MODELLING OF NON-STATIONARY TRAFFIC PROCESSES

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ABSTRACT

A Poisson arrival process is well defined in a strict mathematical sense. Without presenting such a definition here, two main properties are mentioned: the lack of memory and the time-independent intensity. The lack of memory implies that any arrival interval is independent of previous intervals, whereas the time-independent intensity defines a single parameter for the process.

Deviations from a Poisson process may for instance mean a time-varying intensity or a stochastic dependence between arrival intervals, or a combination of both. Whatever is the case, an observation sample over limited time will have some distribution, which in general will be different from the Poisson distribution.

In the present paper it is assumed that any observed sample is taken from a process where the memoryless property applies, whereas the intensity may vary with time. The only limitation is that these variations are "slow".

The arrival process is studied in terms of interarrival intervals (continuous distributions) and call arrivals during fixed intervals (discrete distributions). Formulas for calculation of the distributions and their moments are given and examples of moment matching and loss calculation are presented.

INTRODUCTION

A teletraffic process is often characterized by two parameters only: an arrival parameter (\( \lambda \)) and a service parameter (\( \mu \)), assuming for instance that the arrival process is a Poisson process and the service times follow an exponential distribution. In the following we shall adhere to that assumption as far as the service times are concerned, whereas the arrival process will be assumed to deviate substantially from the Poisson process.

We shall assume the following types of deviations: continuous variations of the arrival rate \( \lambda \) with time \( t \), \( \lambda = \lambda(t) \), even though the lack of memory of the Poisson process is retained; deviations from the memoryless property because of disturbances on the source side, e.g. repetition of unsuccessful calls, with or without change in the long term arrival rate; deviations from the Poisson arrival process through system state dependence, e.g. Poisson traffic offered to primary groups giving non-Poisson overflow to secondary groups.

These types of deviation have the common feature of increasing the variance-to-mean ratio of the distributions of interarrival intervals, of call arrivals during fixed periods, as well as of the offered traffic. Disturbances of the opposite character, leading to reduced variance-to-mean ratios, will not be treated here.

The three types of deviations have different character, and in order to discriminate, we will talk of variations of simple character, repetition character and overflow character.

The simple variation is most easily exemplified by the arrival process coming from a large number of independent and constant sources, where the number of sources varies with time. Typically this is the case in a business environment with arriving and departing users (morning arrival, afternoon departure, lunch hour absence).

The repetition process may be realistically modelled by an additional group of sources with a much higher call rate than the ordinary sources, and the size of this group being dependent on the system state [1].

The overflow process has a much more abrupt character than the other types, since the calls tend to come in bursts, with idle periods between the bursts. An approximate model is the interrupted Poisson process, where the arrival process alternates between zero and a constant intensity [2].

The intensity rate of change with time can be of great consequence for the mathematical models to be used. This has been shown by Conny Palm [3] who solved the fundamental partial differential equations for a process with time varying intensity. Palm introduced the terms slow and rapid fluctuations (träge und schnelle Schwankungen), depending on the time model for an intensity change compared with the time constant of the process. The simple variations as well as processes with substantial repetition components will usually have only slow fluctuations, whereas rapid fluctuations will often occur in overflow cases.

In the following we shall mostly restrict our attention to the slow fluctuation cases, and only use rapid fluctuations for references and then with special mention.

The main objective of this paper is to develop a model that takes into account not only the stochastic fluctuations around a constant arrival rate, but even fluctuations of the arrival rate itself. The way this is done is to assume that an actually measured sample of traffic (or arrivals) from a limited period (e.g. busy hour) of a single day has a distribution equal to that of a traffic generated by a memoryless arrival process with an intensity that varies with time. If this function of time can be determined from matching of a measured and a calculated distribution, it is also possible to calculate the accumulated losses during the observation period, and use this as a basis for a more adequate dimensioning than what is obtained by the constant mean assumption.
Assume a Poisson process with arrival intensity \( \lambda \). The interarrival intervals, \( T \), follow a negative exponential distribution, whereas the numbers of call arrivals, \( N_t \), during time \( t \) follow a Poisson distribution:

\[
P(T < t) = P(t) = 1 - e^{-\lambda t} \quad (1)
\]

\[
P(N_t = r) = p_r(t) = \frac{(\lambda t)^r}{r!} e^{-\lambda t} \quad (2)
\]

The distribution of \( \lambda \) itself is

\[
P(\lambda < \lambda_1) = G(\lambda_1) = \begin{cases} 0 & \text{for } \lambda < \lambda_1 \\ 1 & \text{for } \lambda \geq \lambda_1 \end{cases} \quad (3)
\]

which means that \( \lambda = \lambda_1 \) is constant.

If \( \lambda \) is no longer constant, but has a distribution function \( G(\lambda) \) different from (3), the process is no longer a Poisson process. If, however, the process is generated by a set of sources, all of which have Poissonian character, and the number of sources varies with time, the memoryless character of the total process is retained, since the probability at any instant of a call arrival in a time \( \Delta t \) (\( \Delta t > 0 \)) is only dependent on \( \Delta t \) and the instantaneous value of \( \lambda \). The arrival rate actually varies in steps, as the number of sources is an integer, but approaches a continuous function as the number of sources approaches infinity and the individual rate approaches zero.

We will now assume that \( G(\lambda) \) is a step function, and that unlimited time is spent on each step.

Then distributions (1) and (2) are defined. If we combine the results from all steps, (1) and (2) will be replaced by new distributions expressed by sums with the proper weighting [4]. These sums may be replaced by integrals when the number of steps is increased towards infinity, which implies an infinite time of second order.

There may be two dual approaches, one considering the distribution of \( \lambda \) [3], and the other considering \( \lambda \) as a time function [4]. We shall here concentrate on the time function approach, assuming an observation time \( \tau \) and an arbitrary variation of \( \lambda = \lambda_0 \) with time \( \theta \), Fig. 1. This choice is not obvious, since the distribution is in a way more general. However one might want to see directly how various time functions influence the properties of the total distribution and the grade of service (loss).

It is obvious that any variation of \( \lambda \) with time contains an implicit distribution which is unique, whereas there is an unlimited number of different time functions having the same distribution. There is, however, a unique monotone non-decreasing (as well as one monotone non-increasing) function, having this distribution. For this function \( \lambda_0 \) we have the distribution density \( g(\lambda_0) \):

\[
dG(\lambda_0) = g(\lambda_0) d\lambda_0 = \frac{d\theta}{\tau} \quad (4)
\]

Assume that \( \tau \) is subdivided in \( n \) intervals of length \( \tau/n \) and that \( \lambda \) can be considered to be constant during each interval. In interval no. \( i \), \( \lambda = \lambda_i \), and we have for an interarrival interval \( T_i \):

\[
f_i(t) dt = P(T_i < t) dt = F_i(t) - F_i(t - dt) = \lambda_i e^{-\lambda_i t} dt \quad (5)
\]

\[
1 \quad 2 \quad \ldots \quad i \quad \ldots \quad n
\]

\[
\text{Time}
\]

\[
\text{Intensity}
\]

Fig. 1. Arrival intensity \( \lambda \) varying with time.

Step approximation

Summing up the contributions from all intervals we obtain a distribution

\[
f(t) dt = P(t < T < t + dt) = \sum_{i=1}^{n} a_i [F_i(t + dt) - F_i(t)] = \frac{1}{\lambda_0} \int_{1}^{\lambda_0} \frac{\int_{0}^{\lambda_0} e^{-\lambda t} d\lambda}{\lambda_0} \quad (6)
\]

since the expected number of arrival intervals in any subperiod \( i \) is proportional to \( \lambda_i \), and the weighting coefficient therefore is

\[
a_i = \frac{\lambda_i}{\lambda_0}. \quad \text{For a continuous function we obtain}
\]

\[
\psi(t) = \frac{1}{\lambda_0} \int_{0}^{\lambda_0} f(t) = \frac{1}{\lambda_0} \int_{0}^{\lambda_0} \frac{\int_{0}^{\lambda_0} e^{-\lambda t} d\lambda}{\lambda_0} \quad (7)
\]

In a similar way we can express the distribution of the number of arrivals during a fixed interval \( t \) by

\[
p_r(t) = \frac{1}{r} \sum_{i=1}^{n} \frac{\int_{0}^{\lambda_i} e^{-\lambda t} d\lambda}{\lambda_0} \quad (8)
\]

and for the continuous case

\[
p_r(t) = \frac{1}{r} \int_{0}^{\lambda_0} \frac{\int_{0}^{\lambda_0} e^{-\lambda t} d\lambda}{\lambda_0} \quad (9)
\]

There is no weighting factor in this case, since only equal intervals are considered and the distribution in fact gives a relative number of equally long intervals rather than a relative number of calls. For the special case \( r = 0 \) we get

\[
p_r(t) = \frac{1}{\tau} \int_{0}^{\tau} e^{-\lambda t} d\lambda \quad (10)
\]

The interpretation of \( n_r(t) \) is the probability of no calls during any arbitrary interval \( t \), or in other words the probability of at least a time \( t \) until next call from any arbitrary instant. This can be compared with

\[
\psi(t) = \int_{0}^{\lambda_0} n_r(t) dz = \frac{1}{\lambda_0} \int_{0}^{\lambda_0} \int_{0}^{\lambda_0} e^{-\lambda t} d\lambda \quad (11)
\]
found by integrating (7), which gives the probability of at least a time \( t \) between two adjacent calls. \( \pi_s(t) \) and \( \psi(t) \) are survival functions. \( \pi_s(t) \) has a corresponding density function

\[ \eta(t) = -\pi'_s(t) = \frac{1}{t} \int_0^t \lambda x \exp(-\lambda x) \, dx \]  

(12)

For \( \lambda = \text{constant} \) (10) and (11) become identical \( \pi_s(t) = \exp(-\lambda t) \), whereas for any non-constant \( \lambda \) they are different.

The simplest way of solving \( \psi(t) \) and \( \psi(t) \) is to integrate once more to get the same integral as in \( \pi_s(t) \)

\[ H(t) = \int_0^t \psi(z) \, dz = \frac{1}{t} \int_0^t \lambda \exp(-\lambda z) \, dz \]  

(13)

and afterwards find \( \psi(t) = -H'(t) \) and \( \psi(t) = -\psi'(t) \).

**DISTRIBUTION MOMENTS**

The distributions given on integral form can be solved analytically for linear time functions, whereas second and higher degree approximations lead to incomplete gamma functions. These can of course be solved numerically if needed, but often it is sufficient to determine a number of moments. For this purpose we introduce two generating functions:

\[ g_1(z) = \int_0^\infty e^{zt} \psi(t) \, dt \]  

(14)

\[ g_2(z) = \int_0^\infty \frac{1}{t} e^{-zt} f(t) \, dt \]  

(15)

where \( \psi(t) \) is a continuous density function and \( f(t) \) is a discrete frequency function.

Applying (14) to (7) and (12), we obtain (16) and (17) respectively:

\[ g_1(z) = \frac{\int_0^\infty \lambda \exp(-\lambda z) \, dz}{\int_0^\infty \lambda \exp(-\lambda z) \, dz} \]  

(16)

\[ g_2(z) = \frac{1}{t} \int_0^t \lambda \exp(-\lambda z) \, dz \]  

(17)

Since the \( n \)-th ordinary moment may be expressed by

\[ m_n(\psi(t)) = \int_0^\infty t^n \psi(t) \, dt = g_{1+}(z=0) \]  

(18)

we obtain from (16) and (17)

\[ m_n(\psi(t)) = \frac{n!}{t^n} \int_0^\infty \lambda \exp(-\lambda z) \, dz \]  

(19)

and

\[ m_n(\eta(t)) = \frac{n!}{t^n} \int_0^t \lambda \exp(-\lambda z) \, dz \]  

(20)

These integrals can be solved for a wider range of functions than what is possible for the distributions (10) and (11).

In a similar way we apply (15) to (9) and obtain

\[ g_1(z) = \frac{1}{t} \int_0^t \lambda ^{n-1} \psi(t) \, dt \]  

(21)

The \( n \)-th factorial moment is

\[ F_n(\pi_s(t)) = \int_0^\infty (\int_0^t \lambda ^{n-1} \psi(t) \, dt) \, d\theta = \frac{1}{t} \int_0^t e^{-\lambda \theta (1-z)} \, d\theta \]  

(22)

giving

\[ F_n(\pi_s(t)) = \int_0^t (\lambda \theta)^n \, d\theta \]  

(23)

From the factorial moments the ordinary moments can be calculated [5] by

\[ m_n = \sum_{k=1}^n \alpha_{n,k} \cdot m_k \]  

(24)

where \( \alpha_{n,k} \) are the Stirling numbers of the second kind, defined by

\[ \alpha_{n,k} = \frac{1}{k!} \sum_{i=0}^k (-1)^{i-k} (k+1)^i \]  

(25)

The central moments \( \mu_k \) can further be calculated by

\[ \mu_k = \int_0^\infty (x - \mu_1)^k \, f(t) \, dt \]  

(26)

Other useful definitions are variance-to-mean ratio (peakness) \( \omega \), coefficient of variation \( v \), skewness \( s \), excess (kurtosis) \( \epsilon \) and relative moments \( M_n \):

\[ \omega = \frac{\mu_2}{\mu_1} \]  

(27)

\[ v = \frac{\mu_2}{\mu_1^{1/2}} \]  

\[ s = \frac{\mu_3}{\mu_1^{3/2}} \]  

\[ \epsilon = \frac{\mu_4}{\mu_2^2} - 3 \]  

\[ M_n = \frac{\mu_n}{\mu_1^n} \]  

**APPROXIMATION BASED ON INTERVAL DISTRIBUTION**

In Fig. 2 are depicted various types of time functions. One group has an initial value \( h \) and increases in one or more linear phases or non-linearly towards a final value. The other group has a period \( \tau \) of zero intensity, and then increases towards maximum. (only linear case shown). It is an obvious advantage to choose a model that can cover the whole rectangle determined by the diagonal running from initial to final value. This can be done by a two-piece linear model, which on the other hand has an "unnatural" abrupt change in the rate of increase. A power function of \( \theta \) is also a feasible choice, to be treated later.

The general case of \( q \) straight line pieces gives the following results, using (13) and (20):
Fig. 2. Various types of time functions.
Parameter notations.

From \( H(t) \) it is straightforward to find \( \psi_0(t) \) and \( \phi_0(t) \) as well as \( \psi_q(t) \) and \( \phi_q(t) \).

For \( n=1 \):

\[
m_1, n (\eta(t)) = \frac{1}{\tau} \sum_{i=1}^{q} \frac{1}{\tau_{i-1}} \ln\left[ \frac{k_i (\tau_i - \tau_{i-1})}{h_{i-1}} + 1 \right]
\]

For \( n > 1 \):

\[
m_n, n (\eta(t)) = \frac{n!}{\tau (n-1)} \sum_{i=1}^{q} \frac{1}{\tau_{i-1}} \left[ \frac{1}{h_{i-1}^{n-1}} - \frac{1}{h_{i}^{n-1}} \right]
\]

From (19) and (20) we have

\[
m_n+1 (\omega(t)) = \frac{\tau (n+1)}{D} \cdot m_n (\eta(t))
\]

where \( D \) is the denominator of (28).

Matching to the first two moments to determine \( a \) and \( h \) can be done by means of the expressions in Table 1, though an explicit solution is not available because of the logarithm in \( m_2 \). It is in fact simpler to
use $m_1$ and $m_3$, which gives the simple explicit solution:

$$a = \frac{m_3 - 6m_1 + \sqrt{m_3(m_3 - 6m_1)}}{3m_1^2}$$

$$\frac{1}{h} = \frac{m_3 + \sqrt{m_3(m_3 - 6m_1)}}{6m_1}$$

For $a$ and $h$ to be real, we must have $m_3 > 6m_1$.

Thus all intensity functions are contained within a one-by-one square with the initial value $h$ at the lower lefthand corner (Fig. 2). Using equations (33) and the last equation of (27) we obtain

$$m_1 = h\left[\frac{a}{(1+b-c)+2}\right]$$

$$M_2 = \frac{M_4}{m_1} = \frac{\frac{1}{a}[a(1+b-c)+2]}{\frac{1}{b}\ln(ab+1)+\frac{1}{c}\ln(ab+1)}$$

$$M_3 = \frac{M_4}{m_1} = \frac{\frac{1}{a}[a(1+b-c)+2]^2}{(ab+1)(a+1)}$$

$$M_4 = \frac{M_4}{m_1} = \frac{\frac{1}{a}[a(1+b-c)+2]^3}{(ab+1)^2(a+1)^2}$$

No explicit solution of (39) is possible. The equations have been solved numerically by using a procedure minimizing the function

$$F = w_2(M_3-M_1)^2 + w_3(M_4-M_1)^2$$

where $M_3$ and $M_4$ are the corresponding expressions in (39) and $M_1$, $M_2$ and $M_3$ are given numerical values. $w_2$, $w_3$, and $w_4$ are arbitrary weighting factors.

Fig. 5 shows arrival intensity time functions for the fixed values $m_1=1$, $\mu_2=1.15$, corresponding to a coefficient of variation $\nu = M_2 - 1 = 1.15$ and with various values of $M_3$ and $M_4$, as indicated.

The minimization procedure has some convergence problems, and it is difficult to vary $M_3$ and $M_4$ independently to any great extent. However, there is quite a large range of variation of $M_3$ and $M_4$ for a fixed set of $m_1$ and $\mu_2$, corresponding to distinctly different intensity time functions and hence intensity distributions.
This seems to indicate that it is important to take into account the third moment in addition to the first and second, whereas the fourth moment seems to give little additional information.

The losses can be calculated by

\[ A_s = \frac{1}{r} \int_0^\infty A_s E[A_s] d\theta \]  

(41)

where \( A_s \) is the Erlang loss formula. If the holding time is taken as time unit, \( A_s = \lambda_0 \).

In reference [4] measurements are reported on 10 busy hours, where the scaled average results for the six most uniform days has a very good fit to the exponential distribution. However, one of the six days has a coefficient of variation of \( v = 1.11 \) and two less uniform days have \( v = 1.15 \). These values correspond however, to the smallest third moment \( (M_3) \) in Fig. 5, the loss is 3.8%. For lower loss values more than a tenfold loss increase may occur when \( v = 1.15 \).

APPROXIMATION BASED ON CALL NUMBER DISTRIBUTIONS

The approach of measuring call arrival intervals is a fundamental one. However, it is not very often used in engineering practices. Counting of calls is more often used, besides traffic load measurements. It is therefore good reasons to revert to equation (9) giving in integral form the distribution of call arrivals \( r(t) \) during time intervals \( (t) \).

It is well known that in the Poisson arrival case the distribution of call arrivals during the average holding time is identical to the distribution of the offered traffic, which is equal to the traffic carried on an infinite group of servers. This increase may occur when the number of calls arriving during an average holding time is equal to the instantaneous traffic load on this group at any instant during that holding time. So there is no strict relation between instantaneous values even though the distributions are identical. The different characters of the call arrivals and the offered traffic is even more clearly understood when assuming sudden jumps in call rates, which is known to have coefficient of variation \( v = 1 \) and excess \( e = -1.2 \), may be observed. These are also the limiting values for \( \lambda_0 \) as \( a = 0 \) increasing and \( m_1 \), as can be determined by moment calculations using (23)–(27).

Omitting the details, a two-moment match is obtained by

\[ h = \frac{m_1 - 3(\omega - 1)}{1 + \frac{\omega}{m_1}} \]  

(44)

\[ h(\omega) = 6(\omega - 1) + 2\sqrt{2}(\omega - 1) \]  

(45)

Real values of \( h \) and \( kT \) require \( \omega \frac{m_1}{m_3} > 1 \).

The linear case can be obtained as a special case of a more general function. The most obvious choice would be to assume a polynomial

\[ \lambda_0 = a_0 + a_1(\omega - 1) + a_2(\omega - 1)^2 + \ldots \]  

(46)

A three-moment match will then lead to a parabola. The main limitation of a parabola in this respect is its limited coverage of possible variations of the arrival rate. A better choice is a power function

\[ \lambda_0 = h + k(\omega - 1) \]  

(47)

For simplicity, and since there is no subdivision in segments, we will assume \( t = 1 \), so \( \theta \) is relative time. The three parameters are \( h, k \) and \( x \). For great variance-to-mean ratios it may happen that the matching will require \( h = 0 \).

This means that the intensity is zero during part of the observation period, and the initial point is at some point \( t' \) on the ascissa axis in Fig. 2. An alternative function must be used in this case, and a convenient form is

\[ \lambda_0 = \left\{ \begin{array}{ll}
0 & \text{for } \theta < t' = 1 - \zeta \\
k(\theta - 1) & \text{for } \theta > t' = 1 - \zeta
\end{array} \right. \]  

(48)

with \( \zeta \) replacing the parameter \( h \).
The distribution itself, given by (49) is an incomplete gamma function, and cannot be solved:
\[ \pi_t = \int_0^1 \frac{1}{r!} (h + k \theta)^r e^{-(h + k \theta)} d\theta \]  
(49)

For simplicity we have omitted \( t \), assuming that \( t \) is the time unit. The distribution will then be the distribution of the offered traffic \( A_0 = \lambda g \), assuming only slow fluctuations.

The moments can be calculated by means of (23)-(26) to give for (47)
\[ m_1 = h + \frac{k}{x+1} \]
\[ \mu_2 = h + \frac{k}{x+1} + \frac{k^2 x^2}{(x+1)^2} \]
\[ \mu_3 = h + \frac{k}{x+1} + \frac{3k^2 x^2}{(x+1)(x+1)^3} + \frac{2k^3 x^2 (x-1)}{(x+1)(x+1)^5} \]
\[ \mu_4 = h + \frac{k}{x+1} + \frac{3h^2 + 6hk + 6hk^2 x^2}{(x+1)(x+1)^3} + \frac{k^2 (7 + \frac{2}{x+1})}{(x+1)^2} + \frac{6k^4 (\frac{1}{3x+1} - \frac{2}{(2x+1)(x+1)} + \frac{1}{(x+1)^3})}{(x+1)^3} + k^4 (\frac{1}{4x+1} - \frac{3}{(3x+1)(x+1)} + \frac{6}{(2x+1)(x+1)^3} - \frac{3}{(x+1)^5}) \]
(50)

and for (48)
\[ m_1 = \frac{\zeta k}{x+1} \]
\[ \mu_2 = \frac{\zeta k}{x+1} + \zeta k^2 (\frac{7}{2x+1} - \frac{\zeta}{(x+1)^3}) \]
\[ \mu_3 = \frac{\zeta k}{x+1} + 3\zeta k^2 (\frac{1}{2x+1} - \frac{\zeta}{(x+1)^3}) + \zeta (3x+1) (x+1) \]
\[ \mu_4 = \frac{\zeta k}{x+1} + \zeta k^2 (\frac{7}{2x+1} - \frac{4\zeta}{(x+1)^3}) + 6\zeta k^4 (\frac{1}{3x+1} - \frac{2\zeta}{(2x+1)(x+1)} + \frac{\zeta}{(x+1)^3}) + \zeta (4x+1) - \frac{4\zeta}{(3x+1)(x+1)} + \frac{6\zeta^3}{(2x+1)(x+1)^3} - \frac{3\zeta^3}{(x+1)^5}) \]
(51)

The two equation sets (50) and (51) have a common point where
\[ x = x_0 = \frac{\omega - 1 + \sqrt{(\omega - 1)(\omega - 1 + m_1)}}{m_1} \]
(52)

For \( x \geq x_0 \):
\[ h = \frac{m_1}{x+1} \frac{(x+1)(\mu_2 - m_1)}{\mu_2} \]
(53)
\[ k = \frac{x+1}{x} \frac{(x+1)(\mu_2 - m_1)}{\mu_2} \]
(54)

where \( x \) can be found explicitly from the cubic equation
\[ x^3 - \frac{9}{8} x^2 + \frac{3}{4} x - \frac{9}{8} x^3 = 0 \]
\[ x = \frac{\mu_2 - 2m_1}{(\mu_2 - m_1)^{3/2}} \]
(55)

For \( x < x_0 \):
\[ \zeta = \frac{m_1 (x+1)^2}{(\omega - 1 + m_1)(2x+1)} \]
(56)
\[ k = \frac{(\omega - 1 + m_1)(2x+1)}{x+1} \]
(57)
\[ x = \frac{2(\beta - 1) + \sqrt{\beta (\beta - 1)}}{4 - 3\beta} \]
(58)

The matching procedure should start by finding \( x_0 \) from (52), determined by \( m_1 \) and \( \mu_2 \). That gives the applicable alternative to determine \( x \) and thereafter \( h \) and \( k \), viz. \( \zeta \) and \( k \). The loss for any number \( n \) of servers can then be determined by equation (41) in a numerical form. Fourth and higher moments can be calculated.

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In Fig. 7 is shown the loss ratio as a function of variance-to-mean ratio ($\omega$) and skewness ($s$) for various numbers of servers. The offered non-stationary traffic averaged over the observation period is $m_1=10$ erlangs. The well-known significance of the second moment in relation to losses is clearly demonstrated. What is less often stated is that even the third moment has a substantial influence. This may in the author's opinion be due to the fact that the second moment is most frequently considered in overflow and grading cases, when the variations of the third and higher moments are much dependent on the variations of the second moment, and thus carry little extra information. This is not necessarily the case when second and higher moments vary because of non-stationarity of the traffic sample, when the form of the function of time is important.

CONCLUSION

The present study of non-stationary arrivals with slow fluctuations is claimed to cover most traffic situations except cases where overflow traffic or traffic smoothed by overflow is a substantial part.

The study suggests that true (measured) distributions of offered traffic rather than accumulated averages should be used to establish a dimensioning basis. Mathematical models developed confirm the well known significant influence of the second moment on losses. They further indicate that the third moment can vary over a wide range with the intensity time function, and that this may also influence the losses substantially. Further studies involving live traffic data are recommendable.

REFERENCES