In the study of the congestion processes, when the input process can be considered other than Poisson type, the calculation of both, call and time congestions Probability are too difficult to calculate and even more to estimate them for practical purposes.

To avoid this strong restriction, as is well-known, several sophisticated elucidations and techniques of calculation has been carried out from C. Palm (1943) and perhaps the more accepted would be that of the imbedded Markov Chains.

This work deals with the possibility to use the microprocessor to find a distribution function of the Congestion Process in time of assume one theoretical distribution specified by parameters like mean, standard deviation etc. That is, we look for the m-extremes based on the great possibilities of the microprocessor to study every transition points of both arrival process and congestion process, in a similar way to those employed in the imbedded Markov Chains.

1. METHODOLOGY APPLIED IN THE WORK AND IN THE EXPOSITION

1o. How the problem appears in the daily task of the Operation Area and utilization of the Extreme Value Theory or Ordered Statistics to prove, up to which point, for short time intervals, the arrival process to a set of devices could be considered as a random renewal type.

2o. Use of the properties of GI/M/R System to compute and classify the number of events (peg) by comparing the state of the Systems in consecutive scannings. This Philosophy of Computation makes a great simplification in the hardware of the measurement equipment, a lower cost device and provides a great accuracy in the collected data.

3o. Presentation of the final results, comparing them with those of the classical measurement methods and the interpretation of the observed differences showing the advantages of the proposed procedure.

2. GENERAL CONSIDERATIONS

Among the aspects and parameter that can be considered in the study of Stochastic Processes, from a practical viewpoint, without doubt, the most frequently used is the Congestion concept. To confirm this affirmation it is only necessary to remember the definition of G.O.S.

Accordingly, the "goodness" of many models is expressed in terms of its accuracy to estimate the Probability of Congestion.

Nevertheless, it is not easy to get an agreement about the degree of goodness required or, in other words, it is very difficult to find a reference value to judge, by comparison, if the precision of the estimation of the G.O.S. is adequate or not.

My personal interpretation of this circumstance, looking from the perspective of the Operation Area of a Telephone Administration is briefly exposed below:

Among the Teletraphic topics that affect more directly the nature of the problems in the Operations Area stand out the subjects of: Traffic Measurement, Forecasting, Planning, Dimensioning and Traffic Administration.

The information flow between them can be synthesized as follows (fig. n° 1).

In Forecasting, planning and even in Dimensioning, the input data to the models resulting from the projection of the traffic measurements, or estimated by indirect means will certainly have a lack of precision grade that, if we leave aside the philosophical considerations, it would make it paradoxical to look for an elaborate formula for the Congestion Function, because the input parameter imprecision, in the congestion formula, would void the eventual theoretical rigor of the mathematical expression.

In this sense, we could say that the better known classical formulas, as ERLANG'S loss or waiting models, have enough accuracy and in addition they are an abbreviated way of expressing equipment needs.

However, in traffic administration, considering this subject as a feedback of the above mentioned topics, the decisions are far more critical, first because they affect directly the G.O.S., and secondly because an inaccurate estimation or
TRAFFIC MEASUREMENTS

OPERATION (EXPLOITING)

FORECASTING

G.O.S. INSIDE SPECIFICATION?

YES

OPERATING NORMAL PROCES

NO

TRAFFIC ADMINISTRATION

PLANNING

SIGNIFICANT DEVIATION?

YES

INSTALLATION (DIMENSIONNING)

NO

FIG 1. FLOW CHART OF MAIN TELETRAFFIC TOPICS IN THE OPERATION AREA OF A TELEPHONE ADMINISTRATION

3. EVALUATION OF THE ABOVE CONSIDERATIONS AS VIEWED FROM THE ACTUAL STATE OF THE ART IN THE TRAFFIC MEASUREMENT FIELD

From the viewpoint of the Traffic measurement it can be said that, with the modern large measurement System, in which the periodic scanning method is exclusively used, it must not have errors to taking in account for most of the practical applications when values as Traffic volume, Traffic intensity, M.H.T. are provided; in such a way that, the best thing to do, is to get the shorter period between successive traffic data to make the Forecast and Planning periods as short as convenient and also to have the most recent values for the Dimensioning parameters.

Nevertheless, in Traffic Administration, considered this matter as a feed-back for the just abovementioned topics on one side, and on the other, like one of the more appropriate tools to know up to which point the commitment to the subscribers is fulfilled, it is not enough with the knowledgement of this kind of parameters; it is necessary that the used parameters give an estimation as accurate as possible for the G.O.S. and if possible for the Quality of Service.

In other wards, when a determined quantity of switching equipments is in service, it is within the objetives to have the G.O.S. offered to the subscribers is within of the objetives? When the load of a group is near of the theoretical limit, how can be really detected if the problem is either of G.O.S. or Quality of Service? What can be said about the G.O.S. when the load variation are large enough to doubt the likelihood of the hypothesis of the arrival process being stationary? This and similar questions are considered in this work and a practical solution is shown.

4. HEURISTIC INTERPRETATION OF THE PROBLEM

In order to get an adequate approach in the specification of Congestion process, when this seems to be other than Markov chain, a lot of works has been done since C. Palm (1.943) who tried to solve the problem using the Renewal Type Integral Equations. The impetus for this is the need felt for more accurate computation of blocking probabilities. As far as the author knows, the results are obtained in so strong abstraction of the reality that, sometimes, the practical applications are very rare, and other times the complexity of the final formulas makes their applications too arduous.

In the course of the work carried out in the Operation Area, in the Traffic Administration and in the traffic measurement fields, have been able to confirm that in most cases the situation could be represented as shown by the table below when a small exchange or a overload situation was concerned.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small exchange</td>
<td>Load near of theoretical limit</td>
</tr>
<tr>
<td>Overload situation</td>
<td>Load variation large enough</td>
</tr>
</tbody>
</table>

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GROUP OF 7 REGISTERS

<table>
<thead>
<tr>
<th>NOS</th>
<th>TOMAS</th>
<th>S. OCUP</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>000003</td>
<td>00198</td>
</tr>
<tr>
<td>01</td>
<td>000043</td>
<td>09472</td>
</tr>
<tr>
<td>02</td>
<td>000151</td>
<td>09688</td>
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<tr>
<td>03</td>
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<td>08756</td>
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<td>000310</td>
<td>08776</td>
</tr>
<tr>
<td>05</td>
<td>000298</td>
<td>08466</td>
</tr>
<tr>
<td>06</td>
<td>000119</td>
<td>09218</td>
</tr>
<tr>
<td>07</td>
<td>000027</td>
<td>00840</td>
</tr>
</tbody>
</table>

Table no 1

Where:
NOS = No of devices
TOMAS = Events (peg) = 1,121
S. OCUP = Holdin times in seconds

As can be seen in the following, the theoretical Erlang Loss (by using a well known formula) either loss or waiting models, gives values for the -- G.O.S. quite different of the empirical ones.

Following the specification of the manufacturer the theoretical model for dimensioning would be Erlang Loss, and B (7; 3.13) = 2,5 % while - the empirical time congestion probability is -- Q (7) = 1,1 %.

The persistency of situations like the just above shown, brought me to the idea that it must not be too difficult to see the reason of this difference.

So, I thought that in a very high load level situation (high congestion probability), the arrival process was forced or constrained and, consequently, the stationary character and the time homogenous hypothesis could be unlikely.

Of course, the idea was not new and, among the literature written on this subject, it can be seen similar ideas; so, W. Feller [2] to justify the likelihood of the Markovian character of - the input process to a system, said: "The underlying physical assumption is that the forces and influences governing the process remain constant, so that the probability on any particular event is the same for all time intervals of duration t, and is independent of the past development of the process".

If the forces governing the process seems to be not constants due to the "positive feedback" - between the primitive sources (subscribers) and the system load, the process could not be Markovian.

In view of the similarity, in some aspects which I was analyzing, between Reliability Theory and Teletraffic, I decided to use Pyke’s Test [3] (very similar to that well known of Mann-Kendal) [4] and, as can be seen in the following example, there was no reason, at least as far as the statistical tests could arrive to reject the hypothesis of randomness and, if this is so, more difficult would be to find an unquestionable method to discuss the Statistical Equilibrium axiom.

![Arrival Epochs and Spacings in Seconds](image)

FIG. 2 ARRIVAL "EPOCS". SPACINS IN SEC.

The figure sows the detected arrival "epochs" (in Feller sense of the word epoch) to a subscriber’s group of a small exchange in a very high overload situation.

The mean value of the variable Y_n [5] is \( \bar{Y} = 213 \)

\[ Y_n = \begin{cases} \left( \frac{2(n-1)}{4} \right) + \left( \frac{2n \cdot 3n^2 \cdot 5n}{72} \right) \rightarrow N_1 (217.5; 28) \end{cases} \]

The value 213 fall within the interval \( \mu \pm 0.16 \sigma \)

\[ Y_{\epsilon} = \sum_{i=1}^{n} Y_i \text{ where } Y_i = \begin{cases} 1 \text{ if } D_{n+1} \neq D_n \text{ normalized spacings } \epsilon \text{ from a } \epsilon \text{ distribution on a sample from a } \epsilon \text{ distribution of rate } \lambda \end{cases} \]

\[ D_1, D_2, \ldots, D_n \text{ normalized spacings } \epsilon \text{ from an } \epsilon \text{ distribution } \text{ which can occur with either parametrical or non-parametrical test}, \text{ and this is not an inconvenient in this work due to the way we use it.} \]

It can be argued that, in spite of this test being considered like one of the most powerfull among the non-parametrical type [6], it can not detect some subtle violations as it can occur in Teletraffic, but this can occur with either parametrical or non-parametrical test, and this is not an inconvenient in this work due to the way we use it.

The question is: If it is difficult to detect the non-Markovian character of the arrival process in short intervals, it would be not bad to take advantage of the situation; and in this way, it was added to the possibility of the microprocessor to estimate the G.O.S., the idea of make a good use of the property that \( \text{Pr} > 1 \text{ (at)} = 0 \text{ (at)} \) \(^*(m)\), in counting the number of event (peg) by comparing successive

\((m)\) The equipment designed can avoid some eventual violations of this hypothesis.
Again, it can be argued that the whole arrival - process along the normal measurement interval is not completely taken into account with tests like above mentioned, but my answer to this question is that the overall process, during the complete measurement interval, can be considered as a succession of random renewal processes of different rates and the system is studied by the device in each of them, and the information is stored in - such a manner that the whole process along the T.C.B.H. is considered as a GI/M/R, in which are studied, based on the possibilities of the microprocessor, both, the ergodic Markov chain defined by the arrival process,Z and the congestion process,Z(t).

The whole process can be studied as long as it is needed from a practical viewpoint, due to the fact that the limit to specify the state of the system in each characteristic point is imposed by the noise on the relay's contacts, in such a way that if the system is really a GI/M/R, both the regeneration points and "death points" are detected and analyzed.

5. PHILOSOPHY OF THE MEASUREMENT PROCEDURE AND DERIVED ADVANTAGES

We deal with the possibility to obtain, beside the typical values provided by the conventional equipment (number of events - peg- and elementary interval of holding time -usage-) another more characteristic values to define and control the system.

By comparing successives states of the system, the measuring of classical pegs are obtained, and by accumulation of the elementary holding intervals in registers associated to each state, the classical measurement of traffic intensity are obtained. Besides, enough information to determine the probability distribution P(j) of the arrival process, Z(t), is obtained and also the stationary distribution function of the Congestion Process. For obvious reasons, the equipment has been designed to calculate automatically the mean and standard deviation.

Besides the logical advantages of computing the number of peg by comparing successives states from the viewpoint of Teletraffic, in other aspects are also reasons to do that, because the quick velocity of comparison (scanning interval), in the order of 10^5 times in the normal measurement period and the number of changes of states to detect (app. 2,000-2,500 maximum per each 1,000 subscriber group in the point of highest concentration in the conventional switching equipment) is that in most scanning cycles there is no change of state to detect; so, the microprocessor, which basic work is around of the measurement period, can do some other work to increase the accuracy of the results.

Among the checks is the discrimination between actual signal and noise; by using noise filtering algorithm, simplifying the interfaces (to the exchange) - hardware, self test function, etc.

The data presented below have been obtained with a prototype unit which block diagram is shown on the fig. n° 3.

The system software is built in a modular way as shown in the following figure (bottom of fig. 3), in which:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
</tbody>
</table>

The data tables supplied by the equipment are in the form:

Table n° 2.

<table>
<thead>
<tr>
<th>i</th>
<th>K_1n_{i-1}</th>
<th>K_2Q(i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>K_0</td>
<td>Q(0)</td>
</tr>
<tr>
<td>1</td>
<td>K_1n_0</td>
<td>K_2Q(1)</td>
</tr>
<tr>
<td>j</td>
<td>K_1n_{j-1}</td>
<td>K_2Q(j)</td>
</tr>
</tbody>
</table>

\[ K_1 = \frac{1}{\sum n_1} \]

\[ P_j = \frac{n_{j-1}}{\sum n_1} \]

\[ n_i = \text{arrived calls in state } i \]

\[ T \text{ measurement interval in seconds} \]

From this basic data it is quite easy to obtain \( E[Z(t)] \) or traffic carried, and any moment \( a_\nu = \sum z^\nu Q(i) \) of the stationary distribution function of the Congestion Process.

The nomenclature used is from Siskin in such a way that:

\[ Z_r = Z(t_r) \] where \( t_r \) is the epoch of the \( r \)th arrival

\[ P_{ij} = \text{Call Congestion} \]

\[ P_{ij} = \lim_{T \to \infty} \frac{1}{T} \int_0^T \delta_{j}^{Z(t)} dt \]

\[ Q(j) = \lim_{T \to \infty} \frac{1}{T} \sum_{i=1}^{n} \delta_{j}^{Z(t)} \]

\[ Z_r = Z(t_r - 0) \] where \( t_r \) is the epoch of the \( r \)th arrival

\[ \delta_{j}^{Z(t)} \]

\[ P_{ij} = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \text{Pr}[Z_r = i] \]

\[ P_j = \lim_{n \to \infty} \frac{1}{n} \text{Pr}[Z_r = j] \]

\[ P_r = \text{Call Congestion} \]

In the same way, it can be seen that:

\[ Q(j) = \text{Time Congestion} \]

The data tables supplied by the equipment are in the form:

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6. INTERPRETATION OF THE DATA

The result must be interpreted as follow: The device can be programmed to realize three different tasks, one of each can be selected in an automatic way.

Task 1.- TCBH calculation: As this is a well known subject and available in other equipments we will not cover it in this paper.

Task 2.- Simultaneous measurement of events and traffic during TCBH: This has been the main goal for the development of this equipment, and will be explained in greater detail. Also has been incorporated the possibility to study up to 32 measuring points (either of one or more groups) individually (ICUP) in order to measure G.O.S. under some tail-end conditions.

Task 3.- ICUP: (Individual Circuit Usage and Pag) . This task has been developed for maintenance purposes and, as in task 1 case, will be not cover in this paper. Only indicate that in this choice an ICUP report, for every measuring points, is printed out and circumstances like killer trunk (K), retained device (R), etc. are stood out.

In relation with the task 2, three data set are shown:

1st: Set of data. - HEADER: To identify the measurement conditions. (See translation on the report table n° 3).

2nd: Set of data. - Traffic Parameters. If we make reference to table n° 1, as can be said;

a) The empirical time congestion probability $Pr[1] = Q(7) = 1.1 \%$.

b) $E[Z(t)] = \text{Mean Carried Traffic by the group is estimated (measured as}$

$$\sum_{i=1}^{n} i Q(i) = 3.1 \text{ Erlangs}$$

c) Empirical Call Congestion Probability $Pr[C_i]$ can be really measured if it is zero like in the group of sender $Pr[C_i] = P_4 = 0$ (See table n° 4).

But in some cases, like in the register group, it can be estimated approximately by

$$Pr[C_i] \approx \frac{\text{Number of offered calls during the time congestion}}{\text{Total number of calls}}$$

$$\sim\frac{3.1}{(40/9.92)} \sim 1.09 \% \approx Pr[1] \approx P_5$$

d) In table n° 5 (a group of 20 Intertall Trunk) in can be seen that offered Traffic intensity is 9.5 E. and $Q(18) = Q(19) = Q(20) = 0$ while the respective Erlang Losses are in the order of -- 5 \%, 2.3 \% and 1.1 \%
Q(17) = 35/3.600 ± 9.7 % while B(17, 9.5) ≈ 8.9 %
(It can be seen that in normal load conditions - the theoretical estimations are better).

As mentioned before in task number 2 we have incorporated the possibility of measuring up to 32 input points belonging to one or more groups.

This option allows, when measuring critical load groups, to indicate, by one side, the proper working (or not) of the individual organs within - the group and, by the other, the great difference between the theoretical G.O.S. and the empirical G.O.S. when failure condition appears.

In the example shown (see tables no 6) as three registers (connected to ICUP points 22, 24 and 25) were purposely busied, the interpretation of the G.O.S. as Q(7), which would correspond to a E_i(7; 5,03) if the ICUP information was not available. It would be, in reality compared to E_i(4; 2,03).

Q(7) = 23/300 = 7.6 % while B(7; 5,03) > 12 % (m), E(4; 2,03) > 9.5 %.

In a similar way we could detect failures marked as K (for killer), or L (for lose) which permits the measurements of the G.O.S. under actual failure conditions.

Similar results have been obtained in the measurement carried out on a subscribers group to prove the accuracy of Engset fitting.

7. ADVANTAGES OF THIS PROCEDURE FROM THE POINT OF VIEW OF THE TRAFFIC THEORY

Apart from the advantages already mentioned, there is another one from the point of view of the traffic theory. Is the following. In the classic literature, the G.O.S. is estimated assuming a theoretical distribution for the Congestion Process, and estimating, by means of certain parameters that specify the distribution of the Process, the area of the tail or the probability of a particular value. Excepting rare occasions (which do not precisely correspond to the Congestion - Process) the fitting of the Theoretical to the empirical distributions is not accurate enough for most of the cases.

The goodness of fitting is so critical due to \( F_n(x) \) (empirical distribution function), being \( n \), in Teletraffic, as large as desired, is really a distribution to which converge strongly the empirical distribution of eventual smaller number samples; and, consequently, there is no reason to look for a fitting using the theoretical distribution of a determined type. The inconvenient to fit the tail of a dist. Funct. is avoided in our measurement procedure obtaining directly the area of the tail or the estimate of the probability of a particular value of the process without assuming too complicated hypothesis, about neither mean values nor extremes values.

<table>
<thead>
<tr>
<th>TABLES</th>
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<tbody>
<tr>
<td>NOS</td>
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<td>00</td>
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ACKNOWLEDGEMENT

A good like to thanks sincerely to all my colleagues and collaborators in the daily task in Traffic Measurement, Traffic Administration and Traffic measuring devices design, their unpayable — help.

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Summary of Questions/Answers

Date: 09 June 1983
Session: 1.4
Paper: 4

Q.1 (J G. Kappel)

In tables 1, 6 and 5 the column labeled "S. OCUP" seems to indicate the number of seconds per hour in which N Servers (NOS) are busy, but what is the meaning of the "TOMAS" column? If this is "Arrivals during the state N Busy," as it appears, then none of the illustrated cases seem to be in equilibrium.

A.1 (L.F. Nombela)

"TOMAS": means number of events or peg but, as can be seen on Table No. 2 n{i-1} are the number of arrival calls when there are i-1 busy devices; and then (i-1) is the number of times the system goes from the state (i-1) to state i. This is the reason to put this table K, n{i-1} at the same level than Q (i) and i.