cdot V[S]) / n^2 = V[S] / n \]

Since the mean value of \( Y \) is obviously unaffected by the random variable N, and the actual value of n is resulting from the measurement, the conditional variance is to be considered as

"variance of \( Y \) especially applying on a measurement in which n holding times have been recorded".

The conditional variance \( V[Y|N=n] \) is thus, in the place of \( V[Y] \), the appropriate basis for the calculation of confidence intervals.

6.3 Results in the Case of Poissonian Arrivals and Exponentially distributed Holding Times

If the holding times are exponentially distributed (with mean \( T \)), then

(1) \[ V[H] = E[H]^2 = \tau^2 \]

is valid, and the modulo transform (see 8.) yields

(2) \[ V[D] = \tau^2 \cdot (S \cdot \epsilon_{\tau^2} - 2) \]

which can be written as

(3) \[ V[D] = \tau^2 \cdot (\epsilon_{\tau^2} \cdot G - 2) \]

In the case of a Poisson arrival process, we obtain on this basis by aide of 6.2(13)

(5) \[ V[S] = \tau^2 \cdot (\epsilon_{\tau^2} - G - 1) \]

and applying 6.2(15)(17), the variances

(6) \[ V[A] = \frac{\tau^2}{T} \cdot G \quad A = \frac{\lambda T}{T} \cdot G \]
(7) \[ V[Y|N=n] = \frac{\tau^2}{n} \cdot (\epsilon_{\tau^2} - G - 1) \]

For the relative standard deviations result from this the formulas

(8) \[ \sigma[A]/A = \sqrt{T} \cdot \sqrt{G} \quad \text{(Hayward)} \]
(9) \[ \sigma[Y|N=n]/T = \sqrt{T} \cdot \sqrt{G} - 1 \]

The symbol n in (7)(9) represents the actually resulting count of seizures (N).

For practical applications the unknown traffic parameters \( \lambda \) and \( T \) (which applies to G in each case) may be replaced by the adequate results of the measurement.

As to be expected, for \( A \) applies the well known formula by Hayward (if truncations are taken into account, a formula due to the Waterberg distribution for \( \lambda T / \sigma \) is obtained).

Since (5)(9) may apply as well as to a measurement only for traffic intensity resp. only for mean holding time, it may be seen that the combined acquisition does not imply an increased accuracy (as far as to be expressed by formulas). This is due to the fact that the relevant part of the information given by the count \( N \) is adequate to the approximate gained for the mean holding time.

However, the additional information permits the
unknown parameter \( r \) in (9) to be replaced by actual data instead of estimates.

7. BASIC CONSIDERATIONS ON A PARTICULAR HOLDING TIME

7.1 Geometrical Correlations

We consider a particular holding time recorded by scanning with a scan interval \( Q \) (Fig. 1). The duration of the holding time is denoted by \( H \), and the location of its beginning within the scan interval (defined as the distance from the last scanning point) is denoted by \( U \) (of \( U < Q \)).

\( H \) and \( U \) are independent random variables.

The location of the termination of the holding time within the scan interval (defined as the distance from the last scanning point) is dependent on \( U \) and \( H \), and given by

\[
V = (U + H) \mod Q \quad (0 \leq V < Q)
\]

Applying the modulo transform

\[
R = H \mod Q
\]

we obtain the equation

\[
V = (U + R) \mod Q \quad (0 \leq V < Q)
\]

which is equivalent to (1), but simplified in that way that, with respect to the modulo function, we have to distinguish only between the cases \( U + R < Q \) and \( U + R \geq Q \).

The recorded duration of the holding time is given by

\[
S = JQ
\]

where \( J \) denotes the number of scanning points covered by the actual holding time \( H \).

If we define

\[
D = S - H
\]

the equation

\[
D = U - V = - ((U + R) \mod Q) \quad (-Q < D < 0)
\]

holds in any case.

The independent variables \( U \) and \( R \) constitute a space of events illustrated in Fig. 2. This space may be divided into two regions, in each of which \( V \) resp. \( D \) is a continuous function of the variables \( U \) and \( R \).

Due to this division we distinguish two cases:

Case \( D < 0 \): equivalent to \( 0 \leq U < R < Q \)

in this case

(7) \( V = U + R \) is valid.

Case \( D > 0 \): equivalent to \( 0 < U + R < Q \)

in this case

(8) \( V = U + R - Q \) is valid.

It is to note that the modulo transform applied on \( H \), in connection with the probability distribution of \( D \) corresponds to Iversens stochastic transform /3/.

7.2 Probability Functions of the Dependent Random Variables

As seen in Fig. 2, the event \( \{ D = x \} \) is equivalent to \( \{ R = x, U + x \} \) in the case \( D < 0 \) resp. \( \{ R = x_i, U + x \} \) in the case \( D > 0 \).

Since \( U \) and \( R \) are independent, it follows that

\[
dP(D = x | R = r) = \begin{cases} 
\frac{dP(U = Q - r)}{dP(R = r)} & \text{for } x = Q - r \\
0 & \text{else}
\end{cases}
\]

Under the condition \( R = r \) (of \( 0 \leq r < Q \)), only \( D \geq r \) is possible. Thus the conditional probability is given by

\[
dP(D = x | R = r) = \begin{cases} 
\frac{dP(U < Q - r)}{dP(R = r)} & \text{for } x = Q - r \\
in U = x
\end{cases}
\]

Considering the variable \( V \), only its relation to \( U \) will be of interest. We find that, under the condition \( U = u \) the equation \( R = (V - u) \mod Q \) holds in any case. Thus follows

\[
dP(V = v | U = u) = \begin{cases} 
\frac{dP(R = v - u)}{dP(U = u)} \text{ as long as } u + x \\
\frac{dP(R = v - u)}{dP(U = u)} \text{ as long as } u + x < Q
\end{cases}
\]

7.3 Stochastic Interrelations between \( D \) and \( H \)

We may suppose that \( U \) is equally distributed in \( [0, Q) \). As shown in 9., this assumption holds in any case, provided

- the inter-arrival time is continuously distributed
- the arrival process tends to infinity.

The distribution of \( H \) may be arbitrary, for sim-
Since $Q(Q) \sim (Q)$, it follows that
$$p(D=x) = \begin{cases} p(R=-x)(1-\frac{1}{2}) & \text{for } x \leq 0 \text{;} \quad p(R=x)(1+\frac{1}{2}) & \text{for } x > 0 \text{.} \end{cases}$$

From (7.2): (1) then follows that
$$E[D | R=r] = \begin{cases} - \frac{1}{2} & \text{for } x \leq 0 \text{;} \quad \frac{1}{2} & \text{for } x > 0 \text{.} \end{cases}$$

On this base we conclude:
- $D$ is dependent on $R$, thus on $H$, but:
- $D$ and $R$ are uncorrelated. For from (2) it follows that
$$E[D] = 0, \quad E[DR] = 0, \quad E[R] = E[D]E[R]$$

Since the distribution of $R$ is of no relevance in (2)(3), for all $k=0,1,\ldots$ will result
$$E[D|r=k]=0, \quad E[R] = E[D]E[R].$$

Due to the linearity of $E[D]$ and $E[R]$, it follows from (4) we find:
$$E[D] = E[D]E[1].$$

Based on (3)(4) we find:
$$\begin{align*}
V[S] &= E[H] \quad V[S] = E[H] + V[D] \\
\text{The variance of } V[D] &\text{ may be found by means of:}
E[D^2] = \int_0^\infty E[D^2 | R=r] \cdot P(R=r) \, dr.
\end{align*}$$

It yields, on account of (3):
$$V[D] = E[H] - E[R^2].$$

### 7.4 Stochastic Interrelations between $D$ and $U$

On basis of 7.2: (3) the conditional expectation $E[D | U=u]$ is obtained by the integration
$$E[D | U=u] = \int_0^\infty E[D | U=u] \cdot dP(U=u)$$

We find:
$$\begin{align*}
(1) \quad E[D | U=u] &= GP(R \geq u) - E[R] \\
(2) \quad E[DU] &= \frac{1}{2} E[RU] - E[D^2].
\end{align*}$$

Since $E[D]=0$, we find $E[DU]=E[D]E[U]$, i.e., $D$ and $U$ are correlated.

### 7.5 Stochastic Interrelations between $V$ and $U$

Let $U$ be equally distributed in $[0,Q]$. As shown in 9., the $V$, too, is equally distributed. Hence:
$$\begin{align*}
(1) \quad E[U] &= E[V] = \frac{1}{2}Q \\
(2) \quad E[UV] &= u + E[R] - GP(R \geq u) \quad E[RV] = \frac{1}{2}V[D].
\end{align*}$$

With regard to (1) there follows that
$$E[VU] = E[V]E[U], \quad \text{i.e. } V \text{ and } U \text{ are correlated.}$$

### 7.6 Correlations between Scanning Errors in Sequence

As shown in 7.3, the scanning error does not correlate with the actual holding time. However, due to the results of 7.4,7.5, two particular scanning errors are to be supposed as being correlated, which implies that 3.3: (3) is not valid. This follows because the locations $U_1, U_2$ of two consecutive arrivals are corresponding to the inter-arrival time in the same way, as do the beginning and the termination of a holding time to its duration $H_0$.

As not to be explained in detail, there follows
$$\begin{align*}
(1) \quad E[D_1D_2] &= -E[R_1]E[R_2] \\
(2) \quad p(R_0=r) &= 1/\omega
\end{align*}$$

In this case 7.5:(4) resp. 7.6:(1) yields
$$\begin{align*}
(3) \quad E[U_1U_2] &= \frac{Q^2}{4} = E[U]E[U_2] \\
(4) \quad E[D_1D_2] &= 0 = E[D_1]E[D_2]
\end{align*}$$

### 8. Stochastic Representation of the Modulo Transform

Let be:
$$\begin{align*}
(1) \quad R &= H \mod \omega \quad \text{the modulo transform of a particular holding time } H \\
\text{Then:}
\begin{align*}
P(R=r) &= \sum_{k=0}^\infty P(k\omega+\omega < r < (k+1)\omega) \\
&= \sum_{k=0}^\infty P(R=k\omega+r) - \sum_{k=0}^{\infty} P(H=k\omega) \quad \text{where the sums exist as long as } E[H]\text{ is finite.}
\end{align*}
\end{align*}$$

If we define
$$\begin{align*}
(2) \quad Q(r) &= \sum_{k=0}^\infty P(H=k\omega+r), \\
\text{then the following representations are obtained:}
\begin{align*}
(3) \quad E[R|R=r] &= Q(r) - Q(0) \\
(4) \quad p(R=r) &= Q'(r) \\
(5) \quad E[R] &= \int_0^\omega dR \cdot P(R=r) = \int_0^\omega Q(r) - Q(0) \\
(6) \quad E[R^2] &= \int_0^\omega dR \cdot P(R=r) = 2\int_0^\omega Q(r) - Q^2(0)
\end{align*}
\end{align*}$$

The variance of the scanning error (see 7.3) then follows by
$$\begin{align*}
(7) \quad V[D] &= \sigma^2[E[R] - E[R^2]] = \sigma^2 - \frac{1}{2} V[D]
\end{align*}$$
In the case that $H$ is exponentially distributed:

$P(H > x) = e^{-x/\tau} \quad \mathbb{E}[H] = \tau$,

in (2) is obtained

$P(H > t) = \int_{t}^{\infty} e^{-\xi/\tau} d\xi = 1 - e^{-t/\tau}$

From this base (7) yields

$V[D] = \mathbb{E}[(\frac{Q}{\tau} + 1) e^Q/\tau - 2]$.

9. GENERAL ANALYSIS ON THE DISTRIBUTION OF ARRIVAL LOCATIONS IN A FIXED INTERVAL

This section deals with the properties of the distribution of the termination location $V$ and its relations to the distribution of the arrival location $U$ (referring to $7$).

However, as $U$ and $V$ can be understood as to be locations of consecutive arrivals within any regular interval of the length $Q$, and $H$ as to be the inter-arrival time, the investigation applies generally to the arrival point process.

The modulo transform $H = H \mod Q$ is necessarily required as to be continuously distributed, i.e. there exists the density function $p(R = r)$ ($0 < r < Q$).

We define

(1) $g(r) = p(R = r \mod Q)$

and note that

(2) $\int_0^Q g(r+a) \, dr = 1$ for any $a$.

Statement 1: $p(V=v)$ exists for any arbitrary distribution of $U$

Proof:
Because $p(R=r)$ exists, in 7.2;(4) is obtained

$p(V=v|U=u) = p(R=(v-u) \mod Q) = g(v-u)$.

This yields an existing

$p(V=v) = \int_0^Q g(v-u) \, dP(U=u)$

Statement 2: If $p(U=U')$ exists, then $p(V=v)$ is closer to an equality distribution than $p(U'=U')$, except $p(U'=U')$ is an equality distribution itself.

Proof:
Let be

(3) $p(U=U') = \frac{Q}{Q} + a(u)$

where $a(u)$ indicates the deviation from the equality distribution. Then:

(4) $p(V=v) = \int_0^Q g(v-u) \, dP(U'=U') = \frac{Q}{Q} + a(u) + \int_0^Q g(v-u) \, a(u) \, du = \frac{Q}{Q} + b(u)$, where

(5) $b(u) = \int_0^Q g(v-u) \, a(u)$

Case 1:
If $U$ is equally distributed (i.e. $a(u)=0$), then follows obviously $b(v)=0$, i.e. $V$, too, is equally distributed.

Case 2:
If $U$ is not equally distributed, then

\[
\min(a(u)) < \int_0^Q g(v-u) \, a(u) < \max(a(u))
\]

i.e.

(6) $\min(p(U=U')) < p(V=v) < \max(p(U=U'))$

Applied on an arbitrary arrival process, statement 2 says that arrival locations within a fixed interval $Q$ are equally distributed, if the process tends to infinity.

The interval $Q$ may as well be understood as the fixed measuring period.

Let $U'$ denote the distance from any particular arrival point to the termination point of a fixed time interval $Q$ (the arrival point is not necessarily located within this interval).

Then $U'[U'=0,Q]$ also is equally distributed within $[0,Q)$:

(7) $p(U'=x|U'=0,Q) = 1/Q \quad (0 < x < Q)$

May $W$ denote the amount of time, a holding time $H$ originating within the interval $Q$ will survive this interval (see Fig. 3). Then follows:

(8) $P(W>t|U'=0,Q) = \int_0^Q dP(H>t)$

Since $\mathbb{E}[H] = \tau$, for the limit $Q \to \infty$ results the well known formula

(9) $P(W>t|W=Q,U'=0,Q) = \frac{1}{\mathbb{E}[H]} \int_0^Q dP(H>t)$

as to be valid for any regular arrival process.

Fig. 3: Surviving time $W$ depending on the holding time $H$ and the location $U'$ of the originating point.

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