REDUCTION OF THE RANK OF THE SYSTEM OF LINEAR EQUATIONS TO BE SOLVED FOR THE EXACT CALCULATION OF THE LOSS PROBABILITY OF LINK SYSTEMS

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ABSTRACT

This paper deals with a special problem - the reduction of the rank of the system of equations for the state of probability -, which occurs by exactly determining the loss probability.

The reasons for a possible reduction of the rank of the system of equations for the state of probabilities are the symmetries with regard to the structure of the link systems and the traffic carried by the multiples. The reduction was facilitated by the introduction of a new kind of state description, the so-called "state vector".

The principle of the reduction algorithm is illustrated by means of the "two-stage link system". As indicated in /6/ the method described in this paper is also applicable to multitstage link systems.

The essential result of this paper is that the simple reduction of the number of unknowns has made it possible to vary the structure of the link systems within wider limits than before.

1. INTRODUCTION

The state "a call incoming at the inlet of the link system cannot be connected with the desired outlet" is called state of congestion or, in brief, congestion. The congestion probability connected therewith can be determined by
- simulation
- approximation methods or
- exact calculation.

Up to now, it has in particular been G.P. Basharin and W. Lörcher /1,3/, who have been concerned with the exact calculation of link systems. In the papers of Basharin and Lörcher algorithms are indicated for setting up the equations for the state of probability of a link system. Moreover, the publications of V.E. Benes /2/ deal with a special type of link systems with \( k_1 = 1 \), i.e. the multiples have the same number of inlets and outlets.

The number of unknowns (probabilities for the state patterns) occurring with the exact calculation are great. Under specific conditions, which are dealt with in detail in item 4, different probabilities have, however, one and the same numerical value (for reasons of symmetry) /3,6/. For the two-stage link system with the structures \( G_1 = 5 \), \( k_1 = 3 \) and \( k_2 = 3 \) there are, for instance, a total of 12 576 different patterns of occupancy. Assuming

- that each multiple of stage 1 carries the same load and
- that, in the case of path finding for the through-connection of a call, a free outlet of a multiple of stage 1 is selected at random,

there is, however, the same probability of occurrence of a great number of state patterns. In the example considered above, there exist, for instance, only 123 groups (classes) of states. Each of these classes comprises state patterns, whose probability of occurrence is the same. As a result, the determination of the state probability and the parameters (loss, etc.) to be derived therefrom is considerably facilitated.

In /6/ a generally applicable algorithm (for two-stage or multitstage link systems) was derived for this reduction of the number of unknowns (in the system of linear equations to be solved). The principle of this algorithm is described in this paper. The two-stage link system is used as an example. However, the basic idea of this algorithm is also applicable to s-stage link systems /6/.

The reduction of the number of unknowns (rank of the system of equations) - by means of the algorithm mentioned above - now enables the exact calculation of greater link systems than those calculated by /3/.

2. LINK SYSTEM STRUCTURE, HUNTING METHOD, TRAFFIC

2.1 STRUCTURE

Figure 1: s-stage link system

where

\[ I_1 = \text{number of inlets per multiple of stage 1} \]
\[ K_1 = \text{number of outlets per multiple of stage 1} \]
\[ G_s = \text{number of multiples of stage } s \]
\[ k_r = \text{number of outlets of a multiple of stage } s \text{ in group } r \]

Similar to the above, \( I_2, K_2, G_2 \), etc.
The algorithm derived in /6/ allows the link systems to be divided into link blocks. However, this paper deals only with the following structures.

Let the wiring between the stages be sequential /3/. In the following, the number of links \( l \) from one multiple of stage \( v \) to one multiple of stage \( v + 1 \) is always assumed to be \( l = 1 \). This results in the relation

\[
G_{v+1} = K_v \quad \text{and} \quad I_{v+1} = G_v
\]  

(1)

2.2 HUNTING METHOD

Two hunting methods are considered, viz.

- sequential hunting for the outlets of the multiple of stage \( v \) from a home position (in brief called "sequential hunting") and
- random hunting for a free outlet of the multiple of stage \( v \) (in brief "random hunting") /3/.

2.3 TRAFFIC

The following type of traffic is dealt with below:

- Pure chance traffic of type 1 (PCT 1)
  
  An infinite number of sources produces the traffic offered; the call intensity \( \lambda \) is constant and independent of the number of occupied sources (Poisson input).
  
  - The distribution of the holding time is assumed to be negatively exponential with the mean value \( h \).

3. DESCRIPTION OF THE STATE OF TWO-STAGE LINK SYSTEMS WITH GROUP SELECTION

3.1 PRELIMINARY REMARKS

The terms

- state matrix and
- state vector

used below describe the state of occupancy of a link system and have no mathematical significance. The term "state matrix" is used in traffic theory /1,3,5,6/. Analogously, the term "state vector" was introduced in /6/ as a one-column and one-row state matrix, respectively.

3.2 THE STATE MATRICES \( S \) AND \( S^* \) AND THE STATE VECTOR \( S \)

\begin{figure}[h]
  \centering
  \includegraphics[width=\textwidth]{Figure2}
  \caption{Figure 2}
\end{figure}

Let there be a two-stage link system for group selection as shown in Figure 2. This figure shows the numbering of the multiples of stages 1 and 2 and the numbering of the outlets of a multiple of stage 1. The following definition is introduced.

Definition A

State matrix \( S \) denotes unequivocally the state of occupancy of the links /1,3,6/. The dimension of the matrix is \( G_v \times K_v \), equal to the number of links. Its elements are to be designated by \( s_{i,j} \).

\[
S = \begin{bmatrix}
  s_{1,1} & s_{1,2} & \cdots & s_{1,j} & \cdots & s_{1,K_v} \\
  s_{2,1} & s_{2,2} & \cdots & s_{2,j} & \cdots & s_{2,K_v} \\
  \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
  s_{G_v,1} & s_{G_v,2} & \cdots & s_{G_v,j} & \cdots & s_{G_v,K_v} \\
\end{bmatrix}
\]

with \( i \in \{1,2,\ldots,G_v\} \) and \( j \in \{1,2,\ldots,K_v\} \).

The number of rows of \( S \) is equal to the number of multiples in stage 1, i.e. equal to \( G_v \). The number of columns of \( S \) is equal to the number of outlets of a multiple of stage 1, equal to \( K_v = G_v \). The element \( s_{i,j} \) designates the state of occupancy of outlet \( j \) of multiple \( i \) of stage 1. Let

\[
s_{i,j} = 0, \text{ if outlet } j \text{ of multiple } i \text{ of stage } 1 \text{ is not occupied and } s_{i,j} = r, \text{ if outlet } j \text{ of multiple } i \text{ of stage } 1 \text{ is occupied in group } r.
\]

For further considerations it is now advisable (see \( S^* \) below) to describe the state

"outlet \( j \) of multiple \( i \) of stage 1 occupied" in such a way that the value of the element \( s_{i,j} \) depends on column \( j \). Therefore, a new state matrix, matrix \( S^* \), is defined.

Definition B

The state matrix \( S^* \), which has also the dimension \( G_v \times K_v \), denotes the state of occupancy of the \( G_v \times K_v \) outlets of stage 1. Hence

\[
s^*_{i,j} = 0, \text{ if outlet } j \text{ of multiple } i \text{ of stage } 1 \text{ is not occupied and } s^*_{i,j} = r, \text{ if outlet } j \text{ of multiple } i \text{ of stage } 1 \text{ is occupied in group } r.
\]

with \( i \in \{1,2,\ldots,G_v\} \) and \( j \in \{1,2,\ldots,K_v\} \).

\[
S^* = \begin{bmatrix}
  s^*_{1,1} & s^*_{1,2} & \cdots & s^*_{1,j} & \cdots & s^*_{1,K_v} \\
  s^*_{2,1} & s^*_{2,2} & \cdots & s^*_{2,j} & \cdots & s^*_{2,K_v} \\
  \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
  s^*_{G_v,1} & s^*_{G_v,2} & \cdots & s^*_{G_v,j} & \cdots & s^*_{G_v,K_v} \\
\end{bmatrix}
\]
Hence, there is the relation
\[ s^*_{i,j} = s_{i,j} \cdot r^{i-1} \]  
(2)
between the elements \( s^*_{i,j} \) of matrix \( S^* \) and the elements \( s_{i,j} \) of matrix \( S \).

In addition, \( IS \) holds:
If the state of occupancy of the links of a two-stage link system is denoted by state matrix \( S^* \), the sums of the \( G \) rows alone also clearly denote the state of occupancy of the links, in other words: the state of the \( G \cdot K \) outlets of the multiples of stage 1. This fact leads to definition C.

Definition C
The sum of row \( i \) of state matrix \( S^* \) is assumed to be \( s^*_{i} \). The sequence of numbers of the sums of row \( s^*_{1}, s^*_{2}, \ldots, s^*_{G} \) is called state vector \( s^* \) with \( i \in \{ 1, 2, \ldots, G \} \) and \( s^*_{i} \in \{ 0, 1, 2, \ldots, r^{i-1}\} \).

As can be learnt from the following, this representation is very advantageous.

There is the following relation between the elements of the state vector \( S^* \) and the elements of the state matrices \( S \) and \( S^* \) (transformation relation):
\[ s^*_{i,j} = \sum_{i=1}^{K_i} s_{i,j} \quad \text{and} \quad s_{i,j} = \sum_{i=1}^{K_i} r^{i-1} s^*_{i,j} \]  
(3)

3.3 THE NORMED STATE MATRIX \( SN \) AND THE GENERAL STATE VECTOR \( T \)

Sometimes it is only of interest to know whether a link is occupied or not, but it is not necessary to know the group in which the link has been seized. This is, for instance, true if the general state vector \( T \) is to be determined or in the case of certain transitions for the setting up of equations for the state probability with the BL algorithm. A binary state matrix \( SN \) is introduced for this purpose.

Definition D
The following holds for group selection:
The (binary) normed state matrix \( SN \) denotes the state of occupancy of the links. However, \( SN \) does not indicate in which group \( r \) the link concerned is occupied. The dimension of the matrix is \( G \cdot K \). Its elements shall be designated by \( s_{n,i,j} \).

\[ SN = \begin{bmatrix}
    s_{n,1} & s_{n,2} & \cdots & s_{n,1} & \cdots & s_{n,K_i} \\
    s_{n,2} & s_{n,2} & \cdots & s_{n,2} & \cdots & s_{n,2,K_i} \\
    \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
    s_{n,1} & s_{n,2} & \cdots & s_{n,1} & \cdots & s_{n,1,K_i} \\
    s_{n,0} & s_{n,0} & \cdots & s_{n,0} & \cdots & s_{n,0,K_i} \\
\end{bmatrix} \]

\[ \overline{T} = \{ t_1, t_2, \ldots , t_1, \ldots t_{K_i} \} \]  
(4)

3.4 PERMITTED STATE PATTERNS

Definition E
State patterns which satisfy the above condition (4), are "permitted state patterns". Permitted state patterns are all technically reasonable patterns.

3.5 EXAMPLE

The state matrices \( S, S^* \) and \( SN \), the general state vector \( T \) and the state vector \( S^* \) are plotted below for the indicated pattern of occupancy.

4. REDUCTION OF THE NUMBER OF UNKNOWNS

4.1 THE STATE PROBABILITIES (UNKNOWNs)
The congestion probability \( B \) is a function
\[ B = f (\text{traffic offered} \ A, \ \text{structure}, \ \text{hunting method}, \ \text{group of trunks}). \]
The arrangement for the exact calculation of this function is the so-called statistical equilibrium [3-page 38]. Here, the case of the stationary traffic offered is considered. By means of the statistical equilibrium according to K. Erlang, a system of linear equations is obtained for the state probabilities \( P(\{x\}) \), where \( \{x\} \) is a state pattern which contributes to the state \( k \) links occupied". The nomenclature was chosen according to /3/.

This arrangement is identical with the Chapman-Kolmogoroff equation for the stationary birth-death process.

Basharin and Lörcher /1,3/ have indicated algorithms which enable the system of equations for the state probability to be set up for the link system concerned. In /6/ the abbreviation BL-algorithm (Basharin-Lörcher) was introduced for these algorithms.

4.2 PREREQUISITES FOR THE REDUCTION OF THE NUMBER OF UNKNOWNS

The basic prerequisites for a possible reduction of the number of unknowns in the system of linear equations to be solved are

I. the symmetrical arrangement of the link system and
II. the uniform load carried by the multiples in stage 1.

In this context, symmetrical arrangement of the link system means that the \( G \) multiples in a stage \( v \) have all uniform \( I_v \) inlets and uniform \( K_v \) outlets. Most of the link systems used in practice meet this requirement.

Based on these two basic prerequisites, the number of unknowns can be further reduced if

III. 1) the outlets of the multiples of stage 1 are "hunted at random" or if
III. 2) the outlets of the multiples of stage 1 and the outlets of the multiples of stage 2 are "hunted at random" etc. up to the link system where

III. \((s-1)\) the outlets of the multiples of stage 1 to stage \( s-1 \) are "hunted at random".

In this case, the reason for the possible reduction of the number of unknowns is as follows: If prerequisites I and II are fulfilled, random hunting for the outlets of the multiples of stage 1 to stage \( v \) leads to an even load on the multiples of stage \( v + 1 \).

4.3 EQUIVALENT STATE PATTERNS

The state patterns \( \{x\} \) can be described by the state matrix \( S \) or the state vector \( \vec{S} \). The general state of occupancy (number of seizes per multiple) is described by the general state vector \( T \).

The prerequisites for the reduction of the number of unknowns (state probabilities) indicated in item 4.2 above lead to a system-inherent equality of state probabilities of these state patterns, which are transferable from one matrix to another one by "rearrangement" of the patterns \( \{x\} /6/.

This enables the state patterns to be divided into classes of the same state probability. For each class of equal state probability only one state pattern is required to represent this class. In item 4.4 below, this pattern is called characteristic state pattern. Now the following definition is introduced:

Definition G

State patterns, which can be transferred into another matrix by rearrangement, are called equivalent patterns (matrices, vectors) /3,5,6/. Equivalent patterns belong to one and the same class of equal state probabilities. The classes of the same state probability are called equivalence classes.

The state patterns \( \{x\} \) can be rearranged as follows /6/:

a) by the interchange of rows in the state matrix \( S \) (equivalent to the interchange of elements for the state vector \( \vec{S} \));

b) by the interchange of columns for the state matrix \( S \);
c) by the interchange of elements for the general state vector \( T \).

Example

The following three patterns are transferable from one matrix to another by an interchange of rows:

\[
\begin{pmatrix}
1 & 1 & 0 \\
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

5. CHARACTERISTIC STATE PATTERNS

5.1 PRELIMINARY REMARKS

The representative of the patterns of an equivalence class is called characteristic (ch) pattern /6/. An algorithm for determining the ch patterns was derived in /6/. The principle of this algorithm is described below for a two-stage link system. An algorithm for an s-stage link system is given in /6/.

As can be learnt from /6/, it is advisable to reduce the number of unknowns step by step. The introduction of the vectors allows simple criteria to be indicated for the calculation of the ch state patterns.

5.2 THE CHARACTERISTICS OF THE CH STATE PATTERNS IN THE CASE OF "SEQUENTIAL HUNTING" AND EQUAL VOLUME OF TRAFFIC CARRIED BY THE MULTIPLES OF STAGE 1

In the event of sequential hunting and an equal volume of traffic carried by the multiples of stage 1, all state matrices whose patterns are transferable by an interchange of rows, are equivalent to one another. Now, those patterns are choosen as ch state patterns, to whose state vectors \( \vec{S}_a \) (\( a \): serial number) the following condition applies

\[
\begin{align*}
S_{a,1} & \geq S_{a,2} \geq \ldots \geq S_{a,i} \geq \ldots \geq S_{a,s} \\
& = (6)
\end{align*}
\]

with \( a \in \{1,2,\ldots, NE\} \) and \( NE \): number of permitted patterns.

The state vectors \( \vec{S}_a \) which fulfill the above condition (6), are called characteristic vectors \( \vec{C}_a \). The set of the state patterns \( \vec{C}_a \) is \( \vec{Q}_a \).

The relation (6) leads to the term "combination with repetitions" of \( G \) elements \( c : \{0,1,2,\ldots, r^{ne} - 1\} \). The above is known from the theory of combinations. With (6) we obtain

Theorem a

The sequences of the elements \( c \) of the ch
vectors $C$ can be interpreted as "combinations with repetitions" of $G$, elements $c_i \in \{0,1,2,\ldots, r^m-1\}$. The number of ch vectors is equal to the number $NC$, the number of "permitted combinations with repetitions" of $G$, elements.

Example

For this link system we have $N_5 = 49$ permitted state patterns. By checking which of these 49 state patterns meet condition (6), we obtain the patterns indicated in Table 1.

Starting from a given ch vector $C_B$, the following theorem b holds for the state vectors $S_G$ being equivalent to $C_B$ ($\beta$ serial number of the ch vectors):

$$t_1 \leq t_2 \leq \ldots \leq t_1 \leq \ldots \leq t_{K_1} \leq t_{K_1}$$

The elements $t_i$ are the elements of the general state vectors $T_i$ which belong to the state matrices $S_G$ (see item 3.3).

The subset of the state patterns of $\Omega^{(p)}$, to which the above relation (7) is applicable, is designated by $\Omega^{(p)}$ and the vectors of the set $\Omega^{(p)}$ are designated by $C_{V}$ (Y serial number).

While the interchange of rows is carried out completely, the interchange of columns by means of relation (7) is incomplete. Hence, in the set $\Omega^{(p)}$ defined above, there are still some state patterns which can be transferred to another matrix by the interchange of columns. By setting up the equations for the state probability by means of RL algorithms it is, however, advisable not to eliminate these patterns /see 6/. According to 6 - theorem 4 - f, page 120/ these remaining equivalent patterns can be eliminated, too.

5.4 CARDINALITY OF THE STATE PATTERNS

Theorem b applies to the cardinality $M_{\beta}^{(p)}$ of the ch state vectors (ch state patterns) $C_{\beta}$. The cardinality $M_{\beta}^{(p)}$ of the vectors $C_{\beta}$ is equal to the product of $M_{\beta}^{(f)}$ and $M_{\beta}^{(r)}$. Cardinality $M_{\beta}^{(f)}$ is the cardinality of the general state vector $T$ which belongs to $C_{\beta}^{(f)}$. Theorem b holds analogously for $M_{\beta}^{(r)}$ (because of (7)).

6. DETERMINATION OF THE CH STATE PATTERNS

Let the structure of the system $G_i, K_1$ and $K_2$ be given. The characteristics of the ch state patterns are known (see item 5). There are two possible approaches to determine the ch patterns (ch vectors).

Approach I

All state vectors $S_G$ which meet condition (6) are given and those vectors which do not meet condition (5), i.e. which are not permitted state patterns, are eliminated. In /6/ this algorithm was called BU algorithm.

Approach II

The general state vectors $S_T$ are given and the state vectors $S_G$ - which belong to the general state vectors $T$ and meet condition (6) - are determined. In /6/ this algorithm, which makes use of this principle, is called "generating algorithm".

The following characteristic of the ch vectors $C_{V}$ which result from the general state vector $T$ is of importance for the "generating algorithm".

Theorem c

The elements $c_{V,Y}$ of the ch vectors $C_{V,Y}$, which are derived from the general state vector $T_V$, from "partitions of the length n without regard to order (with given sum $S_P$)". Relation (8) applies to the sum $S_P$. Let $S_{P}$ be the number of partitions which are derived from $S_P$. $NT$ is assumed to be the total number of the general state vectors determined by the structure of the system $G_i, K_1$ and $K_2$. For the indices there holds

$$\sum_{V=1}^{Y} t_{V,Y} = \sum_{i=1}^{\max (t_{V,Y})} c_{Y,i} \leq N_T$$

For the indices for which $\max (t_{V,Y})$ is equal to the minimum number of elements $c_{V,Y}$, unequal to zero.

$$S_{P} = \sum_{k=1}^{K_1} S_{P_{V,k}} \sum_{j=1}^{r^m-1} c_{V,j} = \sum_{i=1}^{K_2} c_{V,i} \cdot r^m - 1 \cdot \sum_{i=1}^{r^m-1} \max (t_{V,Y})$$

In the event of "random hunting" in particular, approach II has a great advantage because "random hunting" requires only those state patterns which meet condition (7). In this case, the number of general state vectors to be given is considerably smaller than that needed for "sequential hunting".

7. ORDER HIERARCHY FOR LINK SYSTEMS

An adequate classification scheme for link systems was derived in /6/. Here, the result is given for two-stage link systems.
be the order matrix for the characteristic vectors \( G \) (partitions) of the given link system with the structure of the system \( G, K, \) and \( K_2 \). The elements of \( \Phi \) are designated by \( \phi \). They are subsets of \( \Omega(\bar{G}) \). The ch vectors \( G \), which belong to \( \Omega_n \), have the following characteristics.

1) The partitions have exactly the length \( n \), i.e. exactly \( n \) elements of the ch vectors \( G \) are unequal to zero, in other words: seizures take place in exactly \( n \) multiples of stage 1.
2) For one element \( t_j \) of the general state vector \( T \) at least there applies

\[
t_j = m.
\]

For the other \( K_1 - 1 \) elements of \( T \) there applies

\[
t_j \leq m.
\]

Hence, at least \( m \) elements of \( G \) are unequal to zero. In other words: seizures take place at least in \( m \) multiples of stage 1. Exactly \( m \) outlets of one multiple \( j \) of stage 2 are occupied.

In the order matrix \( \Phi \) (see Definition H, page 6) there are two parts within which the sets \( \Omega_n \) are empty, i.e. there are no further ch vectors \( G \). Let us first consider part I.

**Part I of \( \Phi \), i.e. \( m > n \)**

From characteristics 1) and 2) in Definition H, page 6 there follows that part \( m \leq n \) has only ch states. In part II there is no state pattern at all (technically impossible).

The following abbreviation is introduced

\[
f_m = K_1 \cdot m = G_1 \cdot m
\]

with \( m \in \{1, 2, \ldots, K_1\} \)

**Part II of \( \Phi \), i.e. \( m > f_m \)**

In part II the sets \( \Omega_n \) are also empty because in column \( m \) of matrix \( \Phi \) a maximum number \( K_1 \) of elements \( t_j \) of \( T \) can have the value \( m \). The maximum number of seizures of the state matrices belonging to class \( m \) of \( \Phi \) is \( f_m = K_1 \cdot m \).

These \( f_m \) seizures can at the most seize \( n = f_m \) multiples of stage 1 (partitions of the length \( n \)).

**Definition I**

The matrix with the dimension \( G \cdot K_2 \), whose elements \( \Omega \) indicate the cardinality of the subsets \( \Omega_n \), is assumed to be the cardinality matrix of rank \( \Theta \).

Although there are permitted state patterns in part II, the ch vectors of these state patterns are in part \( n \leq f_\Theta \) of \( \Phi \).

**Definition I**

The matrix with the dimension \( G \cdot K_2 \), whose elements \( \Omega \) indicate the cardinality of the subsets \( \Omega_n \), is assumed to be the cardinality matrix of rank \( \Theta \).
8.2 NUMBER OF UNKNOWNS IN THE CASE OF "SEQUENTIAL HUNTING"

For the link system shown on this page we obtain the following cardinality matrix of the rank:

\[
\begin{array}{cccccccccccc}
\text{K}_z = m & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
1 & 7 & 6 & 34 & 49 & & & & & & \\
2 & 6 & 22 & & & & & & & & \\
3 & 1 & 4 & 22 & 97 & 91 & & & & & \\
4 & 1 & 34 & 49 & & & & & & & \\
5 & 6 & 100 & 205 & 151 & & & & & & \\
6 & 1 & 61 & 263 & 367 & 232 & & & & & \\
7 & 22 & 232 & 533 & 592 & 337 & & & & & \\
8 & 6 & 142 & 568 & 929 & 469 & & & & & \\
9 & 1 & 61 & 467 & 1109 & 1469 & 1267 & 631 & & & \\
10 & & & & & & & & & & \\
\end{array}
\]

The sum of all \( S_{h,m} \) indicates the number of unknowns (rank) of the system of equations to be solved.

\[ 1 + \sum_{n=1}^{10} \sum_{m=1}^{10} S_{h,m} = 19448 \]

Hence, 19,448 unknowns are obtained for the link system with \( G_1 = 10, K_2 = 10 \) and \( K_1 = 3 \).

From the above cardinality matrix of the rank there follows that for all link systems with \( K_1 = 3 \) and the pairs of values

\[ G_1 \in \{1,2,\ldots,10\}, \quad K_2 \in \{1,2,\ldots,10\} \]

the number of unknowns is known as well. For example, for the link system with \( G_1 = 3 \) and \( K_2 = 2 \) we obtain

\[ 1 + \sum_{n=1}^{10} \sum_{m=1}^{10} S_{h,m} = 71, \]

i.e. 71 unknowns.

8.3 NUMBER OF UNKNOWNS IN THE CASE OF "RANDOM HUNTING"

From the cardinality matrix below follows that for all link systems with \( K_1 = 3 \) and the pairs of values

\[ G_1 \in \{1,2,\ldots,10\}, \quad K_2 \in \{1,2,\ldots,10\} \]

the number of unknowns is also known. For instance, for the link system with \( G_1 = 3 \) and \( K_2 = 2 \) we obtain

\[ 1 + \sum_{n=1}^{10} \sum_{m=1}^{10} S_{h,m} = 1 + 3 + 4 + 7 + 1 + 13 = 29, \]

i.e. 29 unknowns. In the case of "sequential hunting", 71 unknowns are obtained for the same link system.

In the event of "random hunting" and the additional reduction according to /6, theorem 4-F, page 120, 23 unknowns are obtained for the above link system with \( G_1 = 3 \) and \( K_2 = 2 \).
8.4 THE FUNCTION $B = f(Y/N)$

Table 1

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$\sum_{\beta} M_{\beta} = 49$

/1/ Basharin, G.P.: Derivation of equations of state for two-stage telephone circuits with losses. Telecommunications (1960) 1, 79-90


