ESTIMATION OF TRAFFIC SERVICE IN A SYSTEM OF FULL-AVAILABILITY TRUNK GROUPS WITH MUTUAL OVERFLOW

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ABSTRACT

The paper describes the estimation of traffic service with mutual overflow in systems of full-availability trunk groups with Poissonian input processes. The state of works in the domain of mutual overflows, the basic assumptions and the problems for solving are described in the first place. Then follows the description of the investigated system of two full-availability groups, serving traffic with mutual overflow, and for comparison - with one-way overflow too. Served overflow traffic flows were offered to two groups (with own traffic), called the closing groups, of finite or infinite numbers of trunks, enabling overflow calls to reach their destination exchange.

The overflow system was described by a system of equations referring to expanded set of trunk occupancy states. The results of mathematical analysis were compared with the simulation results.

The simulation investigations on CDC 6400 computer were held for combinations of small (8 Erl.), medium (30 Erl.) and heavy (90 Erl.) intensities of Poissonian input processes, while there was variable number of trunks in one of the two groups of the overflow system.

The simulation results (inserted as diagrams and tables) concern the quality of service for all flows (Poissonian and overflow traffic) in cases of mutual and one-way overflows. The analysis is made in order to:

i) distinguish differences in traffic service with both kinds of overflow

ii) prove some new characteristics of traffic service with mutual overflow.

In the conclusion of the paper there are assembled main characteristics of mutual overflow, among which the most important is, that the base purpose of mutual overflow should be improving network reliability. The mutual overflow may be also treated as a weak economical factor in network planning.

GENERAL SYMBOL LIST

- served flow
- service coefficient of A flow
- variance of a traffic flow
- parameters
- confidence level ( =0.95)
- call rate
- disconnect rate (output intensity)
- absolute difference
- (apostrophe) - case of n=A=oo

1. INTRODUCTION

Mutual overflow of telephone traffic in the trunk groups system is a method of calls service, according to which a call offered to one of the groups of the mutual overflow system is:

i) served - if in the own group there is at least one idle trunk

ii) offered to the following overflow groups (in case of a congested own group) and served - if there are (at least one in each) idle trunks in at least one overflow group and in the associated closing group (which connects transit exchange with the destination exchange) iii) offered to the following overflow groups (in case of a congested own group) and lost - in all other occupancy cases.

Up to the present, the problems of the mutual overflow in trunk groups systems are presented by the publications [3], [4], [5]. They describe the problems concerned with determining probability (and thus, as the result - intensity) of traffic overflow in the systems of several full-availability groups, with at most 10 trunks in each group, only for symmetrical cases (groups of the same size). There was assumed that closing groups have infinite number of trunks and therefore have no effect on the parameters of the overflow. The publications have not raised a question of applying mutual overflow in practice, although they [3], [5] treat intentionally a matter of the best building of link networks.

2. GENERAL ASSUMPTIONS

The subject of the paper concerns estimation of traffic service in the class of trunk groups systems with the following characteristics:

1. types of groups - of full-availability

2. input flows - Poissonian processes

3. call service - with no priority, with mutual overflow

4. types of trunks - efficient and failed in a group

5. types of trunk - failed trunks do not failures accept calls for the service

6. service time - exponential distribution

Estimation of the traffic service in the trunk groups system is made according to:

i) the number of trunks in the groups of the overflow system

ii) the number of trunks in the closing groups
A~.AA

Determine the characteristics of traffic service is based on the following premises:
1. Birth and death process describing in-flow and service of the calls is the sta-
mationary one.
2. Overflow system groups may be submitted to the longterm failures.
3. Closing groups do not tend to fail.

As the criterion of estimation of traffic service there has been assumed the average intensity value of component flows served by each group of overflow system and each closing group.

Traffic service in overflow system at the assumption of infinite number of trunks in closing groups is equal to the service of one flow with the intensity A by one full – available group with the number of trunks n, and

\[
A = \sum_{i=1}^{m} A_i \quad n = \sum_{i=1}^{m} n_i
\]

where m - number of groups in overflow system.

Loss factor for each flow A_i is determined then by Erlang formula. This formula can be used for the estimation of traffic service in overflow system at random overload of one or more groups.

3. MODEL OF THE OVERFLOW SYSTEM

Study of characteristics of the overflow system has been made in two stages:
1) first, a simple system of two groups n_1 and n_2 with the mutual overflow, and appropriately associated with them closing groups n_3 and n_4 (presented on fig. 1) was analyzed

![Fig. 1. System of two groups with mutual overflow n_1 and n_2, and closing groups n_3 and n_4](image)

ii) then conclusions concerning the system from fig.1 were expanded to more complicated systems.

Two methods were used during the investigation:
1) state equation method
2) digital simulation method.

In the first one, it was used an original expanding of the set of states of trunk occupancy for the differentiation of own occupancy and the overflow one; a trunk occupancy has been treated as a whole up to now, without analyzing reasons of the occupancy.

During a simulation investigation there have been taken assumptions, that offered traffic flows between exchanges B and C are equal, that is A_3 = A_4 and time congestion coefficients are equal as well, i.e. B_3 = B_4 = const.

Concentrating on the change of B_1, a hypothesis has been put forward that mutual overflow does not disturb a service of the offered flow A_i by the group n_4 at the changes of B_1. This hypothesis is equivalent to taking the assumption that n_4 = \infty. Assuming the hypothesis enabled to generate the smaller number of events during the simulation, (and enabled to limit the cost of the investigation). As far as traffic situation on the real group n_4 is concerned, conclusions can be taken on the basis of the traffic served by the group n_3.

Resulting from the taken assumption and hypothesis, a model of the simulationally tested system has been simplified to the form shown on the fig.2.

![Fig. 2. System of two groups with mutual overflow n_1 and n_2, and the closing group n_3 (n_4 = \infty)](image)

During the simulation this model was treated as:
1) system of two groups n_1 and n_2 serving the traffic with one-way overflow from n_1 to n_2 (n_3 = \infty)
2) system of two groups n_1 and n_2 serving the traffic with mutual overflow (n_3 = \infty)
3) system of three groups n_1, n_2, n_3 serving the traffic with one-way overflow from n_1 to n_2
4) system of three groups n_1, n_2, n_3 serving the traffic with mutual overflow (between n_1 and n_2).

4. MATHEMATICAL SOLUTION

The analysis of traffic service with mutual overflow by the state equation method needs a differentiation of states of trunk occupancy by own connections and the overflow ones. Expanded set of states concern two groups n_1 and n_2 with the assumption that n_3 = n_4 = \infty. Traffic propagation between the groups n_1 and n_2 is shown on the fig.3.

![Fig. 3. Traffic propagation in the system of two groups with mutual overflow (n_3 = n_4 = \infty)](image)
In the expanded set, each state is described by the combination of four variables \( (i_1, i_2, x_1, x_2) \), where \( i \) - number of overflow calls, \( x \) - number of all calls served by the groups - respectively \( n_1 \) and \( n_2 \) \((i_1 \leq x_1, i_2 \leq x_2)\). It is convenient to present a set of states on the surface in the form of a table of states. Fig. 4 presents a structure of the table of expanded set of states. There are marked some changes of states.

Fig. 4. Structure of the table of expanded set of states of two groups with mutual overflow

A system of equations describing changes of states has the following form:

\[
\begin{align*}
\{(i_1, x_1), \varphi(x_1, x_2), p_{i_1, i_2, x_1, x_2}\} = & \; A \cdot \{i_1, i_2, x_1, x_2\} \\
+ & \; x_2 \cdot p_{i_1, i_2, x_1, x_2} - 1 \cdot x_1 \cdot p_{i_1, i_2, x_1, x_2} - 1 \cdot n_2 \\
+ & \; \varphi(i_1, x_1, x_2) - x_1 \cdot \varphi(i_1, x_1, x_2) + \varphi(i_2, x_1, x_2) - x_2 \cdot \varphi(i_2, x_1, x_2) + \\
+ & \; \{(i_1 + 1), p_{i_1, i_2, x_1, x_2} - 1 \cdot \varphi(x_1, x_2) + \varphi(x_2, x_1) - 1 \cdot \varphi(x_2, x_1) + \\
+ & \; \{(i_2 + 1), \varphi(x_1, x_2) - 1 \cdot \varphi(x_1, x_2) + \varphi(x_1, x_2)
\end{align*}
\]

and

\[
\alpha = \begin{cases} 
1 & \text{for } x_2 = n_2, \\
0 & \text{for } x_2 < n_2
\end{cases}
\]

\[
\varphi = \begin{cases} 
1 & \text{for } x_2 < n_2, \\
0 & \text{for } x_2 = n_2
\end{cases}
\]

\[
\beta = \begin{cases} 
1 & \text{for } x_1 = n_1, \\
0 & \text{for } x_1 < n_1
\end{cases}
\]

\[
\gamma = \begin{cases} 
1 & \text{for } x_1 < n_1, \\
0 & \text{for } x_1 = n_1
\end{cases}
\]

Equations of this form and the equation

\[
\sum_{i_2=0}^{n_2} \sum_{i_1=0}^{n_1} \sum_{x_1} \sum_{x_2} p_{i_1, i_2, x_1, x_2} = 1
\]

make a system of linear equations which can be solved by generally known methods. Number \( N \) of equations is obtained from the formula

\[
N = (n_1 + 1)(n_2 + 2)(n_2 + 1)(n_2 + 2)/4
\]

Traffic parameters achieved after the solution of a system of state equations are as follows:

1. mean intensity of the own traffic flow served by the group \( n_1 \)

\[
\lambda_{11} = \sum_{x_1=0}^{n_1} \sum_{x_2} p_{i_1, i_2, x_1, x_2}
\]

2. variance of intensity of the own traffic flow served by the group \( n_1 \)

\[
\sigma_{11}^2 = \sum_{x_1=0}^{n_1} \sum_{x_2} (p_{i_1, i_2, x_1, x_2} - \lambda_{11})^2
\]

3. mean intensity of the own traffic flow served by the group \( n_2 \)

\[
\lambda_{22} = \sum_{x_2=0}^{n_2} \sum_{x_1} p_{i_1, i_2, x_1, x_2}
\]

4. variance of intensity of the own traffic flow served by the group \( n_2 \)

\[
\sigma_{22}^2 = \sum_{x_2=0}^{n_2} \sum_{x_1} (p_{i_1, i_2, x_1, x_2} - \lambda_{22})^2
\]

5. mean intensity of the overflow traffic served by the group \( n_1 \)

\[
\lambda_{21} = \sum_{x_2} p_{i_1, i_2, x_1, x_2}
\]

6. variance of intensity of the overflow traffic served by the group \( n_1 \)

\[
\sigma_{21}^2 = \sum_{x_2} (p_{i_1, i_2, x_1, x_2} - \lambda_{21})^2
\]

7. mean intensity of the overflow traffic served by the group \( n_2 \)

\[
\lambda_{12} = \sum_{x_1} p_{i_1, i_2, x_1, x_2}
\]

8. variance of intensity of the overflow traffic served by the group \( n_2 \)

\[
\sigma_{12}^2 = \sum_{x_1} (p_{i_1, i_2, x_1, x_2} - \lambda_{12})^2
\]

The method of state equations is useful in the case of very small groups (of the order of a few trunks). Its basic importance is in acknowledgment of correctness of simulation program.

5. SIMULATION

Simulation was based on generating of Markov chain approximating real work of the group system. The investigation was made in the way of simulation tests, including several hundreds up to several thousands of events, according to the size of the simulated system. Besides the introductory test which had not been estimated,
a possibility of getting from 5 to 30 estimated tests during a single run of the program was predicted. Treating the average of each test as a realization of random variable with a distribution close to normal, it has been determined - on the basis of Student's distribution - values of confidence interval for a series including at least estimated tests at a confidence level $\Gamma = 0.95$. Obtaining a relative value $\Delta A_{12} s_m/A_{12} s_m \leq 0.05$ for a half of confidence interval $\Delta A_{12} s_m$ or making 30 estimated tests have been assumed as a criterion of the end of the program run. As the result of simulation, a condition $\Delta A_{12} s_m/A_{12} s_m \leq 0.05$ has been satisfied for the cases when $B_1$ is of big value and also $A_1 > A_2$.

6. CONCLUSIONS

Basing on the results of the simulation, following general conclusions have been formulated:

1. With infinite number of trunks in closing groups ($n_2 = n_4 = \infty$) the service of offered traffic flows $A_1$ and $A_2$ with mutual overflow is approximately the same.

2. Offered traffic flow $A_1$ is served better with one-way overflow than with mutual overflow, and flow $A_2$ - just the opposite - it is served better with mutual overflow than with one-way overflow.

3. The effect of finite closing group $n_2$ appears mainly as a reduction of intensity of the served overflow traffic and increase of intensity of the served own traffic by the group $n_2$.

4. Variance of the overflow traffic served by the group $n_2$ is bigger with mutual overflow than with one-way overflow.

5. Variance of all traffic served by the group $n_1$ is smaller with the mutual overflow than with the one-way overflow.

Detailed conclusions are as follows:

6. The hypothesis about possibility of omission the effect of group $n_1$ on traffic service with mutual overflow between $n_1$ and $n_2$, with constant $B_2 = 1\%$ and $B_3 = 1\%$ appeared to be the true one. The service of traffic flow $A_4$ is practically not interfered by overflow traffic from the group $n_2$, and therefore the group $n_4$ does not bring in the limits of service of overflow traffic. In the systems with mutual overflow, in which good quality of traffic service is the same for all groups but one, and closing groups are of similar size, while analysing traffic phenomena one can neglect a closing group associated with the failed group $n_2$ at a practical importance, as for example in lowering of the cost of simulation.

7. Total intensity of the served traffic in the system of groups $n_1$ and $n_2$ is with mutual overflow almost the same as with one-way overflow. Little differences - of absolute value less than 1 Erl. - in service in favour of mutual overflow are too small to be noticed. This is in $\overline{5}$. Hence the conclusion that mutual overflow cannot be applied as a method for increasing capacity of trunk group systems.

8. Small in fact differences (in function of changes $B_1$) in the traffic $A_1$ service with mutual and one-way overflow can be neglected.

9. Differences in the traffic $A_2$ service with mutual and one-way overflow (though bigger than in the case of $A_1$ service) for several cases $A_1$, $A_2$ can be neglected in practice. However, in cases of non-typical overflows, when $A_1 > A_2$ and $30\% < B_1 < 50\%$, mutual overflow helps indeed in improving the service of traffic $A_2$ (in comparison with the one-way overflow).

Big values $\Delta A_{12}$ (even up to 20% - fig. 6) have two causes:

i) first of all, in the case of $A_1 > A_2$ one-way overflow made the traffic $A_2$ service much worse already for $B_1$ over 10%,

ii) mutual overflow gives the possibility for the overflow traffic $A_{21}$ to make the traffic $A_2$ service much worse already for $B_1 = 60\%$.

Therefore it becomes evident that mutual overflow lets to lessen the requirements for the size of an overflow group.

10. Mutual overflows also let to lessen the requirements for the size of a closing group. The basic requirement of neglecting the effect of the closing group on the overflow traffic service is that the closing group should be at least equal to the overflow group. The effect of a group $n_2$ on the traffic service with mutual overflow has been calculated by determining the value of a parameter $m = A_{12} s_m/A_{12} s_m$ in function $B_1$. On fig. 7, confidence interval has not been marked in order not to confuse run of the curves. Configuration of a network seems to be permissible when $n_2 = n_3$. Then, the closing group does not make the service of the overflow traffic worse more than 20$\%$ ($r < 80\%$). When at the same time a condition $A_1 < A_2$ is fulfilled (e.g., $A_1 = 30$ Erl., $A_2 = 90$ Erl., $A_3 = 90$ Erl.), effect of $n_3$ is negligible ($r > 95\%$). It is the same with the effect of $n_3$ which is negligible for $n_3 > n_2$, independently of $A_1$. 

![Fig. 5. Total mean intensity of the traffic served with mutual overflow, by groups $n_1$ and $n_2$ in function $B_1$ at $n_3 < \infty$, for different values $A_1$, $A_2$.](image-url)
Assuming that traffic flows $A_1$, $A_2$, $A_3$ have to be served with the loss equal to about 1%, and basing on the conclusion that there is possibility of neglecting the effect of group $n_3$ on the mutual overflow service, numbers of trunks in groups $n_1$, $n_2$, $n_3$ have been calculated, in the case of traffic service with no overflow (by Erlang formula), with one-way overflow (by BERT method), and with mutual overflow - on the basis of results of simulation. Results are included in the table 1.

<table>
<thead>
<tr>
<th>Examples</th>
<th>Numbers of trunks $n_1 + n_2 + n_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>with no overflow</td>
</tr>
<tr>
<td>$8 8 30$</td>
<td>$15+15+41=71$</td>
</tr>
<tr>
<td>$8 30 30$</td>
<td>$15+41+41=97$</td>
</tr>
<tr>
<td>$8 30 90$</td>
<td>$15+41+106=162$</td>
</tr>
<tr>
<td>$8 90 90$</td>
<td>$15+106+106=227$</td>
</tr>
<tr>
<td>$30 30 90$</td>
<td>$41+41+41=123$</td>
</tr>
<tr>
<td>$30 30 90$</td>
<td>$41+41+106=186$</td>
</tr>
<tr>
<td>$30 90 90$</td>
<td>$41+106+106=253$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Numbers of trunks in groups $n_1$, $n_2$, $n_3$ for different rules of call service with $E_{A_1} = E_{A_2} = E_{A_3} = 1%$

Small advantages of applying a mutual overflow are measurable in the cost which can be saved during the building of a network, as the result of lessening number of trunks in groups of mutual overflow system. The number of saved trunks is around several. It has been proved then, that mutual overflow can be treated as a weak economic factor.

Up to now, as far as reliability is concerned, a long-distance group connecting two exchanges has been divided into several parts (with almost the same number of trunks) put through different trunk routes. These parts here are called component groups.

Mutual overflow enables to increase reliability and lessen the cost of a network as the result of association of groups divided into less component groups, as it is not in the case when there is no mutual overflow (table 2). Values $E_{A_1 mo}$, $E_{A_2 mo}$, $E_{A_3 mo}$ have been determined on the basis of simulation results, values $B_i$ have been taken from the tables. From the point of view of traffic $A_1$ service, division into halves of groups with mutual overflow increases always reliability of a network in comparison with the division of groups into 3 component groups with no overflow. In case of small and medium traffic flows (when $A_3 \geq n_2$ and $n_1 \geq n_2$), division into halves with mutual overflow leads to better service of these flows (with the failure of 50% of trunks) than the division into 4 component groups with no overflow (with the failure of 25% of trunks). At the same time, mutual overflow causes approximately the same loss of all flows ($A_1$, $A_2$, $A_3$). Generally, the loss is not bigger than 20% which can be tolerated when happens not too often and lasts for the short time.

**Fig. 6.** Changes $\Delta S_{A_2}$ in the function $B_i$ with $n_3 < \infty$ for two cases of values $A_1$, $A_2$, $A_3$

**Fig. 7.** Effect of the closing group $n_3$ on the service of offered traffic $A_1$ for different values $A_1$, $A_2$, $A_3$

11. Service of offered traffic $A_1$ is independent of a kind of overflow between groups $n_1$ and $n_2$.
groups (except $n$) enter into the composition of the overflow system, so as the loss factors of the flows are in the system $1 + 5\%$ at the failure of the half of any group.

When planning a network, mutual overflow can be treated in fact as one-way overflow but free-way overflow. Free-way of mutual overflow causes that its importance, which appears only at the failure of trunks, is in lessening of the overload on the failed group, that is, in lessening the effect of failure on traffic service. An efficient network with mutual overflow operates as a network with no overflow but planned for bigger reliability.

Simulation results are the basis for estimating characteristics of traffic service with mutual overflow in the system of three groups $n_1$, $n_2$, $n_3$ (fig.8), disregarding the selection of strategy of routing.

Table 2. Comparison of loss factors $B_{A_1, A_2, A_3}$ with the failures $1/3$, $1/2$, and of the whole groups $n_1$ for chosen values $A_1$, $A_2$, $A_3$

<table>
<thead>
<tr>
<th>$A_1$ [Erl.]</th>
<th>$B_1$ [%]</th>
<th>$A_2$ [Erl.]</th>
<th>$B_2$ [%]</th>
<th>$A_3$ [Erl.]</th>
<th>$B_3$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>5,28</td>
<td>26</td>
<td>5,20</td>
<td>28</td>
<td>5,12</td>
</tr>
<tr>
<td>25</td>
<td>5,28</td>
<td>27</td>
<td>5,20</td>
<td>29</td>
<td>5,12</td>
</tr>
<tr>
<td>26</td>
<td>5,28</td>
<td>28</td>
<td>5,20</td>
<td>30</td>
<td>5,12</td>
</tr>
</tbody>
</table>

Fig.8. Mutual overflow in the system of three groups $n_1$, $n_2$, $n_3$

It is shown in the table 3, how many and what groups (except $n_5$) enter into the composition of a mutual overflow system, so as the loss factors of the flows are in the system $1 + 5\%$ at the failure of the half of any group.

Table 3. Estimation of number of full-availability groups in the system with mutual overflow

<table>
<thead>
<tr>
<th>$A_1$ [Erl.]</th>
<th>$B_1$ [Erl.]</th>
<th>Number of groups $n$</th>
<th>Number of groups $n$</th>
<th>Number of groups $n$</th>
<th>Number of groups $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>15</td>
<td>2</td>
<td>41</td>
<td>1</td>
<td>106</td>
</tr>
<tr>
<td>25</td>
<td>15</td>
<td>2</td>
<td>41</td>
<td>3</td>
<td>106</td>
</tr>
</tbody>
</table>

8. SUMMARY

Precise relations which have been achieved, mainly the results of simulation, let to formulate several practical conclusions concerning the making of a network of mutual overflow groups. The most important of these conclusions are as follows:

1. Mutual overflow can be treated as a weak economic factor, and has limited application as the means for increasing capacity of trunk group systems.

2. The basic aim of using the mutual overflow should be the increasing of resistance of a network to failures. After trunks failures, mutual overflow can be treated as an one-way but free-way overflow, operating only for lessening of the overloads on the failed group.

3. From the point of view of balancing the results of the failure, making of overflow system with the same groups is the best. The system has then the least number of groups. In the case of different groups, overflow system can expand too much (even up to 3 groups).

4. A closing group associated with the failed group of the overflow system can be excluded when considering the results of the failure, because the service of the "fresh" traffic offered to that group does not undergo the limitation which results from mutual overflow.

One general conclusion results from the mentioned above points - the advantage of mutual overflow to one-way overflow because of better traffic service and bigger resistance of a network to failure. Beyond our study, there is a large set of cases of mutual overflow in the systems of limited availability groups, or of full-availability groups with putting only part of groups for overflow service. There are also systems with the shifted busiest hours. However, we can state that definitely most of quality conclusions are correct also for those systems, although quantity proportions can form differently.

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