ABSTRACT

Telephone traffic during peak-load periods has been extensively researched with a view of improving service and limiting investment and cost of operation.

Equally important, but less researched, is how the total (around-the-clock) traffic is generated. Such research is of special interest for the purpose of selecting local telephone tariffs which - in addition to serving the three mentioned goals - also best serve the communications needs of the public, while providing adequate rate of return.

The need to determine subscribers' distribution by their traffic (per billing period) when estimating repression of, or shift in, local traffic caused by tariff changes, is widely recognized. This paper describes various types of such distributions - observed as well as mathematically defined. The effect on subscriber distributions from lines subscribed-to primarily for the purpose of receiving calls is analyzed. Also discussed is the transformation of subscriber distributions caused by measured tariffs. An overview of types of tariffs, and where used, is given. One section is devoted to traffic distributions, which means the distribution of the total traffic volume by subscribers' monthly usage. This is an important subject, because the traffic's distribution function is part of the formula for computation of traffic repression. In this connection is shown in what usage ranges most of the traffic repression occurs.

1. TRAFFIC GENERATED UNDER FLAT RATE IS THE POINT OF DEPARTURE

When flat rate applies, every subscriber's traffic reflects a basic demand for local calls, which is uninhibited by budget restraints. The mean traffic of a subscriber population may be determined empirically or estimated with the use of standard econometry. Mean flat rate traffic is the point of departure for estimating the effect of measured tariffs, which sometimes are called Usage Sensitive Pricing, USP.

2. MATHEMATICAL FUNCTIONS FOR SUBSCRIBERS' FREQUENCY DISTRIBUTIONS

Subscribers' distribution by their monthly traffic may conveniently be illustrated in graphs where, for the purpose of comparison, traffic-values are related to the subscriber population's mean traffic. Resulting curves may be fitted to some mathematical distribution function, such as log-normal, gamma, or Plessing distribution.

3. OBSERVED SUBSCRIBER DISTRIBUTIONS

Frequency distributions of subscribers have been observed in many parts of the world for more than half a century. This enables us to recognize how distributions get different characteristics from such factors as:

- business vs. residence subscribers
- flat rate (FR) vs. measured service (MR)
- developing vs. affluent societies
- homogenous vs. heterogenous populations
- calls vs. minutes of use

When looking at Figure 1, we must keep in mind that distributions may take on an infinite number of shapes through various combinations of the mentioned factors. Yet, some basic traits are there. The following is a brief overview of the illustrated distributions:

(a) A typical distribution at flat rate for residence subscribers in earlier days. It is closely described by a gamma function with b = 2. (Ref. 5, pp. 128/129)

(b) A typical distribution at flat rate for business subscribers in earlier days. It is closely described by a Plessing distribution with \( \alpha = 1.95, \beta = 0.0077, \) and \( k = 10. \) There is a larger spread of business subscribers traffic around the mean than that of residence subscribers, and the distribution is more skewed.

(c) This diagram shows the typical effect that a straight measured tariff, SMR (no allowance), has on a distribution of type (a). According to the principle of diminishing relative savings (Ref. 5, p. 17) the tail section of the distribution becomes more compressed than the low-traffic range. Consequently, the distribution is more concentrated around the mean traffic.

(d) In certain communities, where the telephone system is in a developing stage, some individual business lines have exceptionally large traffic. Sometimes 'borrowing' or 'peddling' of telephone calls is a contributing factor. This makes distribution (d) more skewed than (b); the subscriber population is more heterogenous - from a traffic standpoint.

(e) This distribution indicates an affluent society where many residence lines have zero or little traffic during certain periods; people are away from home because of travel, vacation, second home, etc.

(f) When traffic is measured in minutes-of-use, instead of calls, the distribution will, of course, be more skewed. If the corresponding subscriber distribution by

FREQUENCY DISTRIBUTION OF SUBSCRIBERS BY THEIR MONTHLY LOCAL TRAFFIC

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calls has the shape of (e), the distribution by minutes will take the shape of (f), which may be closely described by a negative exponential function, $e^{-t}$.

(g) Affluence may also be evident among business subscribers. The distribution may take the shape shown in this diagram, because many lines are subscribed-to for the purpose of receiving calls. A description of the composition of this type of distribution is given in Section 4.

4. 'RECEIVING' LINES VS. 'TWO-WAY' LINES

The vast majority of telephones are subscribed-to for the dual purpose of originating and receiving calls. We may call them 'two-way' lines. If all lines were subscribed-to for this dual purpose, they could be expected to be distributed (by originating traffic) according to some commonly used mathematical distribution function.

When the basic monthly charge is low, relative to cost of other goods, one will find - especially in affluent societies - that quite a few telephones are subscribed-to mainly for the purpose of receiving calls. We could call them 'receiving lines'. Most of them are business telephones; for instance second (or third) lines reserved for incoming traffic, or lines to call for information, or to receive emergency calls, etc. Although the main purpose of these lines is to receive calls, some occasional originating traffic will occur.

It is impossible to draw a distinct dividing line between these two kinds of telephone lines, but an arbitrary break-down of subscribers on the two categories enables us to apply mathematical treatment by superimposing their two distributions. Clearly, the frequency distribution for originating calls from 'receiving' lines is a rapidly decreasing function. If we make the arbitrary postulation that any telephone that originates a traffic exceeding one half of the mean traffic be classified as a two-way line - even if it were subscribed-to primarily for the purpose of receiving calls, we may illustrate the superimposed distribution as in Figure 2.

The curve at the top is the gamma distribution

$$h_1 = 4t e^{-t}$$

(1)

Let us assume that 90% of the subscriber population are two-way lines, distributed in this fashion.

The second curve, representing the 10% of the population made up of receiving lines, is hypothetically assumed to be a declining function of the type

$$h_2 = 10(1 - 2t)^4$$

(2)

This is conjecture at this stage, but telephone companies should have no difficulty establishing factual functions.

The third curve represents the total subscriber population and is obtained by superimposing the two curves above. Please note that the superimposition was not made by simply adding the two ordinate readings. A transformation of coordinates was made in order to reduce the composite distribution to unity.

How should this kind of bi-functional frequency distribution be treated in the process of estimating traffic repression? Repression is computed by integrating the product of the traffic distribution function (which is equal to the subscriber distribution function times the relative traffic "$t$") and the repression function.
5. MEASURED TARIFFS’ TRANSFORMING EFFECT ON DISTRIBUTION OF SUBSCRIBERS BY USAGE

In what way a measured tariff transforms subscriber distributions depends on the configuration of the tariff. (Configuration means the geometrical pattern formed by tariff components in a linear payment/usage diagram.) The Configuration Theory is described in ref. 5. Common tariff components are:

- Basic monthly charge (B)
- Rate per call (U)
- Message unit allowance (A)

Tariff structures may have additional features, such as off-peak-discounted usage rates, etc., but in this paper we’ll concentrate on those three.

An overview of various kinds of local tariffs is given in the Appendix, together with a list indicating some places where they are used.

When trying to establish empirically the transforming effect of tariffs one could use either of the following two methods:

- Observation of individual subscribers’ traffic before and after conversion (5.1)
- Analysis of before-and-after distributions (5.2)

They will be described in the following by applying them to a hypothetical case of conversion from flat rate (FR) to a straight measured tariff (SMR), avoiding the complication of a usage allowance.

5.1. INDIVIDUAL SUBSCRIBERS’ BEFORE- AND AFTER-TRAFFIC

This method involves the recording of individual subscriber lines’ monthly originating chargeable local traffic before and after conversion. It should be noted, that recorded monthly traffic ought to be equated to the average number of day-units per month. This is necessary in order to eliminate the disturbing effect of different number of calendar days, holidays and weekends from one month to another. If different calendar months are compared, the seasonal variation in traffic must also be considered.

When analyzing individual subscribers’ traffic we encounter stochastic effects of two different kinds, viz.:

- Normal transientness (5.1.1)
- Different reaction to tariff changes (5.1.2)

5.1.1. TRANSIENTNESS

The author uses the word transientness to describe not only the fact that some subscribers are transient from one community to another (which counts for part of the so-called subscriber-turnover and its concomitant part-month usage). It also describes the fact that subscribers are transient in different usage-brackets from one month to another and from one year to another. The latter could be due to part- or full-month absence, change in number of household members, change of occupation, other activities, habits, etc. The stochastic effect of transientness is present under all normal conditions (even when no tariff change). It has been reviewed in ref. 5, pp. 126, 174–176, and ref 5, Appendix 6. Also thoroughly analyzed by Carl Pavarini in ref. 8.
Figure 3 shows the stochastic effect of transientness. It does not reflect a recorded case. Its sole purpose is to illustrate (in a crude and exaggerated fashion) the character of transientness.

At the top of the figure is a typical subscriber distribution. It has been arbitrarily divided in five segments, 1 to 5, denoting ascending traffic ranges. Since it appears that -- unless some extraordinary circumstances, such as tariff conversion, takes place -- a subscriber distribution remains practically the same from one period to another, we may now analyze some dictates of the transientness phenomenon.

Take segment 1. Not all subscribers in this segment are bona-fide small users. Some of them landed there as transients during this "one month" because of reasons mentioned earlier. Those subscribers will, most likely, be found having higher usage in "another month". The preponderance of movement will be upward, and the distribution of subscribers in segment 1 will be skewed to the right in any other period (prior or subsequent to the "one month").

By the same token, the distribution of subscribers in segment 5 will be skewed to the left in any other period, because those who are not bona-fide high users will have lower traffic. An upward movement is not possible if the tail of the total distribution is to maintain its position from one period to another.

One can conceive of one segment between 1 and 5, for instance segment 3, where there would be about as much upward as downward movement. It follows then, that intermediate segments will be skewed in either direction, segment 2 upwards and segment 4 downwards.

By superimposing the five distributions in the fragmented diagram, we get a distribution for "another month", which is equal to the distribution of "one month", at the top of the figure.

5.1.2. DIFFERENT REACTIONS TO TARIFF CHANGES

In this context we are not talking about the different degree of reaction between various classes of subscribers, or of the difference in reaction by subscribers having unequal usage demands. We are concerned with the fact that a group of subscribers of the same class, which have identical mean flat rate traffic, still may react very differently to a tariff change, causing a stochastic effect. Since no subscriber has any reason to increase his or her traffic because of switch from FR to SMR, all movements caused by the conversion are downwards.

Using the same distribution and segmentation as in Figure 3, we may use Figure 4 to study the effect of the conversion in the same crude manner as for transientness.

In accordance with the principle of diminishing relative savings, subscribers in segment 1 have very little incentive to reduce their traffic. This segment will therefore have a similar appearance before and after conversion. There may be a slight horizontal contraction for reasons described in 5.2.

According to the same principle, the largest relative traffic repression will occur among subscribers in segment 5. Not only will they move to the left in the diagram, but their different power of reaction to the measured tariff will cause a considerable dispersion, as indicated in the figure.

Subscribers in segments 2, 3, and 4 will show the transition from 1 to 5 with gradually increasing dispersion.

By superimposing the five distributions in the fragmented part of this figure, we get the post-conversion distribution at the bottom of the figure, where for comparison, the pre-conversion
distribution is also shown. Both distributions are shown as functions of the flat rate traffic ($T_{FR}$) and illustrate the tariff's transforming effect. This would permit one to deduce the corresponding repression function. (Ref. 5, p. 51)

Because before-and-after studies of individual subscriber's reaction always show the combined effect of transientness and tariff change, the effect of the latter cannot be determined unless the effect of the transientness is segregated. This is a cumbersome task, requiring separate studies of transientness, both under flat rate and measured service. If distributions of the subscriber population by traffic are available for pre- and post-conversion periods, the mentioned complication can be avoided, and the effect of the tariff may be determined directly, as described in the following.

5.2. ANALYSIS OF BEFORE-AND-AFTER TRAFFIC DISTRIBUTIONS

Suppose that we had established the bottom diagram in Figure 4 by observing aggregate distributions, rather than by observing individual subscribers' repression. Since the effect of transientness is washed out with this method, it is possible to deduce the mathematical function that corresponds to this kind of repression. (Repression function means the after-to-before traffic-ratio by subscribers' monthly preconversion usage.)

As a visual aid in this process we'll convert the subscriber distributions in Figure 4 to traffic distributions (which are used in the repression computation). This is shown in Figure 5. (Top curves multiplied by the traffic)

While the areas encompassed by the two curves in the top diagram are equal (same number of subscribers), the areas of the two bottom curves are unequal. The difference between the areas of the two traffic distributions equals the amount of repressed traffic.

A repression function, $u = f(t_{FR})$, that corresponds to this transformation may be fitted by using the following mathematics:

Coordinates are (the abscissa maintaining the Flat Rate scale):

<table>
<thead>
<tr>
<th>Subscribers</th>
<th>Relative Flat Rate Traffic, $t_{FR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{FR}$</td>
<td>$h_{FR}$</td>
</tr>
</tbody>
</table>

Traffic distribution $t_{FR}$, $h_{FR}$, $t_{c}$, $h_{c}$

According to ref. 5, p. 69, the transformation caused by the repression is expressed by:

$$t_{c} = u \cdot t_{FR} \quad \text{and} \quad h_{c} = u \cdot h_{FR} \cdot \frac{du}{dt_{FR}}$$

Since $Y_{FR} = t_{FR} \cdot h_{FR}$, the transformed traffic distribution has the coordinates

$$t_{c} = u \cdot t_{FR} \quad \text{and} \quad y_{c} = u \cdot t_{FR} \cdot \frac{du}{dt_{FR}}$$

which allows a closer fit.

The result of the comparison will be declining repression function, as in Figure 6.

Figure 6. Repression Function for $FR$ to SMR Conversion

Theoretically, there should be no visible repression among the smallest users, because they have practically no economic incentive to reduce their traffic. (The curve must begin at the level 1.0.) In practice, however, there will be an apparent repression even in the lowest usage range. There may be several reasons for this. In the first place, one cannot rule out some psychological effects. For instance, even small users might be more careful not to dial "wrong numbers" when local calls are paid for. Secondly, there is also an indirect effect of transientness, which may best be described by an example.

Let us assume that a lone subscriber has a demand (at flat rate) for 180 calls per month, or about 6 calls per day. Under measured service he reduces the traffic by 1/3, to 120 calls per month, or 4 calls per day. If he has an accident on the third day of a month and spends the rest of it in a hospital, that telephone may have generated only two days traffic of 4 calls, for a total of 8 calls. Had the accident occurred in a flat-rate month, 12 calls might have been generated. One could think of many other reasons for this kind of statistical effect. What we are saying is that: Transients take their reduced calling habits with them when they occasionally land in a smaller usage bracket. The total effect of this may be very small, but a separate study of this kind of transientness would reveal if it is significant or not.

A third reason is that "repression begets repression," just as "traffic begets traffic". To illustrate this phenomenon we will, again, use an example:

A switch has been made from FR to SMR with a residential tariff of $5 per month and 6 $ per call. As a result, mean traffic was reduced by 1/3. A bona fide small user made an average of 10 calls per month under flat rate. About seven of them were calls initiated by the subscriber, but three were "have-him-call-me-when-he-gets-home" type of calls. If we assume traffic to be randomly distributed at the receiving end, this subscriber's mathematical requests for return calls and would thus make one less call per month; not because of a compelling urge to reduce the telephone bill but from $5.60 to $5.34 (1) but because he had one less request.
Appendix

Overview of Various Types of Local Telephone Tariffs with Indication of Where They Are Used

The information was obtained from the following sources:

For U.S.A.

*Exchange Service Telephone Rates in Effect June 30, 1978, Published by National Association of Regulatory Utility Commissioners, Washington D.C.*

For other countries


(The list is not comprehensive, and in some places the tariff configuration may have changed since the dates of these two publications.)

<table>
<thead>
<tr>
<th>Abbr.</th>
<th>Description</th>
<th>Configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 FR</td>
<td>Flat Rate</td>
<td></td>
</tr>
<tr>
<td>#2 SMR</td>
<td>Straight Measured Tariff</td>
<td></td>
</tr>
<tr>
<td>#3 VMR</td>
<td>Variable Measured Tariff</td>
<td></td>
</tr>
<tr>
<td>#4 CMR</td>
<td>Measured Tariff with Ceiling</td>
<td></td>
</tr>
<tr>
<td>#5 AMR</td>
<td>Measured Tariff with Message Unit Allowance</td>
<td></td>
</tr>
<tr>
<td>#6 AVMR</td>
<td>Variable Measured Tariff with Allowance</td>
<td></td>
</tr>
<tr>
<td>#7 BB</td>
<td>Block Tariff</td>
<td></td>
</tr>
<tr>
<td>#8 NR</td>
<td>No-Rental Tariff</td>
<td></td>
</tr>
<tr>
<td>#9 AMR</td>
<td>No-Rental Tariff with Allowance</td>
<td></td>
</tr>
</tbody>
</table>

Various Kinds of Tariff Configurations

Comments on Where Used

1. FR. Flat Rate
   - Non-optional
   - In New Zealand, The Philippines, Hong Kong, Singapore, Saudi Arabia, Kuwait, etc.
   - In small and/or manual systems in Greece, Iceland, Poland, Spain, Turkey, Japan, India, Bangladesh, South Africa, Zaire, Argentina, etc.
   - In Canada for business in small cities and for all residence.
   - Optional
   - In Connecticut for business and residence, all cities.
   - In many large cities in other states in USA, mostly for business.
   - In Canada for business in large cities.

2. SMR. Straight Measured Tariff
   - Non-optional
   - In most cities in Europe, Asia, Africa, and Australia.
   - In Detroit, Michigan.
   - Optional
   - In New York State and Washington, D.C. for the lowest individual residence tariff.
   - In Reno and Carson City, Nevada for 2-party residence service.

3. VMR. Variable Measured Tariff
   - Optional
   - In Chicago, Ill. Exchange, as the lowest individual residential tariff.

4. CMR. Measured Tariff with Ceiling
   - Non-optional
   - Optional
   - For residence in the same cities in Ill.

5. AMR. Measured Tariff with Allowance
   - Non-optional
   - In large cities in Iceland, India, many Latin American countries, etc.
   - In Washington, D.C. and Central New York City for business.
   - In Ohio (optional FR for EAS), California, New Jersey, Rhode Island, and Virginia for business in large cities.
   - Optional
   - In Connecticut for business and residence in all cities.
   - In New York State, except for business in Central New York City.
   - In large cities in some other states in USA for business or residence.
   - In Canada for business in large cities.

6. AVMR. Variable Measured Tariff with Allowance
   - Optional
   - In Pennsylvania for business EAS in large cities.
   - In Chicago + Inner Metro Exchanges, Ill. for business and residence (except as per #5).
# 7. BR. Block Tariff
Non-optional
- In Finland for manual state-owned systems.

# 8. No-Rental Tariff
# 9. No-Rental Tariff with Allowance
- These two types of tariff are not offered publicly, but they are enjoyed by, for instance, the tariff is SMR or AMR and they are exempt from paying the rental.

SUMMARY
It is shown how subscriber distributions take various shapes, depending on a large number of circumstances; one of them being the type of tariff that applies.

A graphic illustration depicts the effect of a simple conversion from flat rate to a straight measured type of tariff.

A graphic illustration is also given of the two stochastic effects one encounters in this process; one describing how the subscribers play musical chairs with their traffic from month to month; the other how subscribers with identical demand differ with regard to their repressed traffic because of different power of reaction.

A method is suggested to determine the mean reaction by subscribers in all parts of the traffic range, without having to specify these stochastic effects. For this purpose, empirical before-and-after distributions are used, and it is pointed out that traffic distributions lend themselves better for a closer fit than do subscriber distributions.

The resulting repression function might have been expected to be solely the result of budgetary restraints, but some secondary effects appear. Because the total repression is - by far - a result of what the large users do with their traffic, the repression caused by the very small users is, in reality, insignificant. But from a subscriber behavior standpoint it is of interest to know what is going on.

In the appendix, a résumé is given of types of tariffs used in various countries. This is done because the configuration of the tariff is an important factor in determining the effect of measured tariffs.

REFERENCES
(2) Gerald Cohen, Experimenting with the Effect of Tariff Changes on Traffic Patterns, paper 325, ITC 8.
(4) - " - The Configuration Theory, paper 323, ITC 8.