ABSTRACT

The problem of optimal allocation of telephone cable drums to construction sites for pulling into underground conduits to cover a given "laying project" is considered. The problem is formulated as a cutting-stock type mathematical program where cable drums of finite length are to be allocated to laying sites with ducts of finite length. A cable can be pulled into one or more sections and is then jointed to other sections of the project. A generalized objective function is defined which takes into account the probability of future use of cable residuals in following laying projects so as to minimize total cost of trimming losses and cable joints.

The combinatorial properties of the process of generating admissible laying plans are exploited to derive a simple heuristic algorithm supported by an exact solution in the final steps of the allocation procedure. The implementation of this combined approach reduced trim losses considerably.

INTRODUCTION

A telecommunication network constitutes a hierarchical structure of multi-wire telephone cables connecting switching centers and subscribers. As the traffic in the network increases and new switching centers and subscribers are added to the system, it becomes necessary to increase the flow capacity of the network by installing new telephone cables. The additional cables have to be pulled into the underground conduits which are connected to each other at junction points known as manholes. Each time period a new "laying project" is planned to meet the needs of the growing network. A laying project is a set of underground sections comprising a connected graph which has to be "covered" by telephone cables which are pulled into the conduits and connected manually to each other at the nodes of the network - the manholes. A cable joint is in general a costly operation that requires several labor days of skilled technicians, depending on the number of wires in the cable.

Telephone cables of fixed length Q (commonly 200 meters) are usually supplied wound on drums. Long and stochastic delivery times along with unavoidable dynamic changes in construction priorities of the telephone network lead to a policy of ordering telephone cables of fixed length. Shorter lengths are later cut from these cables in order to be pulled into the conduits according to the periodic plan.

The periodic problem is to find the "best" allocation of cable drums to cover the sections of the laying project such that the economic losses due to cable residues and cable joints will be minimized.

An additional factor that complicates matters is the technological lifetime of the cables. Cable performance degrades with time and in order to guarantee high reliability cables with age beyond T planning periods are considered obsolete. Thus, when considering the economics of a certain allocation the ages of the cable drums involved should be taken into consideration.

The problem under consideration resembles the classical cutting-stock problem [4,5]. The standard procedure for such a problem is to generate columns of possible combinations of cut patterns. In our case this is equivalent to generating "laying plans" comprised of several adjacent sections of the network such that each laying plan can be "covered" by a single cable drum. In the classical approach residues are considered scraps. However, in our problem, residual cables may be used in future laying projects provided that their length is not smaller than the shortest section in the network, \( \lambda \min \) (about 30 meters), and their age does not exceed T.

Thus, a less naive objective function should be developed which takes into consideration the chances of future utilization of residues as well as the other costs involved.

We start with the basic mathematical programming formulation with a generalized objective function which takes into account the expected losses resulting from not allocating a particular available cable drum to the current laying project. We turn then to derive explicitly the cost coefficients of the objective function and study their properties. These properties are based on the probability distribution function of combined length which we develop in detail. We believe that the scope and implications of this distribution function go beyond the context of the present work. It should be applicable in various situations where a periodic column generation procedure is employed such as in crew, bus or train scheduling, cutting-stock type problems, etc.

The combinatorial difficulties stemming from the huge number of columns generated in cutting-stock type problems are well known. Various methods have been suggested and used to limit the size of the problem. (For examples see [1] and [6].) We use here a combined heuristic and exact solution procedure. The heuristic procedure exploits the combinatorial properties of the problem and turns the curse of enormous laying plans into a blessing having high chances of finding almost an exact match to any available cable drum. At a stage when the dimensions of the problem are reduced, an exact solution - using the mathematical programming formulation - is obtained. This method has been actually implemented and resulted in considerable savings in trim losses and cable obsolescence.

THE PROCESS OF GENERATING LAYING PLANS

Let \( M = \{1, 2, 3, \ldots, m\} \) be the index set representing the sections in the entire laying project. The project may be regarded as a connected graph with m arcs. This graph is always a tree, and hence has \( m + 1 \) nodes (see Figure 1 in the sequel).

A laying plan is an \( m \)-dimensional vector whose 1-th component, \( a_1 \), is either 1 or 0, depending whether section 1 is contained in the plan or not.

Let \( a_k \) be the length of section i (1 = 1, 2, ..., m), and let \((a_1, a_2, \ldots, a_m)\) be a particular laying plan. Then, the total length of such a plan is \( \lambda = \frac{1}{2} \sum a_k \). A cable drum with length Q may be allocated to a plan with length \( \lambda \) if and only if \( \lambda \leq Q \).

Two main characteristics relating to the economics of the project are associated with each laying plan: (i) its total length, and (ii) the number of cable-joints involved. Both characteristics are factors in the costs resulting
from the allocation of a particular drum to a given laying plan. If $Q_k \leq r < L_{\text{min}}$, then the residue $r$ is considered a scrap. Otherwise, the length $r$ determines the associated cost in a manner which will be developed later. Obviously, laying plans with a single joint are preferable to plans with several cable joints. Therefore, in order to reduce the problem's dimensions, we initially generate only a subset of all possible plans. This subset contains all plans having a single joint only. Plans consisting of more than one joint will have to be generated only if the post-optimality analysis requires so. It will become apparent in the sequel that the chances of such an event are remote.

Define a chain as a sequence of adjacent sections between a given pair of two mandatory joints which have no other mandatory joints between them. An admissible laying plan is a sub-chain consisting of one or more adjacent sections within a chain provided that the total length of the plan is not greater than $Q$. Consequently a laying plan requires joints only at its two end points. However, for the book-keeping of the costs resulting from connecting such a plan to the tree-project we debit the a plan by the cost of a single joint only.

We now generate the basic list of all laying plans following the criteria mentioned above. Such a list is an $m \times n$ matrix $A = [a_{ij}]$ where each column corresponds to a particular laying plan and each row to a section in the tree. In this matrix $a_{ij} = 1$ if section $i$ is contained in laying plan $j$, while $a_{ij} = 0$, otherwise.

As an example consider the graph in Figure 1.

![Figure 1](image_url)

Figure 1: An example of a cable laying project.
- indicates a manhole
- indicates a manhole where a joint is mandatory
i represents the section index (i=1,2,...,35)

Figure 1 consists of 35 sections and 36 manholes of which 12 must contain mandatory joints. This graph contains 11 chains as follows:

<table>
<thead>
<tr>
<th>Chain No.</th>
<th>Sections in the chain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 2, 3</td>
</tr>
<tr>
<td>2</td>
<td>4, 5</td>
</tr>
<tr>
<td>3</td>
<td>6, 7, 8, 9</td>
</tr>
<tr>
<td>4</td>
<td>10, 11, 12</td>
</tr>
<tr>
<td>5</td>
<td>13, 14, 15</td>
</tr>
<tr>
<td>6</td>
<td>16, 17, 18</td>
</tr>
<tr>
<td>7</td>
<td>19, 20, 21</td>
</tr>
<tr>
<td>8</td>
<td>22, 23, 24, 25</td>
</tr>
<tr>
<td>9</td>
<td>26, 27</td>
</tr>
<tr>
<td>10</td>
<td>28, 29, 30, 31</td>
</tr>
<tr>
<td>11</td>
<td>32, 33, 34, 35</td>
</tr>
</tbody>
</table>

The matrix $A$ is now generated from the above exhaustive list of chains. For each chain we generate all admissible laying plans whose length is not greater than $Q$. For example, if $Q = 200$ and the lengths of sections 1, 2, 3, ..., 8,9 are 80, 72, 110, 65, 90, 80, 50, 85, respectively, then the basic list of laying plans corresponding to the first three chains above is given in the following submatrix of $A$.

<table>
<thead>
<tr>
<th>Laying plans</th>
<th>Sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chain 1</td>
<td>1 80</td>
</tr>
<tr>
<td>Chain 2</td>
<td>2 72 1 1</td>
</tr>
<tr>
<td>Chain 3</td>
<td>3 110 1 1</td>
</tr>
<tr>
<td>Chain 4</td>
<td>4 65 1 1</td>
</tr>
<tr>
<td>Chain 5</td>
<td>5 90 1 1</td>
</tr>
<tr>
<td>Chain 6</td>
<td>6 80 1 1</td>
</tr>
<tr>
<td>Chain 7</td>
<td>7 50 1 1</td>
</tr>
<tr>
<td>Chain 8</td>
<td>8 85 1 1</td>
</tr>
<tr>
<td>Chain 9</td>
<td>9 60 1 1</td>
</tr>
</tbody>
</table>

| Laying-plan length | 80 72 110 152 182 65 90 155 |

In a similar manner, the basic list $A$ can be systematically generated, eliminating all sub-chain combinations whose length exceeds $Q = 200$ meter.

**A MATHEMATICAL PROGRAMMING FORMULATION**

The mathematical programming problem is associated with the optimal allocation of cable drums to a sub group of the (admissible) laying plans such that each section in the project will be covered exactly once, a cable drum will be allocated once at the most; and the total cost of trimming losses and cable joints will be minimized.

In this work we deviate from the classical approach where trimming losses are considered only as a linear function of the lengths of cable residues. Rather, we employ an innovative compound cost function that takes into account the probability of future use of cable residuals as well as the penalties resulting from the decision not to allocate a cable drum. In addition we also blend into the cost function the inventory holding costs of cables incurred from one period to another.
Suppose that \( D \) distinct cable drums are available for allocation to the given project. Assume (as is the common situation) that \( D \) is large enough such that a feasible solution always exists. Let \( Q_d \) be the length of the \( d \)-th drum \((d = 1,2,\ldots,D)\) and denote by \( t_d \) its age. For each cable drum \( d \) we select from \( A \) all laying plans whose length is not greater than \( Q_d \). Let the matrix \( A_d \) denote the subgroup of laying plans so selected. An element \( a_{dk} \) in the \( i \)-th row and \( k \)-th column of \( A_d \) is 1 if section \( i \) is part of the \( k \)-th laying plan in \( A_d \), and 0 otherwise.

The union of all the \( A_d \) matrices generates the allocation matrix \( A \), i.e., \( A = \{ A_1; A_2; A_3; \ldots; A_d \} \). Let \( N_d \) be the number of columns in \( A_d \). Then \( A \) is an \( mn \times d \) matrix, where

\[
D = \sum_{d=1}^{N} N_d.
\]

Define the problem variables \( x_{dk} \) and \( y_d \) \((d = 1,2,\ldots,D; \ k = 1,2,\ldots,N_d)\) as follows:

\[
x_{dk} = \begin{cases} 1, & \text{if cable drum } d \text{ is allocated to the } k \text{-th laying plan within } A_d, \\ 0, & \text{otherwise,} \end{cases}
\]

\[
y_d = \begin{cases} 1, & \text{if cable drum } d \text{ is not allocated to the project } \text{for reallocation to any future laying plan before its age } t_d, \\ 0, & \text{otherwise,} \end{cases}
\]

The mathematical programming problem is given now by

\[
\text{Minimize} \sum_{d=1}^{N} \left( \sum_{k=1}^{N_d} c_{dk} x_{dk} + \phi(t_d, Q_d) y_d \right)
\]

subject to

\[
\sum_{d=1}^{N} \sum_{k=1}^{N_d} a_{dk} x_{dk} = 1 \quad (1 = 1,2,\ldots,m)
\]

\[
\sum_{d=1}^{N} \sum_{k=1}^{N_d} x_{dk} y_d = 1 \quad (d = 1,2,\ldots,D)
\]

\[
x_{dk} y_d = 0, 1 \quad \text{all } d,k.
\]

The cost coefficient \( c_{dk} \) expresses the cost of the cable joints as well as the expected cost of trimming residues. It takes into account the chances of future utilization of residues (in a manner described in the following section). The function \( \phi(t_d, Q_d) \) (which will also be developed in the sequel) expresses the expected cost incurred by not allocating drum \( d \). The cost depends on the length \( Q_d \) as well as on the age \( t_d \) of the cable.

In matrix form the above set of constraints may be presented as

\[
\begin{bmatrix}
A_1 & A_2 & \cdots & A_d & 0 \\
1 & 1 & \cdots & 1 & 0
\end{bmatrix} \times = \begin{bmatrix} x \\ y_d \end{bmatrix} = \begin{bmatrix} 1_m \\ 0 
\end{bmatrix}
\]

where \( 1_m \) is an \( m \)-dimensional column vector of ones; \( x \) is an \( N_d \)-dimensional vector of the variables \( x_{dk} \) and \( y_d \) is a \( D \)-dimensional vector of the variables \( y_d \).

Given the set of coefficients \( c_{dk} \) and the set of functions \( \phi(t_d, Q_d) \), the program can be formally solved as a set covering, or rather set partitioning, problem \([2,3]\).

For small values of \( m \) this is the proper approach. However, in the problem under consideration \( m \) ranges typically between 30 and 100 sections so that the pure mathematical programming procedure becomes computationally very costly.

It seems reasonable to develop a more rapid solution procedure which will combine heuristic and exact methods. This will indeed be our approach. In the early steps of the allocation procedure cable drums are allocated by a heuristic method which is based on the combinatorial properties of the process of generating laying plans. In the final steps of the allocation, when the number of remaining sections to be covered becomes small, we employ the exact mathematical programming procedure formulated above. This combined approach yields excellent results and great savings.

**THE COST COEFFICIENTS \( c_{dk} \)**

Consider cable drum whose length is \( Q_d \) and its age \( t_d \) expressed in units of planning periods - is \( t_d \).

Let \( r_{dk} \) denote the length of the trimming residue obtained by allocating drum \( d \) to plan \( k \) of \( A_d \). \( r_{dk} = Q_d - r_{dk} \), where \( r_{dk} \) is the length of laying plan \( k \). If \( r_{dk} < r_{min} \) the residue is considered a scrap, and the cost incurred is assumed to be \( \gamma r_{dk} \), where \( \gamma \) is the cost of a unit length of a cable. If \( r_{dk} \geq r_{min} \), the residue might be used in the future, though at a cost \( \phi(t_d, r_{dk}) \).

The cost function \( \phi(t,r) \) represents the compound effect of future chances that this residue will be selected for some future laying plan. Thus, the total allocation cost of cable drum \( d \) to laying plan \( k \) of \( A_d \) is given by

\[
c_{dk} = \begin{cases} \gamma r_{dk} + b & \text{if } r_{dk} < r_{min} \\ \phi(t_d, r_{dk}) + b & \text{if } r_{dk} \geq r_{min} \end{cases}
\]

where \( b \) is the cost of a single cable joint. (Note that the cost \( \gamma r_{dk} \) is not brought into the objective function since the total length of the project is a constant.)

**DERIVATION OF \( \phi(t,r) \)**

Consider an arbitrary cable drum of length \( Q_d \), age \( t \) and unit-length cost \( \gamma \), which is allocated to a particular laying plan of length \( L \). The residue is \( r = Q_d - L \).

Let \( S \) be a random variable denoting the length of some future laying plan in a future project, and let \( F_S(s) = P(S \leq s) \) be its probability distribution function. Define \( \theta(1) = 1 - F_S(1) \). We defer the explicit derivation of \( F_S(s) \) to a later stage, but it is clear that it should satisfy \( \theta(0) = 1 - F_S(0) = 0 \) while, for \( r < L_{min} \), \( \theta(r) = P(S \leq r) = 1 \). The probability that the above cable residue, whose residual lifetime is \( T - t \) planning periods, will not be considered for reallocation to any future laying plan before its technological deterioration at age \( T \) is

\[
\theta_j(1-\theta) = \theta_j^{T-t}
\]

In such a case the cable is held for \( T-t \) periods and then becomes obsolete. Thus, the total cost incurred is \( \gamma r + \theta_j^{T-t} \), where \( \theta_j \) is the holding cost of a unit length of cable for one period.

If the residue \( r \) is allocated to some laying plan at some time \( v \), \( t < v < T \), then only the holding cost \( h \gamma r(T-t) \) is incurred. It follows that the expected future costs, \( \phi(t,r) \), incurred by a residue \( r \) having age \( t \) are given by

\[
\phi(t,r) = \gamma r + \sum_{v=t}^{T-t} \theta_j^{v-t} + \gamma \sum_{v=t}^{T-t} \theta(j) \theta_j^{v-t}
\]

where \( \theta_j = (1-\theta_j)^{T-t} \) and \( \gamma \) is the unit-length cost of the cable.
Note that we assume that if the allocation of the residue \( r \) occurs within the lifetime span of the cable, the scrap value after this (second) allocation is negligible. This assumption has been verified by the actual results of allocation.

**SOME PROPERTIES OF \( \phi(t,r) \)**

We first observe that with no technological obsolescence, i.e., when \( T = \infty \), \( \phi(t,r) = h r/(1-h) \) for \( \theta = \Theta(t,r) < 1 \). This is clearly understood since \( [1-\Phi(r)]^{-1} \) is the expected number of periods until a cable with residue \( r \) is considered for allocation.

Next observe that \( \theta(t,r) = \gamma r \) for all \( r \leq Q \), and \( \phi(t,Q) = hQ \) for \( t < T \). The last result follows since, for \( r = Q \), \( \theta(t,Q) = 0 \), i.e., we certainly consider such a drum for the next allocation, and therefore, only the holding costs of moving from one period to the next one are involved.

The effect of age may be examined via the difference \( \phi(t,r) - \phi(t-1,r) \). For \( 0 < t \leq T \) we have,

\[
\phi(t,r) - \phi(t-1,r) = r(\gamma(1-\theta)-h)T^{-t}
\]

If \( r = Q \) then \( \Phi(Q) = 1 \) and, for \( t < T \), \( \phi(t,Q) = \phi(t-1,Q) \) since in both cases we may use the cable drum immediately at the next period.

If \( r < \theta_{\text{min}} \), \( \phi(t,r) = (T-t)hr + \gamma r \) since we would have to hold the cable until its deterioration date. Obviously, in such a case it is preferable to consider right away the residue \( r \) as a scrap and thus we modify \( \phi \) to be \( \phi(t,r) = \gamma r \) for \( r < \theta_{\text{min}} \).

In any other case where \( 0 < \gamma(r) < 1 \), \( \phi(t,r) > \phi(t-1,r) \) if and only if \( \gamma(1-h) > h \). That is, if two residues at ages \( t-1 \) and \( t \) have the same length it pays not to use the younger one if the holding cost rate falls short of the expected "usage" cost \( \gamma(1-h) \).

Finally we observe that, for \( \gamma(r) = 1 - h/r \), \( \phi(t,r) = \gamma r \) for all \( t \) \((0 < t < T)\). It also follows that, for \( r < \hat{r} = F_{S}(h/r) \), \( \phi(t,r) > \gamma r \), while for \( r > \hat{r} \), \( \phi(t,r) < \gamma r \).

Typical graphs of \( \phi(t,r) \) for \( m = 30 \), \( Q = 200 \), \( T = 30 \), \( h = 1 \) and \( \gamma = 10 \) or 40 are presented in Figure 2.

**RELEVANCE OF \( \phi(t,r) \)**

To demonstrate the relevance of the cost function \( \phi(t,r) \) to the problem under consideration, we present the following example.

Suppose the program consists of two cable drums of equal length, \( Q \), and a single laying plan whose length is \( \theta < Q \) such that \( r = Q - \theta \geq \theta_{\text{min}} \). Suppose that the ages of the cables are \( t_1 = T-1 \) and \( t_2 = T \). There are two alternative solutions: (i) \( x_{11} = 1 \), \( x_{12} = 0 \); or (ii) \( x_{11} = 0 \), \( x_{12} = 1 \). If the cost of no allocation, \( \theta(t,r) \), is ignored, the total costs for the two solutions are \( c_{11} = \phi(T-1,r) + b \) and \( c_{12} = \phi(T,r) + b \), respectively. Now, \( c_{21} - c_{22} = \phi(T,r) - \phi(T-1,r) = \gamma(1-h) + Q(\gamma-h) > 0 \). Thus, the second solution is preferable to the first, which is a reversed conclusion from the case where the no-allocation costs are ignored.

To summarize, the inclusion of the "no-allocation cost" component in the objective function is aimed to counter-weight the costs of residual scrap and storage with the expected cost of scrapping good "old" drums due to no-allocation during their entire lifetime.

**THE PROBABILITY DISTRIBUTION FUNCTION OF COMBINED LENGTHS**

It has been shown that the decision whether to allocate a given cable drum to a particular laying plan or not depends heavily on the probability that the total length \( r \) will be available for utilization in some laying plan in the future. This probability is primarily a function of \( F_{S}(r) = F_{S}(r|h) \), which is the probability that in the next project there will be an admissible laying plan with length \( S \) not greater than the residue \( r \). Thus, it is necessary to derive the probability distribution function of \( S \).

Consider a communication network consisting of a large number of single sections. Let \( G_{k}(E) \) be the probability distribution function of single section lengths. For each communication network \( G_{k}(r) \) is known and satisfies \( G_{k}(\theta_{\text{min}}) = 0 \) and \( G_{k}(Q) = 1 \). A project is a random selection of \( n \) sections (lengths) out of the p.d.f. \( G_{k}(\cdot) \). Laying plans are generated by combining randomly several sections from within the sections of the project. Let \( S_{k} \) be the length of a plan comprised of \( k \) sections so selected. Since \( k \) is usually one, two or three (the range of \( k \) is \( 1,2,3,\ldots \)) and \( Q = Q/k \min \), where \( Q \) denotes the largest integer not exceeding \( Q \) then it may be assumed that \( S_{k} \) is the sum of \( k \) i.i.d. random variables \( L_{1}, L_{2}, \ldots , L_{k} \) having the common p.d.f. \( G_{k}(\cdot) \). That is, \( S_{k} = L_{1} + \sum_{i=2}^{k} L_{i} \). The number of k-combinations (combinations with exactly \( k \) sections) is \( m!(m-k)!/(m-k)! \). However, not every combination is admissible. If \( \alpha_{k} = P(S_{k} = Q) \) then there are only \( (1-\alpha_{k})m!(m-k)!/(m-k)! \) k-combinations which may be considered for allocation. It follows that the total number of admissible laying plans that may be generated is \( \sum_{k=1}^{K} (1-\alpha_{k})m!(m-k)!/(m-k)! \) and hence, the probability that an arbitrary admissible laying plan is a k-combination is given by

\[
p(k) = \frac{(1-\alpha_{k})m!(m-k)!}{\sum_{k=1}^{K} (1-\alpha_{k})m!(m-k)!/(m-k)!} \quad (k=1,2,\ldots,w).
\]

Denote by \( F_{S_{k}}(\cdot) \) the probability distribution function of \( S_{k} \). Then, \( F_{S_{k}}(E) = P(S_{k} \leq E) = G_{k}^{k}(E) \), where \( G_{k}^{k}(\cdot) \) denotes
the k-fold convolution of \( G_L(\cdot) \) with itself. It follows immediately that \( \alpha_k = 1 - G_L^k(Q) \).

Let \( S \) be the length of an arbitrary admissible laying plan. Then, for \( m_{\min} < k < Q \),

\[
P(S \leq k) = \frac{\sum_{i=1}^{k} G_L^i(Q) / [(m-k)!k!]}{\sum_{i=1}^{Q} G_L^i(Q) / [(m-i)!i!]}
\]

The density of \( S \), \( \psi_S(\cdot) \), is given by

\[
\psi_S(\cdot) = \frac{\sum_{i=1}^{k} G_L^i(\cdot) / (m-k)!k!}{\sum_{i=1}^{Q} G_L^i(\cdot) / (m-i)!i!}
\]

where \( G_L^i(\cdot) \) is the density of \( G_L(\cdot) \).

If \( m \gg k \), then \( m_1/(m-k) \approx k \). In such a case,

\[
\psi_S(\cdot) \approx \frac{\sum_{k=1}^{m} G_L^k(\cdot)/k!}{\sum_{k=1}^{m} G_L^k(\cdot)/(1-a_1)/(m-1)!1!}
\]

As was indicated above, for the type of projects dealt with in this work, \( m \) is typically between 50 and 100 and \( \omega \) is not greater than 5.

The basic (discrete) frequency distribution of single section lengths (representing \( G_L(\cdot) \)) used for the calculations of \( \theta(r) \) and \( \phi(t,r) \) is given in Table 1.

Representing graphs of \( \theta(r) \) for \( m = 10 \) or 30, and \( Q = 200 \) are drawn in Figure 3.

<table>
<thead>
<tr>
<th>Single section length (meters)</th>
<th>Relative frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.0047</td>
</tr>
<tr>
<td>35</td>
<td>0.0109</td>
</tr>
<tr>
<td>40</td>
<td>0.0337</td>
</tr>
<tr>
<td>45</td>
<td>0.0745</td>
</tr>
<tr>
<td>50</td>
<td>0.0995</td>
</tr>
<tr>
<td>55</td>
<td>0.1090</td>
</tr>
<tr>
<td>60</td>
<td>0.1110</td>
</tr>
<tr>
<td>65</td>
<td>0.1095</td>
</tr>
<tr>
<td>70</td>
<td>0.1085</td>
</tr>
<tr>
<td>75</td>
<td>0.0926</td>
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<td>0.0045</td>
</tr>
<tr>
<td>125</td>
<td>0.0034</td>
</tr>
</tbody>
</table>

Table 1: The basic frequency distribution (representing \( G_L(\cdot) \)) used for the calculations of \( \theta(r) \).

A COMBINED HEURISTIC AND EXACT SOLUTION PROCEDURE

The curse of the combinatorial properties of the process of generating laying plans can be turned into a blessing. The fact that there are so many admissible laying plans can be exploited to derive a simple and direct scanning procedure for allocating cable drums to laying plans. The idea is that since the number of combinations is so large there is always a very high probability of matching a cable drum with a given length \( Q_d \) with some laying plan of almost equal length. Thus, the procedure is as follows:

(i) Rearrange the basic matrix \( A \) of laying plans in an ascending order of total lengths.

(ii) Arrange the available cable drums in an ascending order of length. (Some of the cable drums have full length \( Q \), and the others have smaller lengths being the residues of previous allocations.) If two cables have the same length, list them in a descending order of their age \( t \).

(iii) Pick the first cable drum in the drum list and try to match it with a laying plan from within the matrix \( A \). A match occurs between a cable drum of length \( t \) if \( Q > t \) but \( Q - t \leq 1 \) meter. If a match is obtained, strike out the corresponding drum and plan from their lists. Also strike out from \( A \) all plans that have common sections with the matched plan and delete the section of the matched plan. Repeat step (iii). If a match is not obtained, pick the next cable drum and repeat step (iii).

(iv) If all sections have been matched, stop. A complete (good) allocation of cable drums to cover the project is at hand. Otherwise, if \( m \) is still large, start step (iii) again with the remaining two lists, where a match is redefined at each iteration such that \( r = Q - t \leq 2 \) meters.

(v) Continue with this process until \( m \) becomes small where a match is realigned at each iteration such that \( r = Q - t \leq 3,4,5 \) respectively.

(vi) With small numbers of remaining uncovered sections and their corresponding matrix of laying plans, solve the exact mathematical programming problem considering only the remaining unmatched cable drums.

To conclude, the implementation of the above two-step allocation procedure - heuristic followed by an exact solution - has yielded great savings in trim residues and cost of cable joints.
REFERENCES


