THE SEQUENTIAL PROJECTION ALGORITHM: A NEW AND IMPROVED TRAFFIC FORECASTING PROCEDURE

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ABSTRACT

Recent developments in estimation theory suggested that the quality of the Bell System's traffic forecasts could be improved substantially by using load projections based on Kalman filter theory. This paper reports the development of a single Kalman filter model, the Sequential Projection Algorithm (SPA), for use in projecting loads for all Bell System trunk groups. The model is robust to a broad range of operating conditions and performs substantially better than existing methods.

Specifications have been written for the inclusion of SPA in a standard mechanized forecasting system. A field study of SPA, reported in a companion paper, verified theoretical results and aided implementation procedures.

1. INTRODUCTION

1.1 BACKGROUND

Load projection, the estimation of future traffic loads, is the basis for the planning and administration of the Bell System's trunk network. A high quality projection achieves reasonable levels of two attributes: stability and accuracy. A stable forecast of traffic for a future year differs minimally from view to view. An accurate forecast is correct, on the average, and has the capability to detect and reflect significant changes in traffic patterns. These desirable attributes are often conflicting in the sense that improvement in forecast accuracy may result in poorer forecast stability and vice versa.

Most load projection algorithms in use today by the Bell System were originally developed for manual calculation; they were included in the first mechanized trunk forecasting systems when digital computers became available. The algorithms, some of which were derived in the 1920s, obtain estimates of future trunk group loads by applying some function of end-office growth rates to estimates of current year trunk group loads. However, analysis, simulation, and field observation have shown that the straightforward application of these standard formulas often result in inaccurate and unstable trunk group load projections. Therefore, substantial revisions have often been necessary to improve the quality of the forecasts.

The availability of computerized data processing makes it feasible to investigate the use of modern, statistically-superior projection techniques. The objective is to derive projection algorithms which achieve an inherently superior balance of the accuracy and stability attributes relative to that of the existing methods.

1.2 SUMMARY

Based on the Kalman filter trending model, a new and improved load-projection algorithm has been designed; it is the Sequential Projection algorithm (SPA). The algorithm (i) is computationally efficient, (ii) provides more accurate and more stable forecasts than do existing methods, (iii) performs well in a multitude of operating conditions, and (iv) includes logic for the processing of outlier data. Our studies employed a combination of analysis, computer simulation, and field trial. The analysis provided qualitative insight into algorithm performance, while the simulation made possible a detailed study of the operating environments of engineering interest. The field study, with Illinois Bell Telephone Company (IBT), has allowed us to establish user guidelines, to verify simulation results, to refine the implementation, and to estimate the effort required for the inclusion of SPA in other systems. IBT's implementation of SPA and the field study are discussed in a companion paper.

From analysis and simulation, the estimated potential improvements, relative to existing methods, for one-year 22 percent lower root mean square (RMS) forecast error and a 53 percent increase in forecast stability. Multiyear SPA forecasts have even more impressive results. The field study, with 1 to 4 years of data processed by approximately 20,000 trunk groups, supports these conclusions.

The improved forecasts will result in a significant reduction in trunk requirements for the Bell System. This is because the minimum level of reserve trunk-capacity necessary to compensate for traffic-forecast errors, while maintaining network service at objective levels, is directly related to the accuracy and stability of the traffic forecasts.

Specifications have been written for the inclusion of SPA in the Bell System's Traffic Routing and Forecasting System (now under development). It is expected that SPA will be the standard algorithm used for short-term (1 through 5 years) projections of the Bell System's trunk group loads. We are studying the applicability of this procedure to point-to-point loads and longer-term (greater than 5 years) forecasts.

2. EXISTING BELL SYSTEM PROCEDURES

Most trunk group (except intertoll) loads are projected by a model of the form: 

\[ \hat{A} = \alpha + \beta t \]

where \( A \) is the current-year measured trunk-group load, \( \hat{A} \) is the estimated future load, and \( \alpha \) and \( \beta \) are the trend at \( t = 0 \). This projection factor, derived from trended office growth rates, is applied to the original office, terminating office, and terminating area growth rates. It is assumed that \( \alpha \) is developed from trended office base-year trunk group measurements and, probably, trended office growth rates. Figure 1a illustrates why this technique often results in inaccurate and unstable trunk group load forecasts. We begin by assuming (optimistically) that the office load projections are completely stable so that the estimated values for \( \hat{A} \) remain constant over several forecast years. Even under these ideal conditions, the resulting load forecasts are not only incorrect but also unstable.
conditions, the trunk load forecast is both inaccurate (the forecasts are always higher than the data trend in this example) and unstable (the consecutively numbered views of the fixed target year vary considerably). The inaccuracy is attributable to the application of the aggregate, rather than the trunk group, growth rate. The instability, which would be greater if p were allowed to vary, is caused by the variability of the trunk group's measured loads.

Clearly the accuracy and stability could be improved by trending the trunk group load itself, i.e., using the dashed line in Figure 1b. The reason this approach has rarely been used is that traffic routing and the composition of end offices tend to change frequently, significantly affecting trunk group load levels and growth rates. The instability, which would be greater if p were allowed to vary, is caused by the variability of the trunk group's measured loads.

Thus, the availability of sequential trending algorithms, such as the Kalman filter models, which do not require the storage or manipulation of history files suggests the feasibility of improving the accuracy and stability of trunk group load projections. In fact, simple analysis shows that an equal-weight trend of 3-5 base load measurements can reduce the 1-year forecast RMS error by 25-50 percent.

3. KALMAN FILTER PROJECTION ALGORITHMS

3.1 GENERAL STRUCTURE

Most existing trunk group load projection algorithms assume linear or log-linear relations between the future and current year loads. For example, the equation in Section 2 assumes

\[ x_n = \alpha x_{n-1} + \beta w_n \]

(3.1)

where:

(i) \( x_n \) is an s-vector of true state variables in period n.

(ii) \( \alpha \) is an sxs transition matrix.

(iii) \( w_n \) is an s-vector of random modeling errors, i.e., random deviations of the true process about the assumed linear relation defined by \( \alpha \).

(iv) \( u_n \) is an s-vector of deterministic changes in state.

The one-step projection is given by

\[ \hat{x}_{n+1} = \alpha \hat{x}_n + u_n + w_n \]

(3.2)

where \( \hat{x}_{n+k} \) is an estimate of \( x_{n+k} \) given data vectors \( y_1, \ldots, y_n \) measured in years 1 through n.

The relation which distinguishes the Kalman filter (KF) from other linear estimation procedures is the particular algorithm for computing \( \hat{x}_{n,n} \), i.e.,

\[ \hat{x}_{n,n} = \hat{x}_{n,n-1} + K_n (y_n - H \hat{x}_{n-1,n-1}) \]

(3.3)

The sxs matrix \( H \) defines the relationship between the data variables \( y_n \) and the state variables \( \hat{x}_{n,n} \), i.e.,

\[ y_n = H \hat{x}_{n,n} + v_n \]

(3.4)

where \( v_n \) is a d-vector of measurement errors.

The sxd matrix \( K_n \) can be calculated by the algorithm given in the Appendix or can be pre-specified as a result of offline analysis of the model parameters. The simplicity of relations (3.3) and (3.4) will be apparent when we discuss a simple KF model in the next subsection.

The point here is that \( \hat{x}_{n,n} \), the smoothed estimate of \( x_n \), is derived as the previous one-step projection \( \hat{x}_{n,n-1} \) plus a linear combination (weighting) of the differences between measurements \( y_n \) and the previous estimate (forecast) of these measurements \( H \hat{x}_{n-1,n-1} \). The weights assigned to the difference terms are the appropriate components of the so-called gain matrix \( K_n \). It is also very important to note that \( \hat{x}_{n,n} \) depends on \( \hat{x}_{n-1,n-1}, y_n, H_n \) and \( H_n \), but not explicitly on \( y_1, \ldots, y_{n-1} \). Thus, no data storage is required.

We can give some insight into the effect \( K_n \) has on algorithm performance — without actually describing a specific model. It can be shown that \( K_n \) has terms which are directly proportional to the elements of the covariance \( Q \) matrix, which defines the variability of the true process random vector \( w_n \), and inversely proportional to the elements of the covariance matrix \( R \) of measurement errors \( v_n \). Equivalently, \( K_n \) is directly proportional to the uncertainty (variability) of the true state and inversely proportional to the uncertainty due to measurement variability. Thus, it is the "Q/R" relation which defines the responsiveness of the filter (via the \( K_n \) matrix) to forecast errors (estimated by \( y_n - H \hat{x}_{n,n-1} \)). However, it is the appropriate weight to balance stability and accuracy (detection). As the elements of \( K_n \) increase (by decreasing Q or increasing R) the forecasts become more stable. As \( K_n \) increases (by increasing Q or decreasing R) forecasts become sensitive to (detect and respond to) data which deviate from the assumed deterministic model, represented by the expected behavior of (3.1).

Generally, estimates of R, the covariance matrix of measurement errors \( v_n \), are available. However, Q is not as easily obtained. But, since it is the "ratio" of Q to R which is important, one can select R and empirically determine a Q which produces the desired degree of filter responsiveness to data.

3.2 A SIMPLE TRUNK GROUP MODEL (TG)

We discuss a two-state load projection model, TG. We use the model to illustrate by example the KP mechanism for quantifying the stability versus accuracy tradeoff.

There are two state variables for a given trunk group:

\[ x_n = \begin{bmatrix} x_n \ 0 \end{bmatrix} \]

(3.5)

\[ x_n = \begin{bmatrix} x_n \ 0 \end{bmatrix} \]

(3.5)

Thus, \( s = 2 \). We assume that the true process \( x_n \) behaves (at least in the short run) on the average as a linear process. That is, the future load \( x_{n+k} \) is the current load \( x_n \) plus \( k \) times the slope \( \alpha \). Hence, we define the \( \alpha \) matrix to be

\[ \alpha = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \]
The measurement system is modeled as follows. We assume only one data variable, i.e., \( y_n \) is an estimate of \( y_n+1 \), for \( n = 0, 1, \ldots, N-1 \), with \( y_0 \) and \( y_N \) being the load-measurement error in period \( n \). This implies that \( N = 1 \). Since \( R \) is defined to be the covariance matrix of the measurement error, we can set \( R = \text{var}(y_n) = \sigma^2 \). However, we can normalize \( R \) by any constant (e.g., \( \sigma^2 \)) and then specify \( Q \) in units of that constant. Thus, we can set \( R \) to be \( R \) and it remains to specify the \( 2 \times 2 \) matrix \( Q \) in order that the KF model be complete.

The search for the best algorithm required two steps. The first resulted in the elimination of some "standard" algorithms and some Kalman filter models. Reasons for the initial pruning included computational complexity, lack of data availability, or generally poor quality projections. The second step was a detailed comparison of the good algorithms (remaining from the first step) and the selection of a (single) best algorithm.

4.1 THE APPROACH

It was decided that the selection of the best algorithm could be based on equilibrium (steady-state) performance. (The notion of steady state has no meaning for conventional algorithms because they do not trend data. However, since the Kalman filter is sequential, its performance is sensitive initially to the amount of data processed. Eventually a steady state is reached; that is, algorithm performance becomes insensitive to the amount of data processed and significant weight is given only to the most recent data.) The result is that algorithms are compared at their theoretical best and the testing process is simplified. If the best equilibrium is a Kalman filter, its transfer properties can be determined as a separate design step. (We discuss this step in Section 5.)

Our evaluations focused on those factors which affect trunk group load forecast accuracy and stability. The broadest classification of these factors breaks them down into office characteristics and trunk group characteristics. Specifically, we were interested in the following office parameters: (i) main station (MS) growth patterns, (ii) load variability, (iii) CCS/MS growth patterns, (iv) CCS/MS forecast bias and variability. Also, the following trunk characteristics were of interest: (v) growth patterns, and (vi) measurement error bias and variability. A final key parameter of interest is called (vii) the community of interest factors; it relates the trunk group load in some year, to the trunk group's originating-office load for that year. In summary, it is assumed that trunk group loads (and hence forecasts of these loads) are directly related to the originating and terminating office loads and their associated community of interest factors.

The general approach to evaluating algorithms was to vary end-office, trunk group, and community of interest factors (i)-(vii) over both typical and extreme values for various test networks. By using a simulation to verify analytical results, it was possible to compare the standard algorithms with each other and with Kalman filter algorithms in all operating environments of engineering interest.

4.2 NARROWING THE CANDIDATE LIST

In order to narrow the list of candidate algorithms we developed a network of 106 trunk groups. We computed forecast accuracy (average error), precision (variance), stability (view-to-view variance), and RMS error. These statistics were computed over 30 years of realizations in order to assure statistical significance.

4.2.1 STANDARD ALGORITHMS

The two best algorithms were \( \text{(A+B)/2} \) and \( \text{AB/T} \). However, our studies showed that in most, but not all, cases \( \text{(A+B)/2} \) had significantly lower RMS error than \( \text{AB/T} \). Similar statements, supported by analytical work, can be made about accuracy, precision, and stability. These results are summarized in Figure 4.
forecast (because of the use of aggregate growth rates) and unstable forecasts (because trunk group base loads are not trended).

### 4.2.2 Kalman Filter Algorithms

We considered and tested several Kalman filter models. The more complex models had as many as \( s = 8 \) state variables and \( d = 4 \) data variables. In addition to trunk group load, state and data variables included combinations of (originating and terminating) office loads, office growth rates, and community interest factors. Some models were linear while others were log linear.

In summary, none performed consistently or significantly better than the simple 2-state, 1-data variant of TG. To ensure that the variability of the base load is a significant parameter. To ensure that the variability has a major impact on facility stability, the variation of consecutive forecasts in these cases was clear; outlier data in the first few years had disrupted the trend-

an additional benefit of TG relative to the standard algorithms and those Kalman filters modeling office characteristics is that TG is not susceptible to the effects of office load forecast bias.

Finally, the simple TG equations were assumed to be easier to implement and maintain than the more complex Kalman filter models. Thus, we select TG as the best Kalman filter model.

### 4.3 Comparing TG and \((A+B)/2\)

In order to compare TG and \((A+B)/2\) in more detail, we expanded the test cases and systematized the methods used in the initial screening. Specifically, we focused on two key parameters: correlation and variability.

One of the key assumptions of the standard algorithms (e.g., \((A+B)/2\)) is that trunk group load is a function of the originating and terminating office loads. To the extent that the trunk group growth rate is accurately modeled by the assumed formula, then the performance of the standard algorithm will depend primarily on the correlation, \( \rho \), between the trunk group and its associated offices. Thus, it was necessary that the test cases in the final testing sample a wide range of the possible correlations. We considered \( -0.35 \leq \rho \leq 0.45 \).

The trunk group filter model predicts the future growth for each office from its past trend unless information on planned network changes is available. Since the base load is the only input to TG, the variability of the measured base is a significant parameter. To ensure that TG is tested over a wide range of trunk group variability, a statistic is needed to index the variability of a base load time series. Since growth in the base load would distort a simple variance calculation, a linear least squares estimate (LLSE) of the base load is subtracted from the actual series of base load to give a new series with mean zero. Then this series is normalized by dividing through the LLSE of the base load. The standard deviation of this series is a percentage. We will call this coefficient of variation \( \text{STD} \).

We first tested the assumption that high values of \( \rho \) will give "better" forecasts than will low values when using the existing methods. If true, one would expect to see a decrease in the RMS forecast error of \((A+B)/2\) as \( \rho \) is increased. This was not the case. This seems to be because the variability of the base load is the major contributor to the RMS error of \((A+B)/2\).
are discussed further in [5]. SPA has five major components: algorithm initiation and transient response; outlier detection and correction; estimation of future deterministic events; a data trending (smoothing) algorithm; and a projection algorithm. In addition to the projection and transient response designs combine to provide a capability for compensating for missing data. The logical interaction of these components is illustrated in Figure 7.

5.1 DATA TRENDING AND PROJECTION

The trunk group Kalman filter model TG, described in Sections 3 and 4, comprises the trending and projection functions of SPA. Recall that the one-year forecast of the base load is used a year later to update the data trend. The output of the trending algorithm is the "smoothed" estimates of the base load and the base-load growth increment.

5.2 INITIATION AND TRANSIENT RESPONSE

In some Kalman filter applications the transient period is very short relative to the total time the filter is in operation. In trunk forecasting this is not true since the average length of time a trunk group remains in uninterrupted service (i.e., no rehoming or area transfer) is generally less than 10 years. Moreover, a filter algorithm generally will not remain in uninterrupted operation even throughout this time period; highly variable or outlier data may require that the algorithm be reinitiated. SPA is designed so that its transient response is at least as good as that of \((A+B)/2\) and so that it achieves its steady-state (and gives good variance reduction) as quickly as possible.

5.2.1 INITIATION

Our objective was to assure that the performance of SPA always be at least as good (in the sense of any of the aforementioned attributes) as that of existing algorithms. We achieved this objective by initiating SPA with statistics derived from \((A+B)/2\), the best of the existing algorithms.

5.2.2 TRANSIENT DESIGN

Estimates of the "year zero" trunk group load \(X_{0,0}\) and its increment \(\dot{X}_{0,0}\) must be supplied to the filter in starting values. Moreover, the initial estimation-error covariance matrix, \(M_{0,0}\), is required.

We used a combination of Kalman filter theory and computer simulation to derive estimates of these quantities. The theory detects inconsistencies among modeling assumptions; the theoretical forecast RMS error (derived from the \(R, Q, H, P\), and \(M_{0,0}\), matrices) should not increase as more data are processed. The computer simulation allowed us to compare in detail the transient responses of our various designs for SPA with each other and with \((A+B)/2\). To obtain statistically significant estimates of the forecast accuracy, stability, and RMS error for each time series characterization, these statistics were averaged for each base year across a test network with 84 statistically identical trunk groups. In the transient analysis, only the first 10 years of filter operation were studied.

Parameters crucial to the design of SPA's transient response were

(i) trunk group measurement error variance (ranging from 8-50 percent) and
(ii) office growth rate bias ranging from 0 to 5 percent.

A partial summary of the results is shown in Figures 8 and 9. In Figure 8, we plot the transient instability of the response of the one-year forecasts of SPA and \((A+B)/2\) for a 10-year span. Note that the stability improvement relative to \((A+B)/2\) occurs as soon as two data points have been processed. In this example there has been a 60 percent improvement in the first year alone. In summary, our results show that SPA will give an immediate improvement in forecast stability and a subsequent improvement in forecast accuracy, relative to \((A+B)/2\).

5.2.3 FILTER GAIN SEQUENCE

According to the filter description in the Appendix and in Section 3.2, the gain matrix \(K_n\) must be calculated each time a trunk group measurement is received.

In Section 4.3.2, especially Figure 5, we indicated how outlier data can degrade the quality of TG forecasts. In fact it is the accuracy rather than the precision component of RMS error which is most affected by outlier data. Thus, in order to improve, or keep outlier data from degrading forecast quality, we include this screening capability in SPA.

The technique is to compare each base load measurement with the one-year forecast of that load. Significant differences, typically in excess of 25 percent are initially labeled outliers. Actual thresholds for outliers depend on typical measurement and forecast error characteristics and on the ability of SPA to recover from data included erroneously in the trending process.

If an outlier is detected, special precautions are taken to insure that erroneous data do not have an adverse effect on forecast accuracy. However, outlier data cannot be deleted automatically, since they may signal an actual and significant change in the overall trend (growth rate).

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After a new data point has passed the outlier screen, the trending algorithm updates the estimate of current base-load and load growth increments. The projections of the future loads are then obtained by extrapolating the current estimates and including the effects of any deterministic events.

5.4 DETERMINISTIC EVENTS

Deterministic events fall into two categories: those related to network rearrangements (e.g., rehoomings) and nonnetwork nonrecurring events (e.g., area transfers of main stations). The impact of network rearrangements is determined by the forecaster in exactly the same manner as is done today with existing methods. The impact of nonnetwork deterministic events is determined from main station (HS) forecasts and CCS/MS forecasts. These impacts affect load projects via the \( u \) term in (3.2). The procedures used in SPA are new but similar in philosophy to those used today.

6. CONCLUSIONS AND FUTURE WORK

SPA has been designed to operate with essentially the same data that is used by existing methods. In addition, it provides the forecaster with a tool for detecting and correcting erroneous data and compensating for missing data. Under normal operation, our theoretical and simulation results predict that SPA will perform significantly better in terms of stability and have somewhat better accuracy than the existing methods. Moreover, when outlier data occur, SPA has been designed to perform at least as well as the existing methods.

Specifications have been written for the inclusion of SPA in the Bell System's Traffic Routing and Forecasting System. A field study of SPA at Illinois Bell Telephone Company (IBT) is also underway. IBT completed the implementation of SPA in March, 1978, and they are using it in parallel with their existing procedures. The preliminary field trial results are documented in a companion paper.[5]

SPA is also being tested by IBT for the projection of point-to-point data. However, the general application of Kalman filtering for projection of point-to-point loads is still under study. The problem is similar to trunk group load projection; however, the large proportion of very small, highly variable loads may require filter models which differ from those already analyzed.

7. ACKNOWLEDGMENT

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REFERENCES


APPENDIX A

CALCULATION OF KALMAN FILTER GAINS

The basic equations of the Kalman filter algorithm are given by (3.1) where \( Q = E[\omega_n^2] \), and (3.5) with \( R = E[\epsilon_n^2] \). The standard assumptions are:

- \( E(\epsilon_n) = E(\omega_n) = 0 \) for all \( n \), \( E[\omega_n^2] = 0 \) for all \( n \neq \ell \), and
- \( E[\epsilon_n^2] = 0 \) for all \( n \neq \ell \).

The smoothed estimated \( \hat{\text{x}}_{n,n} \) is calculated for \( n \geq 1 \) as:

\[
\begin{align*}
\hat{x}_{n+1,n} &= \hat{x}_{n,n} + U_n \\
n &\neq n + 1 \\
K_n &= M_{n,n-1}^T (H M_{n,n-1} H^T + R)^{-1} \\
M_{n,n} &= [I - K_n H] M_{n,n-1} \\
M_{n+1,n} &= \hat{x}_{n,n}^T + Q \tag{a} \\
\hat{x}_{n,n} &= \hat{x}_{n,n-1} + K_n (y_n - H \hat{x}_{n,n-1}) \tag{b}
\end{align*}
\]

When \( n = 0 \), the user supplies the (best) initial estimates \( \hat{x}_0,0 \) of the state variables \( x_0 \). In addition, an initial mean square error matrix \( M_0,0 \) is provided, carefully selected so as to minimize the error characteristics of the initiation transients.

![Data Forecast](image1)

![Stability Accuracy](image2)
LOADS MEASURED BEFORE AND AFTER RENOMING OF AREA TRANSFER

FIGURE 2 LOAD PROJECTION PROBLEM

\[ x_n = \text{TRUE SOURCE LOAD AT TIME } n \]
\[ y_n = \text{MEASUREMENT AT TIME } n \]
\[ \hat{x}_{n+k} = \text{AN ESTIMATE OF } x_{n+k} \text{ GIVEN } y_1, \ldots, y_k, k \geq 0. \]

FIGURE 3 LOAD PROTECTION USING THE KALMAN FILTER

\[ \text{TREND LINE } T_n \text{ WITH SLOPE } b_n, \]
\[ y_n = b_n y_{n-1} + a_n \]

FIGURE 4 PERFORMANCE OF (A+B)/2 RELATIVE TO AB/T

\[ \text{PERCENT OF TRUNK GROUPS} \]
\[ \begin{array}{cccc}
60 & 50 & 40 & 30 \\
41\% & (A+B)/2 & = & AB/2 \\
\end{array} \]

FIGURE 5 RMS ERROR OF (A+B)/2 AND TG AS A FUNCTION OF STD

FIGURE 6 INSTABILITY AS A FUNCTION OF STD FOR TG AND (A+B)/2

FIGURE 7 SEQUENTIAL PROJECTION ALGORITHM (SPA)
FIGURE 8 TRANSIENT ANALYSIS-INSTABILITY OF THE ONE-YEAR FORECASTS FOR A TRUNK GROUP WITH 25 PERCENT MEASUREMENT ERROR

FIGURE 9 RMS ERROR OF THE ONE- AND FIVE YEAR FORECASTS OF TG WITH INFINITE AND TEN-YEAR GAIN SEQUENCE.

FIGURE 10 STABILITY IMPROVEMENT: SPA AND TG RELATIVE TO (A+B)/2