MINIMUM-COST MULTIYEAR TRUNK PROVISIONING

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ABSTRACT

In this paper, we describe a new multiyear trunk-forecasting process, called the Trunk Implementation Plan (TIP), that provides a cost-effective schedule for trunk augmentations and disconnects. TIP reflects the current traffic network, demand dynamics, forecast uncertainty, and expense and capital costs. The key result is that TIP provides a systematic and scientific method for planning network capacity by unifying mathematically the major aspects of the trunk network provisioning process.

First, an overview of the planning process is given. Then an analytical solution to the TIP problem for grade-of-service (final) trunk groups is presented. Finally, a heuristic solution is described for hierarchical trunk groups in network clusters.

1. INTRODUCTION

1.1 Background

The Bell System's trunk provisioning process, to be described in more detail below, is composed of the five major subprocesses illustrated in Figure 1: measuring traffic (I), projecting traffic (II), engineering the trunk network (III), planning network expansion (IV), and adjusting the trunk network (demand servicing) (V). The General Trunk Forecast (GTF), the output of subprocess IV, provides forecasts of future network traffic and trunk requirements to various other network provisioning processes: facility, equipment, and switch planning. The GTF is, therefore, a vital part of the Bell System's network provisioning process.

While there have been major improvements in certain aspects of the trunk provisioning process, the planning methods, and the software tools that produce the GTF, do not, at this time, systematically or scientifically account for several important implementation considerations. These considerations include the existing traffic network, the fluctuation of traffic demands from year to year, the uncertainty of the demand, and the economics of maintaining or rearranging trunks. Thus, in practice, the trunk forecasters often modify the preliminary GTF estimates of future trunk requirements to include these considerations so that the final, published schedule of planned augmentations and disconnects is both feasible and economically sensible.

In this paper, a new multiyear trunk provisioning procedure, called the Trunk Implementation Plan (TIP), is recommended for the network expansion subprocess (IV). TIP formally minimizes the impact of forecast uncertainty and demand dynamics on the cost of implementing a traffic network meeting objective-service criteria. A future paper [8] will address mathematical and computational details and methods to incorporate constraints due to budgetary and switch and facility capacity limitations. TIP is a generalization of the Trunk Provisioning Operating Characteristics (TPOC) models described at the Eighth ITC [4]. We consider network clusters, including the case of only-route (direct final) trunk groups, and optimize variables treated only parametrically in the TPOC analysis.

1.2 Overview

For the past several years there has been a large effort to improve the efficiency, accuracy, and consistency of the Bell System's trunk network provisioning process, illustrated in Figure 1. Specifically, results include the increased quality and quantity of traffic measurements (I), improved, standard, and scientifically-based traffic projection algorithms (II) [2], new engineering procedures that reduce capital costs or improve network utilization for a given service objective (III) [1], and decision-theoretic procedures that determine when service objectives are being violated significantly and what demand servicing actions should be taken (V) [9]. This paper describes a key element in the improved planning process: hierarchical-trunk-network capacity expansion algorithms that explicitly account for demand dynamics and forecast errors (IV).

The distinction between the engineering function (III) and the capacity expansion function (IV) is key and is the essence of this paper. In general, we use the term "engineer" to refer to the determination of network capacity required to achieve a desired service objective for a single (static) time period, assuming that the relevant parameters (load, peakedness, day-to-day variation, and so on) are deterministic. In the engineering process, we ignore forecast errors, (multi-year) demand dynamics, modeling errors, and the existing network capacity. Thus, ECCS engineering for high-usage groups [10], the Neal-Wilkinson engineering for final groups [6], extreme-value engineering [5], and modular engineering [3] are classical examples of engineering procedures.

In contrast to the engineering function, the network capacity expansion function, which we refer to as the Trunk Implementation Plan (TIP), is a stochastic, dynamic optimization algorithm. In this capacity expansion function, the models and sizing procedures in the engineering function are combined with considerations of potential service degradation due to forecast error (i.e., the likelihood that demand servicing (V) may be required), the existing network, (forecasts) traffic dynamics (III), and various capital and administrative costs to produce a cost-effective evolution plan for network capacity. It is this function IV that deals with issues of so-called reserve capacity, transition planning, capacity removal (disconnects), and capacity constraints (switch and facility limitations and even modular sizing). It is also this function that formally acknowledges that 5-year traffic forecasts are less accurate than 1-year forecasts and that 20-year forecasts provide very little useful information.

The key step in the formulation of the TIP models, especially those that pertain directly to the grade-of-service (final) trunk groups, is the explicit incorporation of each of the subprocesses in Figure 1 in a mathematical model. In fact, the description of the model, the mathematical results, and the computational algorithms comprise most of the paper. We expect that a trunk provisioning process based on such complete and consistent models will result in an
improved GTF for use in each of the network provisioning subprocesses: switch, facility, and equipment planning.

2. ONLY-ROUTE TRUNK GROUPS

In this section, we present the assumptions and the mathematical formulation of the only-route (direct-final) TIP problem.

2.1 Mathematical Model for Unconstrained Problem

To introduce the only-route TIP formulation, we need the following notation: Let \( N \) be the number of years in the forecast horizon, \( T(k) \) be the number of trunks in service at the beginning of the \( k \)-th year (\( k = 0, \ldots, N-1 \)), \( u(k) \) the number of planned trunk augmentations/disconnections at the beginning of the \( k \)-th year, and \( d(k) \) the peak demand in trunks during year \( k \). Also, let \( F_k \) be the distribution of \( d(k) \). For TIP, it is assumed that the trunk demands are independent random variables with known distribution functions. (While, in practice, the demands are often not independent, we have found through simulation studies that a variety of network performance statistics are robust to this assumption [8].) In year \( k \), the costs are \( c_f(u(k)) \) for capital, \( c_l(u(k)) \) for labor, \( c_m(T(k),u(k)) \) for maintenance, and \( c_u(d(k),T(k),u(k)) \) for underprovision. A brief description of these cost functions is given in Section 2.1.2.

2.1.1 Trunk Group Dynamics

If the peak trunk demand during year \( k \), \( d(k) \), exceeds the current trunk level, then the trunk group is augmented up to the demand on an emergency basis. The trunk group dynamics are modeled by

\[
T(k+1) = T(k) + u(k) + \max\{0,d(k) - [T(k) + u(k)]\} ,
\]

\[
= \max\{T(k) + u(k),d(k)\} .
\]

(2.1.1.1)

The number of trunks in service at the beginning of the \( k+1 \)-th year is the sum of the planned trunk level for year \( k \) and the demand servicing augmentation, if any, during year \( k \). Moreover, since the planned trunk level (the GTF) is derived from I-IV in Figure 1 and demand servicing relies on I and VI, (2.1.1.1) models all key components of the trunk network provisioning process.

2.1.2 Objective Function

The goal of the only-route TIP is to minimize the expected present worth of trunk provisioning costs. If we denote the present worth of the total cost at year \( k \) by \( g_k(d(k),T(k),u(k)) \), then the TIP objective function can be expressed as

\[
\min_{u} \mathbb{E}\left\{ \sum_{i=0}^{N-1} g_i(d(i),T(i),u(i)) \right\} ,
\]

(2.1.2.1)

where

\[
u = (u(0), \ldots, u(N-1)) .
\]

In (2.1.2.1) the expected value is taken over the future demands; that is,

\[
\mathbb{E} = \sum_{d(0)} \sum_{d(1)} \ldots \sum_{d(N-1)} \left[ g(d(0),T(0),u(0)) + g(d(1),T(1),u(1)) + \ldots + g(d(N-1),T(N-1),u(N-1)) \right] .
\]

(2.1.2.2)

The present worth of trunk provisioning costs at year \( k \) is equal to

\[
g_k(d(k),T(k),u(k)) = p^k \left[ c_f(u(k)) + c_l(u(k)) + c_m(T(k),u(k)) + c_u(d(k),T(k),u(k)) \right] ,
\]

(2.1.2.3)

where \( p \) is the discount factor that measures the worth of the next year's dollars in terms of present dollars.

The capital, labor, maintenance, and underprovision costs are assumed to be piecewise linear and defined for \( k = 0, \ldots, N-1 \) by

\[
c_f(u(k)) = \begin{cases} a_f^+ u(k) & \text{if } u(k) \geq 0, \\ b_f^- u(k) & \text{if } u(k) < 0 , \end{cases}
\]

(2.1.2.4)

\[
c_l(u(k)) = \begin{cases} a_l^+ u(k) & \text{if } u(k) \geq 0, \\ -b_l^- u(k) & \text{if } u(k) < 0 , \end{cases}
\]

(2.1.2.5)

\[
c_m(T(k),u(k)) = a_m^+ (T(k) + u(k)) ,
\]

(2.1.2.6)

\[
c_u(d(k),T(k),u(k)) = a_u^+ \max(0,d(k) - T(k) - u(k)) ,
\]

(2.1.2.7)

where

\[
a_f^+, a_l^+, a_m^+, a_u^+ > 0 \text{ and } b_f^-, b_l^- \geq 0 .
\]

Equation (2.1.2.7) states that, if the peak demand \( d(k) \) exceeds the planned trunk level \( T(k) + u(k) \) during year \( k \), then the cost of providing the additional trunks is proportional to the trunk shortage; if \( d(k) \) does not exceed the planned level, then no cost is incurred. Thus, the underprovision cost reflects the demand servicing policy as described by the trunk-group dynamics equation (2.1.1.1).

We observe that the TIP mathematical model can be viewed as a sequential stochastic decision process. The state of the system, the number of trunks in service, varies according to the trunk group dynamics given in Equation (2.1.1.1). At each stage of the process the cost function \( g_k(d(k),T(k),u(k)) \) is defined via (2.1.2.3)-(2.1.2.7). The TIP problem then is to find a set of decisions (augments, disconnects) \( u^* = (u^*(0), \ldots, u^*(N-1)) \), the optimal policy, that minimizes the total expected cost over \( N \) stages; i.e.,

\[
J(u^*) = \min_{u} \mathbb{E}\left\{ \sum_{k=0}^{N-1} g_k(d(k),T(k),u(k)) \right\} .
\]

(2.1.2.8)

2.2 Optimal Strategy for Unconstrained Problem

As is shown in [7], under rather general conditions for the forecast error distributions and for the cost coefficients, there is a simple recursive way of obtaining the unique optimal policy \( u^* \), where \( u^* \) is defined by \( N \) pairs of scalars \( (S(0),S(0)), \ldots, (S(N-1),S(N-1)) \). The pair \( (S(k),S(k)) \) provides the two critical threshold levels for year \( k \). Specifically, at year \( k \), the optimal decision is to augment the number of trunks up to the level \( S(k) \), or disconnect down to the level \( S(k) \); that is,
The derivation shows that the optimal dynamic inventory-type strategy, \( (\xi(k),\bar{S}(k)) \) for each year \( k \), is independent of the initial trunk level \( T(0) \) and is determined by the present and future demand distributions \( (F_k,...,F_{N-1}) \), the present and future cost coefficients \( a_1^k,...,a_2^k,\beta_j^k \) \( (i=k,...,N-1) \), and the discount factor \( \rho \).

2.3 Cost of Underprovisioning

Although capital, labor, and maintenance costs are easily obtainable quantities, the value of the incremental underprovision cost coefficient \( \alpha^k \) is more difficult to quantify. In this section, we discuss the role of the cost functions in the TIP model and explain our interpretation of the cost of underprovisioning.

We see that the various costs have counteracting influences. Underprovision is a positive force for adding trunks and maintaining large trunk groups. Capital and maintenance costs are more easily quantified; these costs can be specified to utilize this approach [8].

In general, the underprovision cost or, equivalently, the unsatisfied demand penalty cost, includes the amount of money spent in demand servicing plus the amount of money lost when the service objective was violated (loss in revenue, good will). Demand servicing involves all of the costs of planned servicing (i.e., implementation of the GTF), capital, labor, and maintenance. Thus, the per trunk underprovision cost, \( \alpha^k \), should reflect the sum of all of the per trunk costs \( a_1^k, a_2^k \), and \( \alpha_3^k \), plus a penalty due to the fact that demand servicing cannot be carried out with the normal planning intervals and orderly procedures associated with planned servicing. It is important to note that, if \( \alpha_4^k \) were not greater than \( \alpha_1^k + \alpha_2^k + \alpha_3^k \), then there would be no reason for the planned trunk augmentation. Since the demand servicing cost depends on a variety of factors such as switch, trunk, and/or personnel availability, the cost of demand servicing per trunk increases substantially when the need for demand servicing over all trunk groups exceeds the existing level of spare capacity.

As we argued above, we must have

\[
\alpha_4^k > \alpha_1^k + \alpha_2^k + \alpha_3^k. \tag{2.3.1}
\]

This inequality suggests that it might be economical to provide some protection against demand uncertainty by building reserve (i.e., a positively biased) capacity for a trunk group. Moreover, we can expect that the TIP reserve capacity level increases as the cost quotient \( \alpha_4^k/(\alpha_1^k + \alpha_2^k + \alpha_3^k) \) increases. The TIP strategy is designed to give the optimal balance between the expected cost of underprovisioning and the capital, labor, and maintenance costs of planned servicing.

2.4 CONSTRAINED TIP PROBLEM

For the only-route TIP model, the optimal solution always exists and is uniquely defined by

\[
\tau = \left\{ \bar{S}(0), \bar{S}(0),...,\bar{S}(N-1), \bar{S}(N-1) \right\}. \tag{2.4.1}
\]

and the initial trunk level \( T(0) \). The properties of the optimal policy \( \tau \) depend strongly on the value of the expected underprovision cost; therefore, the properties of the TIP solution (such as levels of reserve capacity and demand servicing) are determined by the assumed values of the incremental underprovision cost \( (a_1^k,...,a_2^k) \), among other things.

As we mentioned in Section 2.3, the incremental underprovision cost increases substantially when the need for demand servicing exceeds a certain level. Therefore, the linearity assumption for the demand servicing cost is justified only if the amount of demand servicing does not exceed the allowable threshold. In this section we introduce the revised only-route TIP formulation that explicitly limits the amount of demand servicing to an allowable level.

Given the initial trunk level \( T(0) \), we wish to find the optimal control vector \( \mu^* \) that minimizes the expected sum of trunk provisioning costs,

\[
J(\mu) = E \sum_{k=0}^{N-1} \rho^k \left[ c_1^k(u(k)) + c_2^k(u(k)) + c_3^k(T(k),u(k)) \right] + c_4^k(d(k),T(k),u(k)), \tag{2.4.2}
\]

subject to the demand servicing constraint

\[
E \max\{0,d(k)-\bar{T}(k)\} \leq \beta_k E d(k), \quad k = 0,1,...,N-1, \tag{2.4.3}
\]

where \( T(k) \) is defined by the trunk group dynamics equation

\[
T(k+1) = \max\{d(k),\bar{T}(k)\}. \tag{2.4.4}
\]

\( \bar{T}(k) = T(k) + u(k) \), and the expected value in (2.4.3) is taken with respect to all the demands \( d(0),...,d(N-1) \). The new condition (2.4.3) states that the optimal solution, \( \mu^* \), should result in a level of demand servicing at year \( k \) less than or equal to 100% of the expected number of trunks required in year \( k \). (Recall \( E(d(k)) \) is approximated by the mean of \( F_k \).

Now, we show that the original TIP algorithm can be used to find an optimal solution to the problem above. Indeed, let us consider the functional

\[
H(\mu;\lambda) = J(\mu) + \sum_{k=0}^{N-1} \lambda_k \rho^k E \max\{0,d(k)-\bar{T}(k)\}, \tag{2.4.5}
\]

where \( \lambda = (\lambda_0,...,\lambda_{N-1}) \).

Clearly, for any vector \( \lambda \) with nonnegative components, minimizing \( H(\mu;\lambda) \) with respect to \( \mu \) subject to (2.4.4) is equivalent to solving the unconstrained TIP problem with the incremental underprovision cost \( a_4^k \) replaced by \( \alpha_1^k + \alpha_2^k \).

It can also be shown that, if we can find a policy \( \tau^* \) such that \( \tau^* \) is an optimal solution to the original TIP problem with some incremental underprovision costs \( \alpha_4^k \), where \( \alpha_4^k \geq \alpha_4^k \) for all \( k \), and \( \tau^* \) results in expected demand servicing equal to 100% percent, then \( \tau^* \) is an optimal solution to the problem described by (2.4.2)-(2.4.4). An iterative solution to the revised TIP problem can be specified to utilize this approach [8].
Comparing the TIP model in Section 2.1 and the revised TIP formulation, we note that, by replacing the problem of defining the nonlinear model for the underprovision cost by the simpler problem of assigning a (maximum) allowable level of demand servicing, we facilitate the only-route TIP implementation considerably.

In [8], we show how to choose the vector $\lambda$ in such a way as to obtain the optimal solution satisfying the demand servicing constraint (2.4.3). Of special interest and practical importance is the fact that under reasonable assumptions, a constant, $\lambda_0$, can be used to obtain near-optimal solutions to the constrained TIP problem. In [8], we also show how to choose the vector $\lambda$ in such a way as to satisfy a different constraint, a constraint on the expected proportion of groups serviced (as opposed to the constraint on the expected proportion of trunks serviced) and how to incorporate constraints on $T(k)$ or $u(k)$. Finally, in [8], we also illustrate the relationship between the level of demand servicing (including tails of the blocking distribution) and reserve capacity as a function of forecast error parameters. As in the TPOC application in [4], these relationships may be useful in analyzing and synthesizing provisioning practices.

3. Hierarchical Network Clusters
As in the only-route TIP model, the network TIP algorithm is designed to achieve the minimum present worth of the expected cost of planned and demand servicing over a planning horizon subject to a constraint on the amount of demand servicing.

There is a fundamental difference, however, between the only-route and the hierarchical-network TIP problems. The only-route TIP problem is concerned with determining the amount of reserve capacity needed to hedge against forecast uncertainty to satisfy the constraint on the amount of demand servicing and with smoothing the year-by-year trunk requirements to obtain the optimal balance between the cost of maintaining and rearranging the trunks over a planning horizon. In the network case, one must also determine whether reserve capacity should be distributed over all of the trunk groups in the network or only on specific trunk groups.

3.1 Alternate Routing with the Forecast Uncertainty
In order to address the problem of where to introduce reserve capacity in the network, we shall follow the heuristic principles of hierarchical network engineering. We decompose the network TIP formulation into individual cluster TIP problems, where a cluster is defined by a final trunk group and all subtending high-usage (HU) groups that overflow to that final. The current trunk engineering procedure is based on the Truitt’s technique [10] for determining trunk sizes for high-usage groups and Neal-Wilkinson traffic capacity tables for sizing final groups [6]. As we mentioned in the Introduction, these engineering methods consider traffic demand for a single year and assume that there is no error in the traffic forecast.

3.1.1 Mathematical Model
To analyze the impact of the forecast error on the optimal design of hierarchical networks, we formulate a single-year TIP problem for a Truitt’s alternate routing triangle. Referring to Figure 2, we assume that load $\lambda (\ell_1 + \epsilon_1)$ is offered to the direct (HU) group, and background loads $\ell_2 + \epsilon_2$ are offered to the first and second legs of the alternate route respectively, where $\epsilon_1, \epsilon_2$ are the errors in the load forecast. The planned trunk group sizes on the direct and alternate routes are denoted by $T$, $T_1$, and $T_2$ respectively. Then, the problem of determining $T$, $T_1$, $T_2$ to minimize the expected cost of trunk provisioning activities during the year is given by

$$\begin{align*}
\min_{T_1, T_2} & \quad \{ C_D T + C_{A1} T_1 + C_{A2} T_2 + \\
& + C_{S1} \max(0, d_1 - T_1) + C_{S2} \max(0, d_2 - T_2) \},
\end{align*}$$

(3.1.1.1)

where $d_1$ and $d_2$ are the (realized) trunk requirements on the alternate route legs to satisfy the network service objectives; $C_D$ is the incremental cost of adding a trunk to the direct route on a planned basis; $C_{A1}, C_{A2}$ and $C_{S1}, C_{S2}$ are the incremental costs of the planned and demand servicing on the alternate route legs, respectively.

As in the only-route TIP model, we assume that the demand servicing augmentation is performed on the final group only. Our goal is to minimize (3.1.1.1) under the demand servicing constraints given by

$$E_{\max}(0, d_1 - T_1) \leq \beta E_{d_1}$$

and

$$E_{\max}(0, d_2 - T_2) \leq \beta E_{d_2},$$

(3.1.1.2)

where $\beta$ is specified.

3.1.2 Basic Assumptions
We investigated the cases when 10-100 erlangs of traffic are offered to the direct group and 10-500 erlangs are offered to the alternate route. We also assumed that the coefficient of variation of the demand forecast on the HU group, $CV_D$, and on the alternate route, $CV_A$, vary between 0 - .25 and $CV_D \geq CV_A$. Finally, we assumed that the demand servicing threshold, $\beta$, is in the range from 5 to 30 percent. We defer to a future paper some details of the analysis [8].

3.1.3 Major Conclusions
We compared the optimal single-year TIP solution for the network triangle to the solution that defines the size of the direct route by the Truitt’s formula (also known as the ECCS rule) and then sizes the final to guarantee the specified demand servicing level at minimum cost.

Our results show that the optimal trunk requirement on the HU trunk group does not change significantly with the change in the coefficient of variation of the forecast; i.e., the optimal solution accounts for uncertainty by providing reserve capacity mainly on the final trunk group. Moreover, the cost difference between the optimal and the ECCS based solutions is very small. In fact, our numerical experience reveals that the single-year expected cost is less sensitive to the parameter changes than the trunk group sizes themselves. Specifically, in the range of engineering interest the cost difference between the two solutions is always less than 1 percent. Thus, from our numerical study we conclude that the expected trunk provisioning cost function is very flat in the neighborhood of a solution point and the Truitt’s HU solution is relatively close to the optimal HU trunk size.
This result suggests that, in the presence of uncertainty, an economical network may be obtained by sizing the HU trunk groups by the ECCS rule and then sizing the final groups to satisfy the demand servicing constraint. In order to exploit the single-year ECCS design as a basis for a five-year trunk plan on HU groups, we need to address the question of how to adjust the single-year trunk requirements to obtain an economical trunk implementation schedule for a given planning horizon.

3.2 Cluster Solution

This section outlines an algorithm for obtaining the initial TIP solution for a network cluster. The key idea of our solution is based on the results described in Section 3.1.3, which show that a near-optimal TIP solution can be obtained by providing reserve capacity on only the final trunk group.

Accordingly, in the case of HU trunk groups our task is to find an optimal disconnect policy for the single-year trunk requirements that takes into account the demand dynamics. That is, we need to show how to find a policy that minimizes the present-worth of the capital, labor, and maintenance costs over a planning horizon subject to the ECCS trunk requirements. As Figure 3 shows, when the initial five-year ECCS solution is obtained for a primary HU group, the TIP algorithm proceeds by determining appropriate disconnects and then calculates the overflow traffic. Then, it sizes the intermediate HU trunk group using, again, the ECCS engineering procedure and an optimal disconnect policy. After all subtending HU groups are sized, we complete the initial cluster TIP solution by using the only-route TIP model to plan capacity for the final trunk group.

Under certain circumstances the initial cluster TIP solution may provide more trunk capacity on the final trunk group than is necessary to satisfy the blocking objective and the demand servicing constraint. In that case, we can, as illustrated in the last two boxes of Figure 3, improve the initial TIP solution (reduce costs) by reducing the sizes of the subtending HU groups in an economical fashion. In Sections 3.2.1 - 3.2.3 we describe in more detail the steps of the TIP algorithm presented in Figure 3.

3.2.1 Optimal HU Disconnect Policy

We shall use the notions and notation of Section 2.1, except that d(k) now represents the deterministic (rather than random) ECCS trunk requirement at year k.

As explained in Section 3.1.3, we omit the demand servicing constraint while sizing HU trunk groups. Consequently, for HU trunk groups the objective is to implement the ECCS trunk requirements at minimum cost, i.e., to minimize

\[ \sum_{k=0}^{N-1} \rho^k \left[ c_f^1(u(k)) + c_f^2(u(k)) + c_f^3(T(k), u(k)) \right] \]

subject to the constraint

\[ T(k) \geq d(k), \]

where \( T(k+1) \) is defined by

\[ T(k+1) = T(k) + u(k). \]

With certain conditions on the relationships among the cost parameters, one can show that the optimal policy, \( u^*(k) \), has the following form:

\[
 u^*(k) = \begin{cases} 
 d(k) - T(k), & \text{if } d(k) \geq T(k) \\
 \min \left\{ \max \left\{ \frac{d(k+i) - T(k)}{T(k+1) - T(k+1-m)}, 0 \right\}, \text{ if } d(k) < T(k), \right. 
 \end{cases} 
\]

where \( m \) is the largest integer such that

\[
 b_k^1 - b_k^3 + \sum_{i=0}^{m-1} \rho^i a_{k+1}^m < \rho^m \left[ a_k^{1+m} + a_k^{2+m} \right]
\]

(3.2.1.4)

(3.2.1.5)

(Note that, as in Section 2, the parameters \( a, b, \) and \( c \) have superscripts, while only \( \rho \) is raised to a power.)

Condition (3.2.1.5) states that the present worth of money recovered by disconnecting a trunk in year \( k \) and maintaining \( (T(k+i) - 1) \) trunks, \( i = 0, ..., m - 1 \), is less than the present worth of money spent to purchase and install a trunk in year \( k + m \).

3.2.2 Initial Solution on Intermediate HU and Final Trunk Groups

After we size the intermediate HU trunk groups with the ECCS procedure, the TIP disconnect policy, presented in Section 3.2.1, is applied to these trunk groups. Then, to complete the initial cluster TIP solution, shown on Figure 3, we need to find the economical schedule of trunk augments and disconnects on the final trunk group. However, mathematically, the TIP algorithm for final trunks is identical to that for the only-route case. Hence, the methods of Section 2 are applied, assuming as an input the Neal-Wilkinson trunk requirements corresponding to the forecasts of total loads, including those overflowing subtending HU groups.

3.2.3 Reserve Capacity Analysis and Final Solution for HU Groups

In this section we demonstrate that in some instances the capacity of the final trunk groups, as calculated in Section 3.2.2, is greater than that necessary to satisfy the service objective and the demand servicing constraint. In that case, we show that the initial trunk requirements on the HU groups can be reduced in an economical fashion.

As we stated in Section 2, the TIP solution on a final trunk group is defined by the vector of threshold levels \( S(k) \), given in (2.4.1), where \( S(k) \) represents the minimum trunk requirement in year \( k \).

Specifically, if the number of planned trunks at the end of the \( (k-1) \)th year is \( T(k-1) \), then the planned trunks for year \( k \), \( T(k) \), is determined by (2.2.1). Thus, under certain conditions, the TIP solution has the result that \( T(k) \geq S(k) \). Therefore, \( T(k)-S(k) \) represents capacity that could be used to reduce the planned servicing cost for the year \( k \) on subtending HU trunk groups. The amount of extra capacity at year \( k \), \( z(k) \), depends on the realizations of the demand at previous years \( d(0), ..., d(k-1) \), but it is bounded from below by

\[ z(k) \geq T(k) + u(k) - S(k), \]

(3.2.3.1)
where $T'(k), u'(k)$ correspond to the scenario of no demand servicing on the final trunk group, i.e.,

$$T'(k) = T'(0) + \sum_{i=0}^{k-1} u'(i). \quad (3.2.3.2)$$

and $u'(k)$ is defined by (2.2.1).

This extra capacity at year $k$, $z(k)$, is allocated to the subtending HU groups in an optimal fashion. We defer to [8] the formulation and solution of this resource allocation problem. To summarize, it takes the form of a standard allocation problem that we solved using dynamic programming techniques. The dynamic programming formulation allowed us to reduce the computational burden by taking advantage of possible modularity constraints on HU group sizes [3].

4. Summary

We have described a new, multiyear trunk provisioning process, called TIP, that provides a cost-effective schedule for trunk augments and disconnects. The formulation is novel in that it includes explicitly and systematically key considerations, such as forecast uncertainty, demand dynamics, and implementation costs that formerly were accounted for heuristically.

Most of the key results are summarized, but not derived, here. The details of the mathematical analysis, evidence of the computational efficiency of algorithms, sensitivity of results to assumptions, methods for including budgetary constraints and facility and switch capacity constraints, and quantification of the impact of the TIP algorithms on network provisioning practices will be reported in future papers [7,8].

REFERENCES

ROUTE COST OFFERED LOAD TRUNKS IN TRUNKS REQUIRED
AB \( C_{AB} \) \( I + e \) \( T \) \(-\)
AC \( C_{AC} \) \( I + \epsilon_1 + O(I + e, T) \) \( T_1 \) \( \epsilon_1 \)
CB \( C_{CB} \) \( I + \epsilon_2 + O(I + e, T) \) \( T_2 \) \( \epsilon_2 \)

FIGURE 2 ALTERNATE ROUTING UNDER UNCERTAINTY
Q.1 (Peter Farr)

It appears that in the network expansion subprocess, you are assuming that continuous adjustments to the size of groups is permitted. Would your conclusions regarding optimal provisioning be different if the modular engineering principle was applied as is commonly the case. (Ref: Farr J.P., "Modular Engineering of Junction Groups in Metropolitan Networks" ITC - 8)

A.1 (A. Lashper, C.D. Pack, G.C. Varvaloucas)

This paper presents a continuous formulation of the network expansion problem. In a companion paper submitted to BSTJ (Ref: 8), we introduce a discrete problem formulation to derive an optimal expansion policy under modularity constraints.