MAINTENANCE INTERVAL FOR A FULL AVAILABILITY GROUP OF DEVICES IN A LOSS SYSTEM

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ABSTRACT
A full availability group of devices with random hunting in a loss system is considered. There is a fixed time - the maintenance interval - between moments of maintenance. Defective devices remain in service until replacement or repair at the next moment of maintenance, but their holding time distribution differs from that of the operational devices. The probabilities of loss and of seizing a defective device are calculated as functions of time. Depending on the requirements, these probabilities in combination with economic considerations can be used to determine the maintenance interval.

1. INTRODUCTION
The problem of how the traffic performance of a group of devices is influenced by the presence of non-intermittently defective devices has received considerable attention in literature.

We shall restrict ourselves here to full availability groups in loss systems where defective devices remain in service for some time, i.e. they can be seized by a call. They have an average holding time which is shorter or longer than that for operational devices. Defective receivers, for example, may have a longer holding time than operational ones because the normal holding time is usually shorter than the time which elapses before a time-out or a hang-up of the calling party. On the other hand, for a speech device the normal holding time is usually longer than the time which elapses before a hang-up of the caller.

An important aspect is the hunting mode, as Klimontowicz (Ref. 1) and Forys and Messerli (Ref. 2) pointed out. This is especially clear in the case of sequential hunting when there are one or more defective devices in the neighbourhood of the starting position, with a holding time shorter than the holding time for operational devices. The defective ones will then more often be seized than in the case of random hunting or cyclic hunting with a random starting position.

For random hunting and a given number of defective devices, explicit expressions are known (Ref. 3 and 4) for the probability of loss (i.e. the proportion of calls arriving when all devices are occupied) and the probability of occupying a defective device (i.e. the proportion of calls which occupy defective devices). Other relevant quantities can directly be obtained from these probabilities.

When, after maintenance at time $t=0$, all devices are operational, the failure rate of the devices yields a relation between elapsed time and the number of defectives (Ref 5). This enables us to formulate the above-mentioned probabilities as functions of time. Service requirements expressed in these probabilities in combination with economic considerations, then may lead to determination of the maintenance interval (i.e. the time between two consecutive moments of maintenance). Some possible service requirements are discussed and numerical examples are given.

2. MODEL
The following notation is used:

- $n$ = number of lines (devices of the full availability group under consideration)
- $s$ = calling intensity = average number of calls per time unit
- $h_o$ = average holding time of an operational device
- $h_d$ = average holding time of a defective device
- $a$ = traffic offered = $s h_o$
- $a_d = s h_d = a o h_o$
- $D$ = number of defective devices
- $p(i,j|D)$ = probability that $i$ operational and $j$ defective devices are occupied, when there are $D$ defectives
- $\lambda$ = failure rate of a device
- $T$ = maintenance interval (from time $t=0$ until $t=T$)
- $f_t(D)$ = probability that $D$ devices fail within $(0,t)$
- $P'_t(D)$ = probability of loss at time $t$
- $P_D(t)$ = probability of occupying a defective device at time $t$
- $P_{D0}(t)$ = probability of no success at time $t = P_D(t) + P'_t(t)$
- $P(t)$ = probability of all operational devices being occupied at time $t$
- $N_n(a) = \sum_{i=0}^{n} \binom{n}{i} (a^i/\lambda^i)$
- $E_n(a) = (a^n/n!)/N_n(a)$

The dimensions of $s$, $h$, $k$, $\lambda$ and $T$ are such that $a_o$, $a_d$ and $\lambda T$ are dimensionless (If $1/\lambda$ is expressed in the...
number of operations, it should first be converted to time by taking the average traffic per day into account).

The following assumptions are made:
- Poisson traffic of $a_0$ erl. is offered to the group of $n$ devices (as shown in Ref. 4, this could easily be generalized to the Engset, binomial or negative binomial case)
- Random hunting is applied
- Calls finding all devices busy are lost and not repeated
- Calls seizing a defective device are not repeated
- All devices are operational at $t=0$ (the moment of the latest maintenance) and, independent of each other, have a negative exponentially distributed time until failure with mean $1/\lambda$

Ref. 4 shows that, independent of the form of the holding time distributions,

\[ p(i,j|D) = \frac{C_D^{n-D}}{D^{n-D}} \frac{a_0^{i-j} \cdot a_0^j}{i^i \cdot j^j} \]  

(1)

where

\[ C_D = \sum_{i=0}^{n-D} \frac{D!}{i!} \frac{D^i}{i!} \]  

(2)

The last assumption implies that for each device the probability of failing within $(0,t)$ is $1-e^{-\lambda t}$. Hence we have

\[ f_t(D) = \frac{D!}{D^{n-D}} (1-e^{-\lambda t})^D \cdot e^{-\lambda t(n-D)} \]  

(3)

With (1) - (3) we can find the following probabilities at time $t$:

\[ P_L(t) = \sum_{D=0}^{n} f_t(D) \cdot p(n-D,D|D) \]  

(4)

\[ P_D(t) = \sum_{D=0}^{n} f_t(D) \cdot \sum_{j=0}^{n-D} p(i,j|D) \cdot \frac{D^j}{j^j} \]  

(5)

\[ P_{NS}(t) = P_L(t) + P_D(t) \]  

(6)

\[ P_O(t) = \sum_{D=0}^{n} f_t(D) \cdot \sum_{j=0}^{n-D} p(n-D,j|D) \]  

(7)

In practical situations $\lambda T < 1$ and therefore the mode of the binomial distribution (3) lies in the neighbourhood of $D=\lambda T$ for each $t$ in $(0,T)$. This means that the mode often lies at $D=0$ or $D=1$, and then the terms with $D>2$ are usually unimportant in the series (4), (5) and (7) if $h_o/h_d$. This leads to an approximation for $P_L(t)$ and $P_D(t)$ based on the first three terms of the series. Details are given in the Appendix.

A fundamental remark should be made here. Probabilities $p(i,j|D)$ are derived from the equations of state which are valid for the assumed statistical equilibrium in the busy hour. In $f_t(D)$ time $t$ may be any moment in $(0,T)$. In $P_L(t)$ etc. these probabilities are combined so that, strictly speaking, formulas (4) - (7) are valid only for values of $t$ in the busy hours. Of course, from a practical viewpoint this remark is not important for the determination of $T$ and we shall take (4) - (7) as being valid for any $t$ in $(0,T)$.

3. NUMERICAL EXAMPLES

$P_L$, $P_D$, $P_{NS}$ and $P_O$ have been calculated as functions of $\lambda t$ for two cases:

A. A group of 5 lines with loss probability $= 0.001$ at $t=0$. This means that $a_0 = 0.76$ erl. The ratio $h_o/h_d = 0.01$.

B. A group of 10 lines with loss probability $= 0.01$ at $t=0$.

Hence $a_0 = 4.46$ erl. $h_o/h_d = 100$

The results are given in Fig. 1. Usually $10^{-4} < \lambda T < 10^{-1}$ (e.g. $\lambda = 10^{-6}$/hr then implies that $T$ is between $10^2$ and $10^5$ hrs, which is certainly true) and therefore the range of $\lambda t$ has been restricted to $(10^{-4}, 10^{-1})$.

Fig. 1: $P_L$, $P_D$, $P_{NS}$ and $P_O$ as functions of $\lambda t$ for two cases A and B

The curves and formulas (1) - (7) show the following facts:

*) This function is defined to be $\infty$ if both $j$ and $D$ are $n$
Because $P_L(t)$ is monotonically changing from $P_L(0) = E(a) = E(a_d)$ to $P_L(\infty) = E(a)$, we find that $P_L(t)$ increases in case A and decreases in case B. $P_D(t)$ increases monotonically, i.e. from $P_D(0) = 0$ to $P_D(\infty) = 1 = E(a)$. $P_N(t)$ increases monotonically from $E(a) = 1$ to $P_N(\infty) = E(a)$.

Between $t=0$ and $t=T$, $P(t)$ can either be monotonically increasing, as in case A, or decrease first to a minimum and increase to 1 for greater values of $t$, as in case B. The actual set of parameter values determines which of the possibilities applies. The following argument shows the possible decrease of $P(t)$:

At $t=0$ the traffic carried by the operational devices is $n\lambda$ erl. if the loss is negligible. If $h_o = h_d$, after a certain time $t$, the $(n-1)$ operational devices will carry $\frac{n-1}{n} \lambda$ erl. If $h_o > h_d$, more calls (than in the case $h_o = h_d$) will find the defective device idle and hence the traffic carried by the $(n-1)$ operational devices will be less than $\frac{n-1}{n} \lambda$. Thus the relative decrease with respect to $t=0$ in traffic carried by the operational devices is greater than the relative decrease in number of devices. It is therefore possible that $P(t)$ decreases for small values of $t$.

4. SERVICE REQUIREMENTS

First there is the usual requirement for the loss probability which should now be interpreted as:

$$P_L(0) \leq (e.g.) 0.01 \text{ or } 0.001 \quad (8)$$

Besides this, one or more other requirements must be formulated which, together with economic considerations, can determine $T$.

It is clear from example B that $P(t)$ with its possible non-monotonicity cannot be used for these requirements, although it has some intuitive appeal. Furthermore $P_L(t)$ decreases if $h_o > h_d$ and this makes $P_L(t)$ as such also unsuitable.

Some possible requirements seem to be:

a. $P_D(t) \leq H_1$ for all $t \in (0, T)$

b. $P_N(t) \leq H_2$ for all $t \in (0, T)$

Here $H_1$ and $H_2$ are given constants.

If the number of devices is the smallest compatible with (8), and requirement a. and/or b. lead to an uneconomically small value of $T$, it can be investigated whether a moderate increase of $n$ can help to get an acceptable value of $T$. If this is not the case, the requirements or the maintenance philosophy have to be reconsidered.

As an example, a requirement in case B could be:

$$P_N(t) \leq 0.02 \quad \text{for all } t \in (0, T)$$

According to Fig. 1 this implies that $AT \leq 0.0063$. If $\lambda = 10^{-6}$/hr, $T$ thus has to be $\leq 6300$ hr, i.e. $\frac{1}{4}$ months.

5. CONCLUSIONS

A full availability group of devices with random hunting in a loss system is considered. Because defective devices remain in service until the next moment of maintenance, there is a probability of seizing a defective device. This probability is used jointly with the probability of loss to form the basis for service requirements. Calculation of these probabilities then makes it possible to determine the maximum maintenance interval compatible with these service requirements. Some numerical examples are given.

6. REFERENCES

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APPENDIX

Formulas for $C_0$, $C_1$ and $C_2$

From (1) and (2) we derive:

$$C_0 = \frac{1}{n} \sum_{i=0}^{n-1} a_i^{(n)} = \frac{N_n(a)}{N_0(a_o)}$$

$$C_1 = \frac{1}{n} \sum_{i=0}^{n-1} a_i^{(n-1)} = \frac{N_n(a_0)}{N_0(a_o) N_n(a)}$$

$$C_2 = \frac{1}{n} \sum_{i=0}^{n-1} a_i^{(n-2)} = \frac{N_n(a_0)}{N_0(a_0) N_n(a)}$$

When $P_L(t)$ is approximated by the first three terms of (4), we get:

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$P_L(t) = f_t(0) \cdot B(n,o|o) + f_t(1) \cdot p(n-1,1|1) +$

$+ f_t(2) \cdot p(n-2,2|2) + \ldots$

$a_n \frac{a_{n-1}}{o} f_t(1) + C_1 \cdot a_d \cdot (n-1) + \ldots$

$a_2 \frac{a_{n-2}}{o} f_t(2) + C_2 \cdot \ldots$

$= f_t(0) \cdot E_n(a_0) + f_t(1) \cdot a_d \cdot \frac{E_{n-1}(a_0)}{K_1} +$

$+ f_t(2) \cdot a_d \cdot \frac{E_{n-2}(a_0)}{K_2}$ (A1)

where $K_1 = n+a_d \cdot (1-E_{n-1}(a_0))$

$K_2 = (n-1) \cdot (n+2a_d) + a_d^2 \cdot 2 \cdot a_0 \cdot (ad+n-1) \cdot (1-E_{n-2}(a_0)) +$

$+ a_0^2 \cdot (1-E_{n-3}(a_0)) \cdot (1-E_{n-2}(a_0))$

In the same way from (5):

$P_D(t) = f_t(1) \cdot \frac{1}{i=o} \cdot \frac{1}{p(i,0|1) \cdot \frac{1}{n-1} +}$

$+ f_t(2) \cdot \frac{1}{i=o} p(i,j|2) \cdot \frac{1}{n-1-j}$

$= f_t(1) \cdot C_1 \cdot E_{n-1} + \ldots$

$+ f_t(2) \cdot C_2 \cdot \ldots$

$= f_t(1) \cdot \frac{1}{K_1} \cdot 2f_t(2) \cdot a_n \cdot a_d \cdot \frac{1}{n-1} + a_d \cdot (1-E_{n-2}(a_0)) \cdot \frac{1}{K_2}$ (A2)

The coefficients of $f_t(0), f_t(1)$ and $f_t(2)$ in (A1) form a decreasing sequence if $a_d < a_0$, i.e. if $h_d < h_0$.

They are all $= E_n(a_0)$ if $h_d = h_0$, and they form an increasing sequence if $h_d > h_0$. This can easily be proven with the recurrence relation for the Erlang function $E_n(a)$.

For (A2) we can check that the coefficient of $2f_t(2)$ is always smaller than the coefficient of $f_t(1)$ if $h_d < h_0$.

It was stated in Section 2 that in practical situations the terms with $D > 2$ in the distribution (3) are usually (much) smaller than $f_t(2)$. Combined with the two foregoing paragraphs, this means that (A1) and (A2) are satisfactory approximations of $P_L(t)$ and $P_D(t)$, respectively, if $h_d \leq h_0$.