SERVICE PROTECTION FOR SPECIAL TRAFFIC FLOWS

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ABSTRACT.

The introduction of new services with a traffic behaviour characterized by heavy intensity variations requires protection of the traffic with a normal traffic profile. Two systems of service protection, i.e. SPH and PRS, are dimensioned on the basis of performance criteria. The performance criteria are obtained by modeling traffic intensity probability density function (pdf) for the explosive traffic, transforming this pdf with a repeated attempt model, and weighing blocking and loss probabilities through the transformed pdf.

If the explosive traffic intensity is assumed to be a slow varying quantity (quasi stationarity), SPH and PRS have been evaluated on the basis of a fictitious measure: the fractional number of circuits needed. In case this assumption of stationarity does not hold the two service protection methods are compared on the basis of the response of the blocking probabilities in time, when a step function in call intensity is offered to SPH and PRS respectively.

Experiments show that, on traffic theoretical grounds, PRS provides about the same service protection as SPH with generally one circuit less.

1. INTRODUCTION.

In the near future some new services are planned to be introduced in the Netherlands telephone network. For these services (e.g. free phone service) the tariff charging will differ from that of normal traffic. In combination with traffic of other existing services with a special tariff, it will be separated from the normal traffic handling. This traffic will be referred to as SCS traffic, traffic from Special Charging Services. The SCS traffic will be routed to a regional SCS exchange from toll and regional centres (figure 1).

From the SCS exchange the traffic will be sent to their respective destinations. By the nature of the several services, the SCS traffic will consist of parts with a different traffic behaviour. Two types of behaviour can be distinguished. The first type shows a normal traffic profile (e.g. semaphone, time announcement). This traffic will be referred to as non-explosive traffic (EN). The other part of the SCS traffic will show a behaviour with heavy intensity variations (e.g. free phone service, news, weather forecast) to be denoted by the term explosive traffic (X).

The amount of SCS traffic that is expected within a few years, covers a range up to 90 Erlang for the largest toll centre. However, in the nearby future it is expected that the total SCS traffic (EN + X) will be smaller, with a relatively large amount of explosive traffic. In a more distant future the total traffic will correspond with the upper values of the range mentioned, the explosive traffic will relatively have been decreased.

When the two types of traffic are offered to a common trunk (N) during peak values of the X traffic there will be a decreasing traffic carrying capacity for the EN traffic. In other words with extreme X values, the probability of loss for the EN traffic increases (*). As the type X traffic is given a lower priority than the EN one, the service to the EN traffic must be protected against surges in the X type (service protection).

There are several publications in which service protection methods have been discussed ([1], [2], [6]). These studies concern the service protection of final traffic when it is offered together with overflow traffic from other routes to a final trunk.

Our problem is slightly different: the calls from both traffic sources are routed direct to the same trunk, but the intensity of one source shows heavy variations.

In this paper two methods of service protection are compared on the basis of circuits needed (after determining the system parameters of the respective service protection methods).

2. SERVICE PROTECTION METHODS.

The two methods of service protection now under discussion in the Netherlands Administration are SPH and PRS.

* PRS-Priority Reservation System ([1], [2], [6]).

Depending on the actual trunk occupation, calls from the explosive traffic source (X) are denied access to a trunk which they share with calls from the non-explosive traffic source, when the number of seized circuits of the trunk exceeds the value of $N - m - 1$.

(*) The same symbol (e.g. X) is used for denoting the call intensity of a traffic source and the type of traffic itself.
**Figures 2a and 2b:** Block diagrams of PRS and SPH.

The common route L is for EN overflow traffic from trunk H and the explosive traffic X (*). In this study the trunks are assumed to have full availability.

**2.1 MATHEMATICAL DESCRIPTION OF PRS.**

Consider a birth-death process \( k(t) \) for \( t \geq 0 \), where \( k(t) \) is the number of occupied circuits on \( t \). With the following state diagram (***) it is easy to derive the differential equations for the state probabilities.

\[
\frac{dP_k}{dt} = a_{k-1}(t)P_{k-1}(t) - (k+a_k(t))P_k(t) + (k+1)P_{k+1}(t) \tag{1}
\]

with:

\[
a_k(t) = \begin{cases} \text{EN}(t) + X(t) & \text{if } k < N-m \\ \text{EN}(t) & \text{if } N-m \leq k < N \\ 0 & \text{if } k = N \end{cases}
\]

and

\[
P_{-1}(t) = P_{N+1}(t) = 0 \tag{2}
\]

With the vector \( \mathbf{P}(t) = [P_0(t), \ldots, P_N(t)]^T \), eq. (1) can be written in vector notation:

\[
\frac{d\mathbf{P}}{dt} = M(t)\mathbf{P} \tag{4}
\]

where \( M(t) \) is a band matrix with one upper and one lower side band.

If statistical equilibrium is considered then

\[
(*) \text{ The same symbol (e.g., } H \text{) is used for denoting the trunk size and the trunk itself.}
\]

\[
(**) \text{ The holding times are Negative Exponential Distributed with mean } 1. \text{ Throughout the paper the mean holding time is used as time unit.}
\]

EN(t) and X(t) are time independent and \( d\mathbf{P}/dt = 0 \). With eq. (1) and eq. (5):

\[
(1) \quad \frac{X}{N} P_k = 1, \quad k=0
\]

the following blocking probabilities can be derived:

\[
B_{en} = P_N = \left( \frac{(EN+X)^N}{N!} \right) \cdot P_0 \tag{6}
\]

and

\[
B_x = \sum_{k=N-m}^{N} \frac{(EN+X)^N}{k!} \cdot \frac{1}{N!} \cdot P_k \tag{7}
\]

where

\[
P_k = \sum_{k=0}^{N-m} \frac{(EN+X)^{N-m}}{k!} \cdot \frac{1}{N!} \cdot P_k \tag{8}
\]

When the X traffic heavy overloads the system then

\[
B_{en} = \frac{P_e^{N-m}}{N!} \tag{9}
\]

This may be considerably smaller than the Erlang blocking probability \( B_{ER} \). The clue is that not only the last \( m \) circuits can be seized by calls from the EN source, but also any free circuit without competition as long as the occupation state \( k \geq N-m \).

**2.2 MATHEMATICAL DESCRIPTION OF SPH.**

Consider the two dimensional birth-death process \( (h(t), l(t)) \) for \( t \geq 0 \), where \( h(t) \) is the number of occupied circuits on the H-trunk and \( l(t) \) is that number with regard to the overflow trunk L both at time \( t \). With the state diagram in figure 3b it is easy to verify that the differential equations of the state probabilities for \( (h|0,1,\ldots,H) \) and \( (l|0,1,\ldots,L) \) are given by:

\[
\frac{dP_{h,l}}{dt} = (h+l+ah(t)+al(t)).P_{h,l}(t) + \begin{cases} 
\text{EN}(t) + X(t) & \text{if } h < H \\
0 & \text{if } h = H
\end{cases} \tag{10}
\]

with:

\[
a_h(t) = \begin{cases} \text{EN}(t) & \text{if } h < H \\
0 & \text{if } h = H
\end{cases}
\]

and

\[
a_l(t) = \begin{cases} \text{X}(t) & \text{if } l < L \text{ and } h < H \\
0 & \text{if } l = L
\end{cases} \tag{11}
\]

and with

\[
P_{-1,1} = P_{h,-1} = P_{h+1,1} = P_{h,L+1} = 0 \tag{12}
\]
By defining the hypervector \( \mathbf{P}(t) = (P_0, \ldots, P_L)^T \) with \( P_i = (P_{i0}, \ldots, P_{iH})^T \),
\[ \frac{d\mathbf{P}(t)}{dt} = \mathbf{M}(t) \mathbf{P}(t) \] (14)

\( \mathbf{M}(t) \) is a band matrix with two upper and two lower side bands consisting of the coefficients of \( P_{h+1,l} \) and \( P_{h,l+1} \) and the coefficients of \( P_{h-1,l} \) and \( P_{h,l-1} \) respectively.

In the case of statistical equilibrium \( EN \) and \( X \) are time independent and the differential quotient is equal to zero. Instead of using
\[ \mathbf{M} \mathbf{P} = 0 \] (15)
(which contains one dependent equation), in the stationary case
\[ Q \mathbf{P}(t) = \mathbf{r}_0 \] (16)
will be solved. Here eq. (16) equals eq.(15)
when the last \( H+1 \) equations are replaced by:
\[ r_h = \sum_{l=0}^{H} P_{hl} \begin{pmatrix} \text{EN}_h \ & L \\ \text{J}_j & j=0 \end{pmatrix} \begin{pmatrix} H \\\n_j=0 \end{pmatrix} \] (17)
and \( \mathbf{r}_0 \) is defined by:
\[ \mathbf{r}_0 = (\mathbf{0}, \mathbf{0}, \ldots, \mathbf{0})^T \] (18)
with
\[ \mathbf{r} = (r_0, r_1, \ldots, r_H) \] (19)
the Erlang distribution vector.

The blocking probabilities can be found with eq. (20) and eq. (21) after solving the set of linear equations (16) using a Gauss-Seidel iteration procedure
\[ B_{en} = P_{HL} \] (20)
and
\[ B_{x} = \sum_{h=0}^{H} P_{hL} \] (21)

When \( X \) is very large, then calls from the \( X \) traffic will occupy all circuits of the \( L \) trunk and \( B_{en} \) will equal the Erlang blocking probability \( B_{en}(H, EN) \).

For SPH and PRS it is now possible to compute both time varying and stationary probabilities as a function of \( X(t) \) and \( X \) respectively.

The problem arises to model the stochastic behaviour of \( X(t) \), or to model the statistics of \( X \), for which the \( B(t) \)'s resp. \( B \)'s are computed and the service protection methods can be compared.

As there are no measurements on the explosive traffic, it is not possible to model \( X(t) \).

Moreover, as concerns the time varying approach, because of the non-linearity of the systems, there will be no unique time responses to characterize the systems.

For this reason, as a first approach, the variations in the \( X \) traffic intensity are considered to be so slow that for a specified \( X \) value a good approximation of the blocking probabilities can be found with (6) - (8) and with (20) and (21) for PRS and SPH respectively (\(*\)).

The comparison of PRS and SPH will be restricted to the normal busy hour (chapter 4 and 5).

To check the results of this (quasi) stationary approach, in chapter 6, an impression is given of the time dependent behaviour of PRS and SPH on stepwise changes in the \( X \)-intensity.

\(*\) Although approximate calculation methods exist, we prefer exact calculation because the differences between the two methods are expected to be small.

3. QUASI-STATIONARY METHOD OF COMPARISON.

3.1 SIMPLE METHOD OF COMPARISON.

Comparison of PRS and SPH by considering the quasi-stationary blocking probabilities of both traffic sources as functions of \( X \) seems to be the simplest approach.

First an SPH parameter setting is chosen (in agreement with section 3.2) and next a PRS method is selected with the same total number of circuits and a protection parameter \( m \) in such a way that the \( B_{en} \) \( (X) \) and \( B_{x}(X) \) curves of SPH and PRS respectively are "as close as possible" possible.

The results of such a dimensioning method are illustrated in figure 4a and 4b. (\( EN = 18.75 \) calls per unit of time).

Figure 4a: Blocking probabilities \( B_{en} \)-PRS and \( B_{en} \)-SPH as a function of \( X \) \( (SPH: H=22, L=18; PRS: N=40, m=2) \).

Figure 4b: Blocking probabilities \( B_{x} \)-PRS and \( B_{x} \)-SPH as a function of \( x \), (same parameter setting as in figure 4a).

Figure 4b shows that \( B_{x}(X) \) - SPH is far almost the entire region \( X \) greater than \( B_{x}(X) \) - PRS. On the other hand figure 4a shows that for low \( X \) values \( B_{en}(X) \) - SPH is greater than \( B_{en}(X) \)-PRS. For high \( X \) values this situation is reversed. So no unique conclusion is possible. This situation is symptomatic of this way of comparing the two service protection methods.

For this reason a more advanced method has been developed.

3.2 MORE ADVANCED METHOD OF COMPARISON.

A better way to judge the curves of figure 4 is possible when the various curves are weighed with the probability density function of \( X \) i.e. \( f(x) \). As there are no results from measurements on explosive traffic, \( f(x) \) has to be estimated.

Wilkinson [3] showed that a gamma distribution provides a suitable fit to day-to-day load variations in the busy hour of local exchanges. Moreover, he found a relation between the variance and the mean:

\[ \frac{\sigma^2}{\mu} = C_1 \cdot C_2 \] (22)
with $\sigma^2 = \text{variance of traffic load A in the busy hour}$ and $\mu_A = \text{expectation of this traffic load.}$

For the Netherlands telephone network Effting [4] found:

$$C_1 = 0.104 \text{ and } C_2 = 1.360 \quad \text{(23)}$$

This mathematical model actually refers to normal traffic. Here the same model is chosen for the day-to-day intensity variations (at the same hour) of the explosive traffic $X$.

So

$$f_X(x) = \frac{\lambda(x)}{\Gamma(x)} x^{r-1} e^{-\lambda x} \quad \text{(24)}$$

with

$$\mu_X = \int_0^\infty x f_X(x) dx = r/\lambda \quad \text{(25)}$$

and

$$\sigma^2_X = \int_0^\infty (x-\mu_X)^2 f_X(x) dx = r/\lambda^2 \quad \text{(26)}$$

To account for the heavy intensity variations for a specified $\mu_X$, $\sigma^2_X$ is chosen as follows:

$$\sigma^2_X = f\{0.104 \times \text{A}^{1.36}\} \quad \text{(27)}$$

where $f$ is a factor $> 1$ (e.g. 5, 10 or 15).

The purpose of the service protection principle is to keep $B_y$ low (e.g. for the Netherlands Administration $\leq 5\%$), if necessary at the cost of the $X$ traffic. So the blocking probabilities of the $X$ traffic may be considerably high.

As high blocking probabilities will cause repeated attempts, it is to be expected, that for the explosive traffic, a specific call intensity $X$ (customer demand) will transform into a higher call intensity $Y$, actually offered to the system. For the repeated attempt phenomenon the following model is assumed (figure 5).

$$Y = \frac{X}{1 - \alpha B_y(Y)} \quad \text{(28)}$$

and the inverse relation:

$$X = Y(1 - \alpha B_y(Y)) = m(y) \quad \text{(29)}$$

with

- $\alpha$ = the constant probability that a non-successful attempt will be repeated.
- $X$ = call intensity (demand).
- $Y$ = call intensity (offered).
- $B_y(Y)$ = probability that an attempt (fresh or repeated) will be blocked.

It is supposed that a Poisson process with intensity $X$ will be transformed to a Poisson process with intensity $Y$.

The pdf of $Y$ viz. $f_Y(y)$ is related to $f_X(x)$ according:

$$f_Y(y) = f_X(m(y)) \frac{d m(y)}{dy} \quad \text{(30)}$$

where $m(Y)$ is given by (29).

Note that every other protection method or parameter setting of it will transform $f_X(x)$ into another $f_y(y)$ due to a different $B_y(y)$.

In figure 6 this transformation is graphically represented for an SPH service protection method with $H = 22$ and $L = 18$ (mean EN traffic = 18.75 calls / time unit, $\mu_x = 11.25$ calls / time unit, $f = 10$ so $\sigma^2 = 27.96$, $\alpha = 0.8$).

Figure 6: Probability density functions of $x$ and $y$.

As a consequence of the introduction of the repeated attempt model a distinction has to be made between the blocking probability of $Y$ i.e. $B_y(Y)$ and the probability of loss of the original demand intensity $X$, to be denoted as $V_x(Y)$.

$$V_x(Y) = \frac{X - Y(1 - B_y(Y))}{X} \quad \text{(31)}$$

It is now possible to compute the weighted means for instance and use them for the dimensioning of the protection system. Both service protection methods can be compared on the basis of total number of necessary circuits.

3.3 DIMENSIONING CRITERIA AND REQUIREMENTS.

The following quantities are used as dimensioning criteria:

1. $B_{\text{EN}} = \int_0^{B_{\text{EN}}} f_y(y) dy \quad \text{(32)}$

The expectation of the blocking probability is identical to that of the loss probability for the EN calls.

2. $B_{\text{EN}}(Y)$ with $\int_0^{B_{\text{EN}}(Y)} f_y(y) dy = 90\% \quad \text{(33)}$

3. $V_x(Y) = \int_0^{V_x(Y)} f_y(y) dy \quad \text{(34)}$

Indicating the loss probability of the $X$ traffic weighted over 90% of the probability mass of $Y$.

Criteria (1) and (2) give measures for the protection of the EN traffic, whereas criterion (3) is a measure for the service to the X-traffic.

A demand to $B_{\text{EN}}(Y)$ means that $B_y$ must stay below this value in 90 out of a 100 times. This measure is added because of the positive skewness of $f(x)$.

The fact that $V_x$ is a 90% percentile whereas $B_y$ is averaged over the entire $Y$ range indicates the difference in priority between the EN and the X-traffic.

The Netherlands Administration is interested in the following demands with respect to the performance criteria:

1. $B_{\text{EN}} \leq 2\% \quad \text{(35)}$

2. $B_{\text{EN}}(Y) \leq 5\% \quad \text{(36)}$

3. $V_x \leq C\% \quad \text{(37)}$

where $C$ varies from 2 (2) to 6 percent.
4. EXPERIMENTS.

An experiment can be defined by choosing a set of input parameters (mean SCS traffic \( \mu_x + EN \), ratio \( \mu_x / EN \), f (variance multiplying factor), \( \alpha \)) and by choosing the system parameters \( N,m \) or \( N,H \) of the service protection method under study. The output of the system consists of the measures defined in the previous chapter: \( \text{Ben} \) for \( \mu_x \) (figure 7).

![Figure 7: Block diagram of the experiment system.](image)

Before conducting any experiment, the parameters \( \alpha \) and \( f \) are preset to \( \alpha=0,8 \) and \( f=10 \). In the next chapter the sensitivity of the results for variations in \( \alpha \) and \( f \) will be discussed. This reduces the problem to selecting values for the mean of the SCS traffic intensity and the \( \mu_x / EN \) ratio which are likely to occur in the future.

In an early stage the mean of the total SCS traffic intensity may reach a value of 10 calls per time unit. The explosive part of this traffic is expected to be relatively large so \( \mu_x / EN \) is chosen 1 (experiment I). In a more distant period a regularly occurring amount of SCS traffic intensity can be predicted to be 50 calls per time unit. In this situation the non-explosive part is expected to have grown considerably so \( \mu_x / EN \) is chosen 0.2 (experiment III). The parameter setting of the second experiment has been chosen between these extreme values: mean SCS traffic intensity = 30 and \( \mu_x / EN = 0.6 \) (experiment II).

In table:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>SCS traffic</th>
<th>( \mu_x / EN )</th>
<th>assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>10</td>
<td>0.6</td>
<td>( f=10, \alpha=0,8 )</td>
</tr>
<tr>
<td>II</td>
<td>30</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>50</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Parameter setting for the main experiment.

Conducting an experiment means searching for the minimum value of \( N \) for which the demands defined in chapter 3 (\( \text{Ben} \leq 2\% \), \( \text{Ben}(y90) \leq 5\% \), \( \nu \leq 2,4,6\% \)) are satisfied. So each \( N,m \) or \( N,H \) combination, a number of "observations" (computing criteria values) have to be made (leading to figure 8). The lines drawn in this figure are calculated by interpolation and represent the dimensioning criteria at the required value. It is easy to see which parameter setting of a service protection method satisfies which demand (satisfying all the demands is indicated by the shaded area in figure 8).

![Figure 8: The results of experiment II (mean SCS traffic=30, \( \mu_x / EN=0,6 \), \( f=10, \alpha=0,8 \)).](image)

The SPH parameter setting that fulfills the requirements in which \( C=2\% \), is a trunk of 40 circuits from which 22 circuits have been separated (*). The total result of figure 8 can be represented in table 2:

<table>
<thead>
<tr>
<th>C</th>
<th>N</th>
<th>\text{NORM}</th>
<th>\text{En}</th>
<th>\text{Ben}(y90)</th>
<th>\text{Vx}</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPH</td>
<td>0.2%</td>
<td>60</td>
<td>22</td>
<td>1.49</td>
<td>4.24</td>
</tr>
<tr>
<td></td>
<td>0.4%</td>
<td>36</td>
<td>24</td>
<td>1.53</td>
<td>3.32</td>
</tr>
<tr>
<td></td>
<td>0.6%</td>
<td>30</td>
<td>23</td>
<td>1.61</td>
<td>2.95</td>
</tr>
<tr>
<td>PRS</td>
<td>0.2%</td>
<td>39</td>
<td>2</td>
<td>1.54</td>
<td>4.37</td>
</tr>
<tr>
<td></td>
<td>0.4%</td>
<td>37</td>
<td>1</td>
<td>1.67</td>
<td>4.04</td>
</tr>
<tr>
<td></td>
<td>0.6%</td>
<td>36</td>
<td>1</td>
<td>1.62</td>
<td>3.96</td>
</tr>
</tbody>
</table>

Table 2: Results of experiment II.

Figure 8a shows that although the shaded area for \( C=6\% \) reaches to below a trunk size of 37, there are no suitable \( N,H \) combinations at this level (because of the discreteness in \( N \) and \( H \)). So for \( C=4\% \) and \( C=6\% \) the same number of circuits (38) is necessary. An alternative method would be to define a fictitious measure, namely the fractional trunk size corresponding with the lowest point of the shaded area for the several values of \( C \). Following this method, the results of experiment II can be represented in table 3, together with the results of experiments I and III.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>\text{Ben} \text{traffic}</th>
<th>( \mu_x / EN )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>10</td>
<td>0.6</td>
</tr>
<tr>
<td>II</td>
<td>30</td>
<td>0.6</td>
</tr>
<tr>
<td>III</td>
<td>50</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 3: Results of experiments I, II and III. \( (f=10, \alpha=0,8) \).
In this way the results can more easily be validated against each other, by calculating the difference in fractional circuits between SPH and PRS \( (\Delta N) = N_{SPH} - N_{PRS} \). The results of table 3 are graphically represented in figure 9:

![Figure 9: Difference in fractional circuits between SPH and PRS as a function of combinations of the mean SCS-traffic and the ratio \( \nu_x/EN \) at different values.](image)

From this figure it can be concluded that PRS gives better results than SPH.

Taking into account the typical pattern in \( \Delta N \) for the experiments I, II, III, it has to be considered that the results are influenced by two effects: an increase in mean SCS traffic and a substantial decrease in the ratio \( \nu_x/EN \). How \( \nu_x \) and EN individually change in the experiments can be derived from table 1 (EN increases from 5 (I), 18.75 (II), to 41.67 (III); \( \nu_x \) increases from 5 (I) to 11.25 (II) and decreases to 8.33 (III). Note that the pattern in \( \Delta N \) can be also be recognized in the changes in \( \nu_x \).

In section 5.1 more attention is being paid to this object.

The saving in circuits is generally dependent on the severity of the dimensioning demands. This can be seen when C is varied from 2 to 6%. The less restrictive C is chosen, the higher the difference between SPH and PRS becomes.

The construction of figure 8a required about 2 h 10 min CPU time while for figure 8b 0.5 min CPU time was required (DEC system 10).

5. SENSITIVITY ANALYSIS.

5.1 SENSITIVITY OF THE RESULTS FOR VARIATIONS IN \( \nu_x/EN \).

Similar to the experiments in the previous chapter two other experiments (IV, V) are conducted. In combination with experiment II, experiment IV and V make it possible to investigate the variations in \( \Delta N \) if \( \nu_x/EN \) is varied \((\nu_x/EN \text{ is kept constant})\). The parameter \( \nu_x/EN \) is varied from 1 \((\nu_x/EN = 15)\) in experiment V, via 0.6 \((\nu_x = 11725, EN = 18.75)\) in experiment II, to 0.2 \((\nu_x = 5, EN = 25)\) in experiment IV.

For experiments IV and V the results in fictitious measures are given in table 4.

![Table 4: Results of experiments IV and V. (mean SCS-traffic:30, \( f = 10, a = 0.8 \)).](image)

These results are graphically represented in figure 10.

The difference between SPH and PRS decreases when \( \nu_x/EN \) decreases. However, this decrease does not show a parameter setting for which it would be more advantageous to use SPH instead of PRS. In general the difference between SPH and PRS is dependent on the severity of the demand C.

A comparison of the figures 9 and 10 shows that the decrease in \( \Delta N \) as a function of \( \nu_x/EN \) cannot be found in figure 9. In the experiments V, II and IV, \( \nu_x \) is monotonously decreasing and EN is monotonously increasing, whereas in the experiments I, II and III of the previous chapter, \( \nu_x \) increases first (from 5 to 11.25) and then decreases (to 8.33) while EN monotonously increases (5, 18.75, 41.67).

If a metamodel for \( \Delta N \) is needed, it would probably have been more appropriate to use \( \nu_x \) and EN (separately) as variables instead of the quantities \( \nu_x \) + EN and \( \nu_x/EN \). It can be expected that the influence of \( \nu_x \) will be much greater than that of EN (compare experiments I and III: \( \nu_x \) increases from 5 to 8.33 and EN increases from 5 to 41.67 whereas \( \Delta N \) is about the same).

5.2 SENSITIVITY OF THE RESULTS FOR VARIATIONS IN \( f \) (SEE EQ. (27)).

To investigate the influence of variations in \( f \), two more experiments (VI and VII) have been conducted. Then, together with experiments II, there are three experiments in which the mean SCS traffic \((30), \nu_x/EN \text{ (0.6)}\) and \( a \text{ (0.8)}\), are kept at the same value and \( f \) differs from 5, 10 to 15.

For experiments VI and VII the results are given in table 5.

![Table 5: Results of experiments VI and VII. (mean SCS-traffic:30, \( \nu_x/EN = 0.6, a = 0.8 \)).](image)

This is graphically represented in figure 11.
From this figure it can be concluded that for different values of the factor f, the PRS method satisfies the demands with fewer circuits. This difference between the two methods seems independent on the height of f (for the values of f investigated).

For different f values, the saving in fractional circuits is dependent on the severeness of the demand C.

5.3 SENSITIVITY OF THE RESULTS FOR VARIATIONS IN a.

The experiments VIII and IX reveal the differences between SPH and PRS (at various C levels) when the probability of repetition a is varied from 0.7 (experiment VIII) to 0.9 (experiment IX) while the mean SCS traffic (30), \( \mu_x/EN = 0.6 \) and f(10) are kept constant. This results in:

<table>
<thead>
<tr>
<th>Experiment VIII</th>
<th>Experiment IX</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>PRS</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 6: Results of experiments VIII and IV. (mean SCS-traffic (30), \( \mu_x/EN = 0.6, f(10) \).)

Then, in combination with the results of experiment II, figure 12 can be composed.

From this figure it appears that the difference between SPH and PRS increases with increasing a. This difference is in favour of the PRS method.

An additional experiment reveals that, if a=0, there is no difference in fractional circuits between the two methods. So it may be expected that \( \Delta N \) will be positive for \( a \geq 0 \).

For different a values, the saving in fractional circuits is dependent on the severeness of the demand C.

6. COMPARISON OF TIME RESPONSES OF PRH AND SPH.

In this chapter the conclusion of the quasi-stationary approach - there is a slight difference between methods in favour of PRS - has been checked by comparing the time responses of some SPH and PRS systems on a stepwise change in the call intensity of the X-traffic (EN is supposed to be constant). The SPH systems studied correspond with those for which the parameter settings are obtained from experiments I, II and III with \( C=2\% \) (see chapter 4). They are compared with PRS systems with the same total number of circuits, on the basis of the expected number of blocked calls during the transients of the blocking probabilities \( b_m(t) \) and \( b_a(t) \) respectively. On \( t=t_0 \) a step is made from \( X=X_1 \) to \( X=X_2 \). The responses are measured during the transient times, which are defined as follows:

- \( T_{en} \) or \( T_s \) is the time elapsed from the moment of the step till the time that \( b_m(t) \) or \( b_a(t) \) respectively reaches the stationary end value within 5%.
- On \( t=t_1 \), a step downwards of the same magnitude is made.

After solving the set of linear differential equations (14) or (14), the time-varying blocking probabilities \( b_m(t) \) and \( b_a(t) \) respectively can be computed.

For \( P(t=t_0) \) or \( P(t=t_0) \) respectively the stationary distribution corresponding with the constant EN and X, values has been taken, and likewise the stationary distribution corresponding with EN and X for \( P(t=t_1) \) or \( P(t=t_1) \). For the solution of (4) or (14) a fourth order Runge Kutta procedure has been used.

Figures 13 and 14 give an example of the \( B_{en}(t) \) response for PRS (40, 2) and SPH (40, 22).
instance the X source during a step upwards (+) is determined by (38):

$$t_0+T_x^+$$

$$X^+ = \int_{t_0}^{t_0+T_x^+} B_x(t) \, dt$$

\hspace{1cm} (38)

where $T_x^+$ equals the transient time for a step in X upwards. In the same way the expected number of blocked calls from EN and X source are determined during a step in X upwards and downwards (+).

Table 7: Conducted experiments in the time domain.

| exp| SP method | EN $X_t$ | PRS $Y_t$ | EN $B_{EN}(y_0)$ | EN+ EN | EN | X | X |
|----|-----------|---------|-----------|-------------------|--------|-------------------|
| I  | SPH 17.8  | 5       | 1.7,14    | 1.21              | 3.57   | 1.69               |
|    | PRS 17.2  | 5       | 1.7,14    | 0.69              | 2.15   | 1.33               |
|    | difference SPH-PRS | 0.52 | 1.12 | 0.36 | 0.82 | 0.35 | 0.88 |
| II | SPH 40.2, 18, 75 | 5 | 1.19 | 4.24 | 1.58 | 0.78 | 0.26 | 9.47 | 1.17 |
|    | PRS 40.2, 18, 75 | 5 | 1.17 | 3.57 | 1.13 | 0.64 | 0.28 | 6.95 | 1.27 |
|    | difference SPH-PRS | 0.02 | 0.67 | 0.43 | 0.54 | 0.38 | 0.29 | 8.08 | 1.50 |
| III| SPH 65.45, 41, 67 | 4 | 18 | 0.87 | 3.68 | 2.00 | 1.600 | 1.443 | 5.267 | 3.234 |
|    | PRS 60.3, 41, 67 | 4 | 18 | 0.87 | 3.68 | 2.00 | 1.600 | 1.443 | 5.267 | 3.234 |
|    | difference SPH-PRS | 0.58 | 0.56 | -0.02 | 0.470 | 0.039 | 1.162 | 1.162 |

An impression of the difference in functioning between SPH and PRS, on the basis of criteria obtained using the stationary approach is acquired by subtraction of the criteria values of SPH and PRS (e.g. $B_{SPH}$ (SPH) - $B_{PRS}$ (PRS)).

The same impression for the time domain criteria is obtained when the differences in the expected number of blocked calls are computed for EN, EN+ on the one hand and for $X_t$ and $X_t$ on the other hand. If the difference in blocked calls (between the two methods) for the step upwards and the step downwards is compared (for both types of traffic) the following results can be found.

For experiment I and II the PRS method gives a better protection for the EN traffic while the service given to the X traffic is also better. This is in agreement with the results of the quasi-stationary approach.

For experiment III the PRS method provides a worse service with respect to the X traffic. This outcome is also in agreement with the results of the quasi-stationary criteria.

7. CONCLUSION.

In this study two service protection methods (i.e. PRS and SPH) have been compared on a traffic theoretical basis.

Experiments performed, show that the PRS method meets all the demands with respect to the non-explosive and the explosive traffic with generally one circuit less than the corresponding SPH method.

Sensitivity analysis reveals that the differences between PRS and SPH:

- increase, according as the explosive traffic grows in relation to the non-explosive traffic.
- are insensitive to changes in the variance of explosive traffic intensities (in the region studied).
- increase with increasing probability of call repetition.

Comparison of the time responses of some SPH and PRS systems on stepwise changes in the call intensity of the explosive traffic corroborates the conclusion of the experiments with the quasi-stationary approach.

It is recommended to investigate both systems in a wider sense (e.g. flexibility, cost of installation etc.).

8. ACKNOWLEDGEMENT.

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