DESCRIPTION OF INTERPROCESSOR COMMUNICATION BY QUEUEING BREAKDOWN ANALYSIS

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ABSTRACT

Within the present paper, guided by the results on queueing breakdown analysis, a single-processor model is classified with respect to the priority source. An approach to the analysis of preemptive policies in processor models is presented which is based on the study of stochastic processes with absorbing states. The mathematical background of this approach is found to be common to certain probabilistic methods based on the definition of joint transition probabilities and concerned with the description of processor communication. Some performance measures of the processor model in view of the application situation are calculated and discussed.

1. INTRODUCTION

The idea of distributed processing touches many disciplines connected with networking and computer communication. From this perspective it is clear that there exists many possible concepts for distributed systems. For example, the control unit of an exchange system, organized by the hierarchical distribution of processors will often be implemented using the concept of a top-down tree structure which consists of single (redundant pairs) main-processors providing the distribution of the streams of requests to several sub-processors (e.g. corresponding to different sections of a switching network).

Beside the well known advantages of distributed systems, their drawback is mainly the growing amount and complexity of the multiple processes that run in them and which can lead to performance restrictions. These multiple processes, generated by peripheral devices (or users) or caused by interprocessor communications, are, roughly speaking, organized by establishing certain priority rules and service strategies [e.g. background jobs will be interrupted by foreground jobs and queues arising will be serviced with respect to the first come-first served (FCFS) procedure].

A standard guide for the classification of interprocessor communication does not exist. Beside a division based on interprocessor message handling and hardware topology [24], the literature on the performance analysis of processing systems differs mainly with respect to the priority rules and service strategies [12]. In contrast, within the present paper major classifications are derived with respect to the source of the priority requests.

As the modelling of cumulatively occurring preemptive requests by a Poisson source is based on the assumption of an infinite number of different situations forcing interruptions, i.e. based on an unlimited storage of priority items (e.g. event buffer), a finite source might be the appropriate model. Considering certain limits on this finite case, we arrive at processor activities which can be interpreted as directed streams of requests. This agrees with the fact that a message is always unidirectional and hence, two parties of communication in distributed systems, the source and the destination might be considered.

Fig. 1. Processing system for the control of a switching network

The preemption policies applied in processor models can be analysed by the study of stochastic processes with absorbing states. In particular, the trajectory, once having entered the point of state at the instant of preemption, does not return to the former state space. The study of this class of processes can be based on the definition of joint transition probabilities which, in the case presented, satisfy an improper condition. The occurrence of improper probabilities presents a rather mathematical problem which, among others, for stochastic processes in countable spaces referred to as quasi-processes, is presented in [15] (various notations are used in the literature). In this way in the present paper, a rather mathematical problem is examined with respect to its application to queueing theory.

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2. ABSTRACT OF THE GENERAL THEORY

The solution of several advanced problems in applied probability provided by the introduction of joint transition probabilities, is, roughly speaking, based on the idea that an observer may record not only the trajectory of a process but also some other occurrences related (or even unrelated) to the path of this process.

Besides those for other classes of stochastic processes (cf. [2]), appropriate examples for integer-valued processes are, among others, the analysis of non-Markov processes by the method of supplementary variables (cf. [4], [16]) and the description of Markov processes which remain in an absorbing state [10]. Thus, the first example applies to the case of generally distributed service times and the second one is concerned with the occurrence of preemptive priority items.

Let \( N_t \) be a one-dimensional stochastic process in a state space consisting of a finite or countable number of points and in continuous time which in the Markov case is completely described by the transition probabilities \( P_{ij}(s,t) := P(N(t)=j|N(s)=i) \)

We define the joint transition probabilities

\[
\bar{P}_{ij}(s,t,A) := P(N(t)=j,N(s)=i \cap A)
\]

(1)

(A is the random event defined below) and hence, the following equations are obvious:

\[
\bar{P}_{ij}(s,t,A) = P(A|N(t)=j,N(s)=i)P_{ij}(s,t)
= [1 - P(A|N(t)=j,N(s)=i)]P_{ij}(s,t).
\]

(2)

In the case of the supplementary variable method, providing the analysis even the service times are generally distributed, the event \( A \) is concerned with the elapsed time in service, i.e. \( A := T_s < x \) where the supplementary variable \( T_s \) is the service time and \( x \) is a point on the corresponding time axis leading to a state space extension (usually, the absolute probability or their density with respect to \( x \) is presented i.e. \( A := x < T_s < x + \Delta x \) for the latter).

In the case of a Markov process which remains in an absorbing state, the complement of the event \( A \) can be expressed by \( A := s < T_s < t \) where \( T \) is the instant of time when the trajectories reach the absorbing state ("life time"). Certainly both cases can be combined by starting in eq.(1) with an additional intersection.

The following properties of the joint transition probabilities are implied by the properties of \( P_{ij} \)

and eq.(2):

1) \( \bar{P}_{ij}(s,t,A) = 0 \)
2) \( \sum_{j=0}^{\infty} \bar{P}_{ij}(s,t,A) = 1 \)
3) \( \lim_{t \to s} \bar{P}_{ij}(s,t,A) = \lim_{s \to t} \bar{P}_{ij}(s,t,A) = \delta_{ij} \)

(\( \delta_{ij} \) is the Kronecker Delta)

Stochastic processes satisfying the improper condition 2) (which generally can be arrived at by considering the direct product of stochastic kernels concerned with the r.h.s. of eq.(2) [9]) have already been investigated in the first works devoted to this subject of mathematical research (cf. [7]). At that time the interpretation of this phenomena was implied mainly by the Feller-Lundberg condition i.e. the trajectories increase so rapidly that they may reach the "infinite" value within a finite time interval with positive probability [another probabilistic interpretation of the improper condition 2) which is not of interest here has been presented in [5]].

Since the more advanced theory of stochastic processes was established, this "infinite" point in the state space was extended to an arbitrary one, and hence, this point can now act as an absorbing state reached by preemption. The improper condition 2) can then be interpreted as the disappearance of the trajectories from the state space. Thus, the appropriate state space of the Markov chain described by the joint transition probabilities \( \bar{P}_{ij} \) is a semi-infinite lattice of integers obtained by removing the absorbing state from the former state space.

In the following we restrict ourselves to this latter case. Bearing in mind the regularity condition for a Markov chain in continuous time, from eq.(2) we obtain, for suitable small \( \Delta t > 0 \), the relation

\[
\bar{P}_{ij}(t,t+\Delta t) = [1 - q(t)(t) - c_i(t)\Delta t] \delta_{ij} + q_i(t)\Delta t \bar{P}_{ij}(t) + o(\Delta t)
\]

(3)

where \( c_i \) is defined to be \( c_i(t) = \lim P(t<T^+<t+\Delta t|N(t)=i)/\Delta t \) and acts as an intensity and all other quantities are as accepted in the literature (cf. [2]).

By substituting eq.(3) into the Chapman-Kolmogorov equation we arrive at

\[
\bar{P}_{ij}(s,t) = -[q_j(t) + c_j(t)]\bar{P}_{ij}(s,t) + \sum_{k=0}^{\infty} q_k(t) \bar{P}_{kj}(s,t) \bar{P}_{ik}(s,t)
\]

(4)
which is an extended version of the classical Kolmogorov forward equation. The backward equation, adjoint to eq.(4), can be obtained by the same procedure.

By somewhat stronger assumptions, the solution of eq.(4) can be obtained by the method of successive approximations

\[ \bar{p}_{ij}(s,t) = \sum_{n=0}^{\infty} \bar{p}_{ij}^n(s,t) \]  

where \( \bar{p}_{ij}^0(s,t) = \delta_{ij} \exp\left[-\frac{t}{s}[c_j(u)+C_j(u)]du\right] \) and \( \bar{p}_{ij}^n \) for \( n \geq 1 \) follows by iteration.

The solution system in eq.(5) affords the simple probabilistic interpretation of \( \bar{p}_{ij}(s,t) \) as the joint probability that during the time interval \((s,t)\) the trajectories will arrive at the state \( N(t) = j \) by exactly \( n \) jumps under the condition that \( N(s) = i \) and no absorbing state is reached within this time interval.

The solution by successive approximations in eq.(5) includes the proof of its existence and uniqueness. Additionally, the conditions 1) and 3) can be shown. Further, the following relation, which can be deduced from the solution system, proves the improper condition 2) and hence, acts as an existence theorem for the joint transition probabilities \( \bar{p}_{ij} \):

\[ \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} \bar{p}_{ij}^n(s,t) = 1 - \sum_{n=0}^{\infty} \sum_{j=0}^{\infty} t \int c_j(\theta) \bar{p}_{ij}^n(s,\theta) d\theta. \]  

From eq.(6) appropriate performance measures for the cut-off behaviour of preemptive priorities can be deduced. For the cumulative distribution function of the instant of time when the non-priority path arrive at an absorbing state ("life time" of the non-priority path) defined as \( F^+(t) = \Pr{T^+ \leq t} \), we obtain

\[ F^+(t) = \sum_{j=0}^{\infty} \int c_j(\theta) \bar{p}_j(\theta) d\theta \]

where \( \bar{p}_j(\theta) = \int_0^{\infty} \bar{p}_j(o,t) \) \( [p_j(o) \) is the initial probability of the non-priority path]. From eq.(7) the probability density function \( f^+ = dF^+/dt \) as well as the "survivor function" of the non-priority path defined as \( F^+(t) = \Pr{T^+ > t} \) can be deduced. In this way an expression for the interruption rate \( \Lambda \) is obtained:

\[ \Lambda(t) = f^+(t)/F^+(t). \]

For completeness, by an argument similar to eq.(1) we arrive at an expression for the distribution of the absorbing states

\[ \Pr{N(T^+)=j} = \int c_j(\theta) P_j(\theta) d\theta \]  

where the existence of the r.h.s. of eq.(9) is provided by suitable bounds on \( c_j \). Eq.(9) expresses the fact that the absorbing states for the non-priority path is randomly distributed.

It is well known that the application of the Kolmogorov eq.(4) to the description of queueing systems is concerned with the restriction of the Markov chain in continuous time to a birth- and death process. In particular, for the general birth- and death process eq.(4) applies in the homogeneous case to

\[ d\bar{p}_{ij}(t) = -\left(\lambda_j + \mu_j + c_j\right) \bar{p}_{ij}(t) + \lambda_{j-1} \bar{p}_{i-1,j-1}(t) + \mu_{j+1} \bar{p}_{i+1,j+1}(t). \]

From eq.(10) we easily deduce the infinitesimal generator and other characteristics concerned with the birth- and death process presented.

3. THE MODELS

Within the present paper, we consider the problem faced by a single processor (defined as a processor-memory pair) providing service to two classes of jobs. In view of the mathematical description of real time computer systems, an appropriate model for a single processor consists of one service station and queues for each class (cf. fig. 2). In particular, considering certain limits of the priority queue length, we arrive at a model where the priority queue is replaced by a feedback arrangement (cf. fig. 3).

In processor systems, background jobs (e.g. processor communication with a secondary memory) must occasionally be preempted in order to run high priority jobs. Within the present paper the preemption resume (PR) policy is applied. Additionally, queues arising are served with respect to the FCFS procedure. Results dealing with an extension to several classes, other preemptive priorities (e.g. preemptive repeat, preemptive loss) and other service strategies (e.g. last in-first out, random) are available in literature.

In the following, the PR items are assumed to arrive in a Poisson stream with constant mean rates \( \lambda_1 \) and \( \lambda_2 \). The corresponding service times \( T_{s1} \) and \( T_{s2} \) (in isolation) are assumed to be independent and identically distributed random variables according to general service time distributions with finite moments \( \mu_{1,S1} = E(T_{s1}) \) and \( E(T_{s2}^2) \) (E for expectation) where i=1 refers to the priority source and i=2 to the ordinary source of items. The notations

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\( \rho_i = \lambda_i / \mu_i \) for the traffic densities and \( \nu \) for the ratio \( \mu_2 / \mu_1 \) are applied.

As the PR policy determines the class of the processor models presented in the following, the approach described in the foregoing can be applied to analyse each of these models.

### 3.1 The (oo,oo) Model

This model (priority and ordinary source both infinite, cf. [14]) describes a single processor with queues of unlimited length for both the priority and non-priority items. Following [13] for generally distributed service times, the treatment is based on the definition of (several) joint transition probabilities according to eq. (1). For exponentially distributed service times the model prior was presented in [25].

![Fig. 2. Single-processor model with externally generated requests](image)

\[ \lambda_1 \rightarrow \text{FCFS} \quad \text{PR/HOL} \]

\[ \lambda_2 \rightarrow \text{PROCESSOR} \]

### 3.2 The (N,oo) Model (1<N<oo)

The restriction to a finite source of cumulative interruptions which generalize some of the models presented agrees with the assumption of a finite number of situations forcing interruptions of a running background job. On the one hand this is in accordance with the real situation, while on the other hand a loss system for the priority items will be established which might be restricted to certain application situations. Nevertheless, this model and the following one occur in the development of a certain finite case. For the investigation of this model the results on machine interference theory (N machines serviced by one repairman) under the Markov regime presented in [1], [20], [21] are helpful.

### 3.3 The (1,oo) Models

Now the priority items are allowed to occur if, and only if the processor is not busy processing foreground jobs or the processor is idle. A minor modification of this model leads to the case where priority items occur if and only if a background job is running, i.e. the current background job itself generates priority requests (cf. fig. 3). Hence, the priority requests in the latter model might be termed internally generated (thereby following the terminology in [17], cf. active breakdown in [14]). In contrast, requests described by all other models reviewed in the foregoing, if as they occur even the processor is idle, are called externally generated (cf. fig. 2). Considering this terminology and in order to reflect the interaction between source and destination in processor systems, the occurrence of the priority items might be interpreted as directed streams of requests.

![Fig. 3. Single-processor model with internally generated priority requests](image)

\[ \lambda_1 \rightarrow \text{FCFS} \quad \text{PR} \]

Both models for generally distributed service times are investigated by the supplementary variable method in [23]. For Erlang distributed service times, including the limit of constant service times, the internal case is treated in [11]. Results for the external case restricted to exponentially distributed service times are found in [25].

### 3.4 Further Remarks

Neglecting delayed preemption, established in processor systems to meet the conflicting objectives of efficient operation and rapid response to high priority requests (cf. [3]) as well as setup times incurred before processing of interrupted jobs can be continued (cf. [22]), the PR policy presented naturally implies the notation of instantaneous interprocessor communication. In contrast, the head-of-the-line (HOL) policy, which describes the situation where priority requests arriving when a background job is operating must wait at the head of the queue until the background job is completed (cf. [19]), as well as the delayed preemption policy imply the notation of delayed interprocessor communication.

From eq. (8) and eq. (7) we can immediately conclude that delayed preemption is one solution to the problem of finding a balance between the two extremes, the PR and the HOL policies defined above. The approach presented is believed to be an appropriate method for developing policies which provide this balance between rapid response and operating efficiency (e.g. time and state dependent preemption). Further, when analysing the top-down tree structure of a processing system mentioned in the foregoing (cf. fig. 1), the classification presented is found to be helpful.

### 4. RESULTS AND DISCUSSIONS

The simple results obtained under stationary conditions (i.e. deduced from the steady-state Kolmogorov equations), are figured in the following in order to reflect the description of the cut-off behaviour of preemptive priorities by the method presented. As the non-saturation conditions (obtained from the probability that the processor is
idle) vary from $P_1 + P_2 < 1$ for $N = \infty$ to $P_2 < 1/(1+P_1)$ for $N = 1$ where the latter is true even in the internal case, the following figures are calculated according to these inequalities.

For $N = \infty$ the curves converge to the boundary of the well-known triangle. The convergence of the HOL-curve against the PR-curve ($N = 1$) for $v = \infty$ [the HOL-curve tends to an asymptotic value of $1/(1+v)$] and in turn against the triangle for $N = \infty$ is obvious.

In fig. 4, where the regions for non-saturation lie below the curves, the "improvement" in the case of a finite priority source is paid for by establishing a loss system for the priority items (though there might be little sense in a processor system which is, for the most part, interrupted by high priority requests, cf. fig. 7). For $N = \infty$ the curves converge to the boundary of the well-known triangle. The convergence of the HOL-curve against the PR-curve ($N = 1$) for $v = \infty$ [the HOL-curve tends to an asymptotic value of $1/(1+v)$] and in turn against the triangle for $N = \infty$ is obvious.

The interruption rate in fig. 5, as well as the "survivor function" for the non-priority path in fig. 6 reflect the same situation. The curves for $N = \infty$ converge against those for the (oo,oo) model where, for the latter, the interruptions are described by a simple M/G/1 queue which is independent of the non-priority queue. The fact that the internal case lies below (fig. 5) and above (fig. 6) the external case, which can be proved by the corresponding non-saturation condition, expresses that in the former case interruptions occur if and only if the processor is busy with a non-priority job and in the latter case interruptions occur even if the processor is idle.

From the efficiency curves in fig. 7, thereby following a definition introduced in [6] (ratio of the mean queue length of the background jobs without priority items to this mean queue length in the presence of priority items, cf. [17]), we immediately observe the agreement of the internal case with the corresponding external case which simply expresses the vanishing idle periods of the processor for an increasing priority traffic.

5. SUMMARY AND CONCLUSIONS

In this paper a single-processor model was classified with respect to the length of the priority source. In particular, considering certain limits of a finite priority source, we arrived at a model where priority requests are generated by the non-priority jobs. When analysing the simple structure of a processing system mentioned in the foregoing, the classification presented is found to be helpful. An approach to the analysis of the preemptive policies in the single-processor model was presented which is based on the study of stochastic processes with absorbing states. This approach is believed to be an appropriate method for developing
policies which provide the balance between the two extremes, the PR and the HOL policies. The mathematical background of this approach was found to be common to certain probabilistic methods based on the definition of joint transition probabilities and concerned with the description of processor communication. Further, as this approach leads to improper probabilities, a rather mathematical problem is examined with respect to its application to queueing theory. Some performance measures of the processor model which are specifically concerned with the approach described are calculated and discussed.

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REFERENCES

[10] Giglmayr, J., Queueing applications of Markov-processes with absorbing states (to be published)

Q.1 (R. Syski)

In the homogeneous case, equation (10) refers to the well known birth death process killed (disappearing) at state dependent rate $c_j$, and $\bar{p}_{ij}(t)$ are taboo probabilities, with the taboo set consisting of a single adjoint absorbing state, corresponding to $A(T,t)$ where $T$ is time to absorption, assumed finite by (5) - (7). Strictly speaking, equation (9) should be interpreted (in homogeneous case) as: $\text{IP} (N(T - 0) = j) c_j \int \bar{p}_j(t) \, dt$, for $j = 0, 1, 2, ...$

A.1 (J. Giglmayr)

In reply to your question Professor Syski, within the present paper killing of birth and death processes was applied to describe the cut-off behaviour of preemptive priorities. By the definition of the joint transition $\bar{p}_{ij}(t)$ in equation (2) (Taboo probabilities as you indicated) the trajectories of the killed birth and death process are restricted to the half-open time interval $(0,T')$ (in order to simplify the mathematical part of the paper, this and corresponding problems were neglected). In this sense, the expression for the distribution of absorbing states presented in your question is indeed more precise than equation (9), in my paper.
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In the homogeneous case, equation (10) refers to the well known birth death process killed (disappearing) at state dependent rate $c_j$, and $P_{ij}(t)$ are taboo probabilities, with the taboo set consisting of a single adjoint absorbing state, corresponding to $A = (T^* > t)$ where $T^*$ is time to absorption, assumed finite by (5) - (7). Strictly speaking, equation (9) should be interpreted (in homogeneous case) as:

$$IP (N(T^* - o) = j) = c_j \int_0^{T^*} p_j(t) dt ,$$

for $j = 0, 1, 2, \ldots$

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