CONGESTION CONTROL IN PACKET SWITCHED NETWORKS BY GRADUAL RESTRICTIONS OF VIRTUAL CALLS

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ABSTRACT

A congestion control technique for international packet switched networks, which strengthens packet flow restrictions to virtual calls in a congested outgoing route in a switch gradually, is proposed. Basic traffic characteristics of the system under the proposed congestion control are analyzed theoretically. A simple queuing model is developed and state probabilities of the system under the congestion control are obtained analytically. Using the probabilities, equations to evaluate the performance of the congestion control such as throughputs and buffer overflow probabilities are derived. Several numerical examples are obtained and which show the effectiveness of the proposed congestion control technique. Packet flow restrictions can be strengthened from virtual calls with low priorities first. By making the number of virtual calls which are controlled simultaneously small, the frequency of repetitions between the activations and terminations of the controls becomes low. This is preferable from decreasing the amount of processing loads. A mechanism for realizing the proposed technique is also discussed.

1. INTRODUCTION

In packet switched networks no physical circuits are established between communicating data terminals, and network resources such as buffers in switches and transmission links are shared dynamically among all the calls which use a network simultaneously. Hence packet switched networks generally have excellent traffic handling capabilities compared with conventional circuit switched networks. On the other hand, they are very sensitive to traffic overloads, however, traffic characteristics change drastically according to load variations. Congestions which are generated locally in a network usually expand to large part of the network. The influence of network congestions appearing as increases of packet transmission delays and decreases of throughputs, network congestions lead directly to degradations of communication qualities of the calls already in connection. Therefore, in order to maintain high quality of communication services in packet switched networks, various kinds of traffic controls are necessary to be performed at all the switches in the networks.

Traffic control techniques are very important subjects, therefore, in operating and administrating packet switched networks. They are also a very interesting field of study academically, and a lot of research activities(1), (2), (3), (4) are being done on the subject lately. Most of the researches, however, are on the networks in which way of packet handling is datagram-type. There are only few studies dealing with the networks in which virtual circuits are established throughout the networks before actual communications take place, such as international packet switched networks.

This paper presents a congestion control technique for international gateway switches. Packet queue lengths for individual outgoing routes in a switch are observed constantly, and packet flows through virtual calls connected to a congested outgoing route are restricted gradually depending upon the queue length to the congested outgoing route. Traffic characteristics are analyzed theoretically and several problems in realization of the technique are also discussed.

2. TRAFFIC CONTROLS AT INTERNATIONAL GATEWAY SWITCHES

As for the protocol between gateway switches, it is agreed internationally to adopt CCITT X.75. X.75 is a virtual call protocol, and it is necessary to establish a virtual circuit between communicating data terminals before actual data transmissions take place. All the packets belonging to a call are transmitted via the same path through international packet switched networks.

Traffic controls in such networks generally consist of the following four individual controls:

- Call blocking control;
- Routing control;
- Flow control;
- Congestion control.

Call blocking controls and routing controls are concerned with establishing virtual circuits, while flow controls and congestion controls with packet flows through the established virtual circuits.

Call blocking controls are to prevent congestion conditions from getting worse by rejecting new calls to establish. Routing controls are to improve the traffic handling efficiency of the whole network by establishing new virtual circuits avoiding congested parts of the networks. For these controls, techniques established for circuit switched networks can be applied.(5)

In protocol X.75, a link-by-link flow control by window mechanism is specified. At each gateway switch, window sizes are given to all the virtual calls considering data transmission rate of communicating data terminals and transmission delays between neighboring switches. Generally, window sizes are determined so as to enable smooth data transmissions from sending to receiving terminals, and they do not have direct correspondence between the amount of buffers the virtual calls use. Taking into account bursty nature and directionality of flow patterns of data traffic, a considerably large number of virtual calls are established on each physical link compared with its data transmission capacity. Hence it is quite inappropriate from the standpoint of efficient buffer utilization that each virtual call is assigned fixed amount of buffers which is equal to the window size, and generally the whole buffers in switches are shared dynamically among all the virtual calls.

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We can save a lot of buffers with this sharing. Under overload situations, however, throughput degradations occur because of the lack of buffers. It is necessary, therefore, to implement some congestion control mechanisms in order to cope with overload situations quickly and improve the traffic characteristics under congestion conditions.

There are two major types in the patterns of causing congestion conditions. The first is the congestions caused by simultaneous packet flows through a lot of virtual calls, each of them being free from flow controls. The second is the congestions caused by several virtual calls to which flow controls are effected. Because of link-by-link window controls, the packets which belong to such virtual calls stay in the switch occupying buffers which can be as large as the window sizes given to the virtual calls. This leads to the decrease in the number of buffers available to the virtual calls which are not flow controlled, eventually causing throughput degradations.

There is a possibility that the second type of congestions is frequently caused when there are many communications in the network which are continuous transmission of large amount of data from high speed hosts to low speed data terminals. For the following reasons, however, the first type can be considered to be the major pattern of congestions:

- High speed hosts will probably establish several virtual calls between low speed data terminals simultaneously;
- Usually only small window sizes which correspond to data transmission rates of low speed data terminals are given to such virtual calls;
- Generally, high speed hosts are notified of the data transmission rates of low speed data terminals at call establishment phase by throughput class indications which will be offered by networks. Therefore, hosts will control their packet transmissions on end-to-end basis.

In this paper, we will consider only the first type of congestions.

3. DESCRIPTION OF THE PROPOSED CONGESTION CONTROL TECHNIQUE

A queuing model of an international gateway switch is shown in Fig. 1. The number of routes accommodated in the switch is \( R \) and route \( n \) (\( n = 1 \sim R \)) consists of \( S_n \) physical links. The size of the buffer in the switch is \( M \). Let \( n \) be the number of packets waiting to be transmitted to outgoing route \( n \) including the ones in transmission be denoted \( i_n \). The proposed congestion control consists of two stages of packet flow restrictions. Two thresholds, \( t_n \) and \( l_n \), are necessary to be set to the queue length for outgoing route \( n \) for the controls. The queue lengths to individual outgoing routes are observed in the switch. When queue length \( i_n \) exceeds threshold \( t_n \), the first stage control is activated. Control packets are transmitted to the previous switches to hold packet flows through \( 1 \) to \( R \) part of virtual calls connected to route \( n \) in the switch. That is, the amount of packet traffic to outgoing route \( n \) is reduced from \( a_n \) to \( a_n' = r_n a_n \) (\( 0 < r_n < 1 \)) with the first stage control. When the queue length, continuing to grow further under the first stage control, reaches threshold \( l_n \), the second stage control is activated. The packet flows from the rest of virtual calls, which are not restricted with the second stage control. When queue length \( i_n \) decreases to be less than the threshold, \( l_n \) or \( t_n \), the respective control is removed. The virtual calls which are the object of the first stage control can be chosen either randomly or according to priorities if they are given to each virtual call.

It is to be noted that queues are not necessarily formed for each outgoing route by some buffer management schemes actually employed in switches. However, the systems in which the number of packets for each outgoing route existing in the switch is always counted can be reduced to the model shown in Fig. 1. When we design the amount of buffers for a switch actually, several buffers to accommodate packets belonging to virtual calls flows of which are restricted by flow controls or congestion controls dispatched by other switches are also necessary to be considered in addition to \( M \) buffers described above. The occupation of buffers by such packets is rather deterministic and necessary amount of buffers can be determined separately.

4. TRAFFIC ANALYSIS

The state probabilities of the system under the congestion control is obtained first in this section. Then the parameters, which are necessary to evaluate the performance of the congestion control technique such as throughputs, buffer overflow probabilities and average packet queue lengths, are obtained using the state probabilities.

Following assumptions are made for the analysis.

1) Packets in the switch occupy one buffer irrespective of packet lengths. That is, \[ \frac{1}{n} i_n \leq M \quad (1) \]
\[ 0 \leq l_n \leq M, \quad n = 1 \sim R \quad (2) \]

2) Packet arrival process is Poisson. Let the packet arrival rate to outgoing route \( n \) be denoted \( \nu_n \).
Then, \[ \nu_n = \{ l_n, \quad i_n \leq t_n - 1 \}
\[ \nu_n = \{ l_n', \quad \nu_n = \{ l_n, \quad i_n \geq t_n \}
(3) \]
where \( r_n \) is the restriction ratio for the first stage control.

3) \( S_n \) links for each route have the same transmission rate. Packet transmission times are exponentially distributed with mean value \( 1/\nu_n \).

The state of the queuing system can be expressed by vector \( \lambda = (\lambda_1, \lambda_2, \cdots, \lambda_R) \). The state transition diagram is shown in Fig. 2. Let the state probability be denoted by \( \pi(\lambda) \). Referring to Fig. 2, we can obtain equilibrium equations by the local balance method \((1)\), \((6)\), \((7)\) for each parameter \( i_n \) as follows:

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\[ P(i_1, i_2, \ldots, i_R) = q_{11} P(i_1+1, i_2, \ldots, i_R) \]
\[ + q_{12} P(i_1, i_2, i_3+1, \ldots, i_R) \]
\[ + \cdots + q_{1R} P(i_1, i_2, i_3, \ldots, i_R+1) \]  
\[ v_n P(i_1, \ldots, i_n, \ldots, i_R) = q_n \mu P(i_1, \ldots, i_n+1, \ldots, i_R) \]
\[ v_R P(i_1, i_2, \ldots, i_R) = q_R \mu P(i_1, \ldots, i_R+1) \]  
where
\[ 0 \leq i_n \leq s_n-1 \]
\[ s_n = \begin{cases} i_n + 1, & i_n \leq s_n-1 \\ i_n - s_n, & i_n \geq s_n \end{cases} \]  

Each equation shown in Eq. (4) can be solved in terms of individual parameters \( i_1 \sim i_R \) independently. For example, plugging values from 0 to \( i_1-1 \) into parameter \( i_1 \) iteratively in the first equation presented in Eq. (4), we can get the following solutions:

\[ P(i_1, i_2, \ldots, i_R) = \frac{a_{i_1}}{i_1} P(0, i_2, \ldots, i_R), \quad 0 \leq i_1 \leq s_1 \]
\[ P(i_1, i_2, \ldots, i_R) = \frac{a_{i_1} \mu}{s_1 i_1} P(s_1, i_2, \ldots, i_R) \]
\[ P(i_1, i_2, \ldots, i_R) = \frac{a_{i_1} \mu}{s_1 i_1} P(t_1, i_2, \ldots, i_R) \]
\[ P(i_1, i_2, \ldots, i_R) = \frac{a_{i_1} \mu}{s_1 i_1} P(t_1, i_2, \ldots, i_R) \]
\[ P(i_1, i_2, \ldots, i_R) = \frac{a_{i_1} \mu}{s_1 i_1} P(t_1, i_2, \ldots, i_R) \]
\[ P(i_1, i_2, \ldots, i_R) = \frac{a_{i_1} \mu}{s_1 i_1} P(t_1, i_2, \ldots, i_R) \]
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\[ P(i_1, i_2, \ldots, i_R) = \frac{a_{i_1} \mu}{s_1 i_1} P(t_1, i_2, \ldots, i_R) \]
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\[ P(i_1, i_2, \ldots, i_R) = \frac{a_{i_1} \mu}{s_1 i_1} P(t_1, i_2, \ldots, i_R) \]

Similar solutions can be obtained for parameters \( i_2 \sim i_R \). Applying these solutions iteratively, we can get the explicit solution for \( P(i) \). The solution is,
5. NUMERICAL EXAMPLES

5.1 THE THRESHOLD FOR THE FIRST STAGE CONTROL, t_n

Numerical examples of throughputs, buffer overflows, and the first stage control probabilities as a function of the threshold value for the first stage control are shown in Figs. 3 - 6. In these calculations, the number of routes, R, and the amount of buffers, M, are 2 and 30 respectively. The restriction ratios of the first stage control r1 and r2, are both 1/2. The second stage control is not implemented, that is, the thresholds for the second stage control, t1 and t2, are both 30. Figs. 3 and 4 correspond to a normal load situation (a1=a2=0.7), while Figs. 5 and 6 to a situation where route 1 is overloaded (a1=1.2, a2=0.7). It is necessary to determine the threshold value, t_n, in order to improve the performance under overload situations as much as possible without deteriorating the performance under normal load situations.

Fig. 3 shows that the throughput under the normal load situation stay almost constant so long as t_n is not set too small. Fig. 5 gives the throughput characteristics under the overload situation. Throughput T_1 of the congested route gets improved as t_n becomes larger, while throughput T_2 of the normal route begins deteriorating as t_n exceeds a certain value. The point where t_n = 30 corresponds to the case where no controls are implemented, and the control becomes more effective as t_n becomes smaller. Fig. 5 shows, therefore, that appropriate controls improve the throughput of the normal route by limiting the throughput of the congested route and lead to better overall throughput characteristics of the switch.

Figs. 4 and 6 illustrate quantitative tradeoffs between buffer overflow probabilities and the first stage control probabilities. Both buffer overflows and controls lead to reduction in通过puts, however, the difference between them is that, the former has an effect on the throughputs of all the routes, while the latter only on the throughput of the specific route. When t_1 and t_2 are small, P_{ov} is small while P_{t1} and P_{t2} are large. This is because packet flows are restricted by the first stage control before buffer overflows occur. When t_1 and t_2 are large, on the other hand, P_{ov} is large while P_{t1} and P_{t2} are smaller. This shows that the first stage control is no more effective enough to prevent buffer overflows from occurring frequently.

5.2 EFFECT OF THE CONGESTION CONTROL TO BALANCED OVERLOADS

Figs. 7 and 8 illustrate the performance of the congestion control technique as a function of the offered traffic under the condition that a1=a2. The threshold values of the first stage control for both outgoing routes are 20, i.e. t_1=t_2=20.

When the loads are balanced, it is unlikely that the situation where the packets to the specific route occupy most of the buffers occurs. Hence it seems that we cannot expect the improvements of the traffic characteristics under balanced overload situations. As a matter of fact, however, as Fig. 7 illustrates, there is improvement of throughput under this case too. This is because the loads to individual outgoing routes are unbalanced on short time basis even if average loads are balanced.

5.3 EFFECT OF THE CONGESTION CONTROL TO UNBALANCED OVERLOADS

Figs. 9 - 11 show the numerical examples where the amount of traffic to outgoing route 1 varies, while that to outgoing route 2 is kept constant (a1=0.7). Fig. 9 gives throughput characteristics when the second stage control is also implemented in addition to the first stage control. The threshold values of the second control for both outgoing routes are 26, i.e. t_1=t_2=26. Fig. 11 illustrates average packet queue lengths for individual outgoing routes including packets being transmitted.

Fig. 9 illustrates that the throughput for normal route 2,
The degradation of $T_2$ does not occur until $a_1 = 1.4$ in the system with only the first stage control. $T_2$ begins deteriorating gradually, however, for $a_1 \geq 1.4$, because the amount of traffic under the control, which is $1/2$ of the value without control, also becomes too large for $a_1 \geq 1.4$, i.e. $1/2a_1 \geq 0.7$. In the case the second stage control is also applied, the packets to outgoing route 1 cannot occupy more than 26 buffers. Reserving at least 4 buffers for the packets to the normal outgoing route, the system with second stage control can improve performances under heavily overloaded situations. In international packet switched networks, however, traffic controls such as call blocking controls and routing controls are performed at call establishment phase, and overload traffic which is likely to occur under these conditions can be expected to be limited within certain extent. Hence it seems to be possible that, practically, the first stage controls solely realize sufficiently effective congestion control even without the second stage controls.

Fig. 7. Throughput under balanced loading.

Fig. 8. Buffer overflow probability under unbalanced loading.

Fig. 9. Throughput under unbalanced loading.

Fig. 10. Buffer overflow probability under unbalanced loading.

Fig. 11. Average packet queue length under unbalanced loading.
6. REALIZATION OF THE TECHNIQUE

6.1 THRESHOLDS FOR ACTIVATIONS AND TERMINATIONS OF THE CONTROLS

For the theoretical analysis on the congestion control technique in Section 4, it is assumed that the controls are carried out ideally and identical thresholds are used for both activations and terminations of the controls. When the thresholds are the same, however, the shuttling of control actions between activations and terminations might occur. These shuttling phenomena cause the amount of control packet traffic and processing loads of switches to increase. In realizing the technique, therefore, it is necessary to establish the thresholds for terminations to be lower than the ones for activations and moderate the shuttling phenomena of control actions.

Analysis on the shuttling phenomena is given in Appendix. Using the formula obtained in Appendix, the average transition times from activations to terminations and from terminations to activations of the controls can be calculated. For example, when the difference between the two thresholds is four concerning the first stage control, they are calculated to be 165 ms and 305 ms respectively under the condition that the transmission rate of the outgoing line is 48 kbps, the average packet length is 800 bits, \( d_n = 1.2 \), \( r_n = 1/2 \), and \( s_n = 1 \). Average packet transmission times are approximately 40 ms. Therefore, the average transition times are about 10 times and 18 times as large as average packet transmission times. The shuttling phenomena of this extent can be considered to be sufficiently infrequent.

6.2 MECHANISM FOR REALIZING THE CONTROLS

As for a mechanism for flow restrictions, RNR/RR packets are defined for each virtual call in protocol X.75. The length of these control packets is very short, being about 1/40 of usual data packet lengths. In this paper, the average transition times between activations of controls are calculated to be 10 ms and 18 ms respectively under the condition that the transmission rate of the outgoing line is 48 kbps, the average packet length is 800 bits, \( d_n = 1.2 \), \( r_n = 1/2 \), and \( s_n = 1 \). Average packet transmission times are approximately 40 ms. Therefore, the average transition times are about 10 times and 18 times as large as average packet transmission times. The shuttling phenomena of this extent can be considered to be sufficiently infrequent.

In this paper, the analysis is done under the assumption that there is no delay from the dispatch of the controls until they are actually effected. However, when there are long delay links such as satellite links, packet arrivals continue through the virtual calls to be controlled for some time after the dispatch of the controls. The congestion control technique can be applied for these situations by implementing extra buffers enough to accept those packets.

APPENDIX

ANALYSIS ON SHUTTLING PHENOMENA

Here we are going to derive average transition times between activations and terminations of the first stage control theoretically.

Behaviors of queues for each outgoing route can be treated independently from each other as long as buffer overflows do not occur. Let the threshold for the termination of the first stage control to outgoing route \( n \) be denoted \( t_{s,n} \) as Fig. 12 shows. The first stage control to outgoing route \( n \) is activated when the queue length reaches threshold \( t_n \) and terminated when the queue length decreases to threshold \( t_{s,n} \). Generally,

\[ t_n > t_{s,n} \] (A1)

Let the difference between the thresholds be denoted \( d_n = t_n - t_{s,n} \) (A2)

During the transition period between the two thresholds, the queue length does not become zero. Therefore, the number of virtual calls to be controlled simultaneously in these switches is much smaller than that in the switch where the congestion condition is detected.

7. CONCLUSIONS

A congestion control technique for international packet switched networks, which strengthens packet flow restrictions gradually depending upon the queue lengths to individual outgoing routes in a switch is proposed in this paper. Fundamental traffic characteristics are analyzed theoretically and some problems in realizing the technique are also discussed. State probabilities of the switch under the congestion control are obtained analytically first. Using the probabilities, equations which give values of important parameters to evaluate the performance of the technique such as throughputs, buffer overflow probabilities and average packet queue lengths are derived. The effectiveness of the proposed congestion control technique is demonstrated by several numerical examples.

Analysis on the shuttling phenomena is given in Appendix. Using the formula obtained in Appendix, the average transition times from activations to terminations and from terminations to activations of the controls can be calculated. For example, when the difference between the two thresholds is four concerning the first stage control, they are calculated to be 165 ms and 305 ms respectively under the condition that the transmission rate of the outgoing line is 48 kbps, the average packet length is 800 bits, \( d_n = 1.2 \), \( r_n = 1/2 \), and \( s_n = 1 \). Average packet transmission times are approximately 40 ms. Therefore, the average transition times are about 10 times and 18 times as large as average packet transmission times. The shuttling phenomena of this extent can be considered to be sufficiently infrequent.

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6.2 MECHANISM FOR REALIZING THE CONTROLS

As for a mechanism for flow restrictions, RNR/RR packets are defined for each virtual call in protocol X.75. The length of these control packets is very short, being about 1/40 of usual data packet lengths. When RNR packets are necessary to be transmitted to a lot of virtual calls simultaneously, however, the traffic loads caused by these RNR packets contribute to accelerate the congestion conditions in the switch. This problem can be solved by defining C-RNR/C-RR (Collective RNR/Collective RR) packets which are effective to a group of virtual calls. In order to realize this mechanism, a table is necessary to be implemented for each outgoing route in the switch as Fig. 12-(a) illustrates. The tables give for each outgoing route in the next switch the virtual call numbers which are the objects of the first stage control and the second stage control respectively. Fig. 12-(b) illustrates an example of the tables. Switches which receive C-RNR/C-RR packets are required to transfer the controls to the previous switches by transmitting RNR/RR packets individually to all the virtual calls to be controlled. Generally,

\[ t_n > t_{s,n} \] (A1)

Let the difference between the thresholds be denoted \( d_n = t_n - t_{s,n} \) (A2)

During the transition period between the two thresholds, the queue length does not become zero. Therefore, the number of virtual calls to be controlled simultaneously in these switches is much smaller than that in the switch where the congestion condition is detected.

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A congestion control technique for international packet switched networks, which strengthens packet flow restrictions gradually depending upon the queue lengths to individual outgoing routes in a switch is proposed in this paper. Fundamental traffic characteristics are analyzed theoretically and some problems in realizing the technique are also discussed. State probabilities of the switch under the congestion control are obtained analytically first. Using the probabilities, equations which give values of important parameters to evaluate the performance of the technique such as throughputs, buffer overflow probabilities and average packet queue lengths are derived. The effectiveness of the proposed congestion control technique is demonstrated by several numerical examples.
The number of combinations where the queue becomes $d_n$ for the first time at $(j_a+j_d)$th movement under the condition that it moves $j_a$ times upward and $j_d$ times downward can be obtained from the random walk theory as follows:

$$\frac{j_a - j_d}{j_a + j_d} \left( \frac{j_a + j_d}{j_a} \right)^{j_d}$$  \hspace{1cm} (A7)

Combining terms (A5), (A6) and (A7), the probability that the queue grows becomes $d_n$ for the first time at time $t$ under the condition that there are $j_a$ arrivals and $j_d$ departures during interval $t$ can be obtained. Let the probability be denoted $q_{j_a,j_d}(t)dt$.

Then,

$$q_{j_a,j_d}(t)dt = \lim_{\Delta t \to 0} \frac{j_a - j_d}{j_a + j_d} \left( \frac{j_a + j_d}{j_a} \right)^{j_d} \cdot (1 - e^{-\Delta t})^{j_d}$$  \hspace{1cm} (A8)

Using Eq. (A9), $q_{j_a,j_d}(t)dt$ can be simplified as follows:

$$q_{j_a,j_d}(t)dt = \lim_{\Delta t \to 0} \frac{j_a - j_d}{j_a + j_d} \left( \frac{j_a + j_d}{j_a} \right)^{j_d} \cdot e^{-\frac{j_d}{j_a+j_d} \left( A_{1} + B_{1} \right)}$$  \hspace{1cm} (A10)

Considering Eq. (A4), probability $q_d(t)dt$ can be obtained by summing $q_{j_a,j_d}(t)dt$ for all possible values of $j_d$.

$$q_d(t)dt = \sum_{j_d=0}^{\infty} q_{j_a,j_d}(t)dt$$  \hspace{1cm} (A11)

Let the probability that the queue reaches threshold $t_n$ starting from threshold $t_{s,n}$ at least once be denoted $q_{d}(d_n)$.

Then,
When \( d_n \geq 2 \),

\[
q_d(d_n) = \frac{d_n d_n!}{(\lambda_n + S_n\mu_n) d_n!} \sum_{j=0}^{\infty} \frac{(2jd + d_n - 1)!}{j!} \left( \frac{S_n\mu_n}{\lambda_n + S_n\mu_n} \right)^j \left( \frac{\lambda_n}{\lambda_n + S_n\mu_n} \right)^j d
\]

Let the average transition time from the termination to the activation of the first stage control be denoted \( D_u(d_n) \).

\[
D_u(d_n) = \int_0^\infty \frac{q_d(t) dt}{q_u(d_n)}
\]

\[
D_u(d_n) = \frac{d_n d_n!}{(\lambda_n + S_n\mu_n) d_n!} \sum_{j=0}^{\infty} \frac{(2jd + d_n - 1)!}{j!} \left( \frac{S_n\mu_n}{\lambda_n + S_n\mu_n} \right)^j \left( \frac{\lambda_n}{\lambda_n + S_n\mu_n} \right)^j d
\]

Let the average transition time from the activation to the termination of the first stage control can also be derived similarly. Let the average transition time and the probability that the queue reaches threshold \( t_{a,n} \) starting from threshold \( t_{n} \) at least once be denoted \( D_d(d_n) \) and \( q_d(d_n) \) respectively.

\[
q_d(d_n) = \frac{d_n d_n!}{(\lambda_n + S_n\mu_n) d_n!} \sum_{j=0}^{\infty} \frac{(2jd + d_n - 1)!}{j!} \left( \frac{S_n\mu_n}{\lambda_n + S_n\mu_n} \right)^j \left( \frac{\lambda_n}{\lambda_n + S_n\mu_n} \right)^j d
\]

The average transition time from the activation to the termination of the first stage control can also be derived similarly. Let the average transition time and the probability that the queue reaches threshold \( t_{a,n} \) starting from threshold \( t_{n} \) at least once be denoted \( D_d(d_n) \) and \( q_d(d_n) \) respectively.

Then,

\[
q_d(d_n) = \frac{d_n d_n!}{(\lambda_n + S_n\mu_n) d_n!} \sum_{j=0}^{\infty} \frac{(2jd + d_n - 1)!}{j!} \left( \frac{S_n\mu_n}{\lambda_n + S_n\mu_n} \right)^j \left( \frac{\lambda_n}{\lambda_n + S_n\mu_n} \right)^j d
\]

\[
D_d(d_n) = \frac{d_n (\lambda_n + S_n\mu_n) d_n!}{(\lambda_n + S_n\mu_n) d_n! + q_d(d_n)}
\]

\[
D_d(d_n) = \frac{d_n (\lambda_n + S_n\mu_n) d_n!}{(\lambda_n + S_n\mu_n) d_n! + q_d(d_n)}
\]

\[
D_d(d_n) = \frac{d_n (\lambda_n + S_n\mu_n) d_n!}{(\lambda_n + S_n\mu_n) d_n! + q_d(d_n)}
\]

A Numerical Example:

Let the transmission rate of the link be 48 kb/s and average packet length be 800 bits.

Then,

\[
\mu_n = \frac{48000}{800} = 60
\]

Let other parameters be,

\[
a_n = 1.2 \quad i.e. \quad \lambda_n = 72
\]

\[
r_n = \frac{1}{2} \quad i.e. \quad \lambda' n = 36
\]

\[
S_n = 1
\]

\[
d_n = 4
\]

Applying Eqs. (A14) and (A16), we can obtain

\[
D_u(4) = 0.305
\]

\[
D_d(4) = 0.165
\]

ACKNOWLEDGEMENT

The authors wish to acknowledge the continued guidance and encouragement of Dr. Y. Nakagome, Dr. H. Kaji and Dr. H. Teramura of KDD Research and Development Laboratories. They are also grateful to Mr. S. Ando for his helpful discussions on the subject of this paper.

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