ABSTRACT
The terminal reliability of a computer communication network is determined for a given level of traffic between the terminal computers, taking account of the condition that the packet delay must not exceed a given value. This delay is equated to the length of path taken by the packet. We describe two methods to determine the terminal reliability, and show how it is affected by different levels of traffic, and different values of delay. We conclude that the conventional approach to the analysis of network reliability, in which the maximum allowed delay is not considered, may be of limited value. Mention is made of the analogous calculation in circuit switched networks.

1. Introduction
In the probabilistic analysis of network reliability the network is represented by a graph G in which the nodes correspond to computer centres, and the edges correspond to communication links. Each element (node or link) has a probability of being operative.

The object in conventional reliability analysis is to find the probability that there exists at least one path between a specific pair of nodes (terminal reliability) or between every pair of nodes (global reliability).

This measure of reliability may be unsatisfactory because it does not reflect the performance of the network; for example, after the failure of elements in the network, if the terminal capacity remaining between a source node S and a terminal node T is not enough to accommodate the mean traffic between them, then S and T can be considered to be non-communicating. Similarly, the delay between S and T may be too high for effective communication.

In reference (8) four methods were described for calculating the terminal reliability and unreliability, taking into account the level of traffic between the terminal nodes, and the terminal capacity remaining after elements in the network had failed. The terminal reliability was defined as the probability that the terminal capacity > \( Y_{ST} \), and it was shown that if all links have equal capacity, this reduces to the probability that there are at least \( n + 1 \) link disjoint paths between S and T where

\[
n = \left\lfloor \frac{Y_{ST}}{c} \right\rfloor \quad \text{where} \quad c = \text{link capacity}, \quad \lfloor x \rfloor = \text{largest integer} \leq x
\]

In packet switched computer communication networks the main parameter used to measure the performance between terminal nodes is the packet delay, that is the time taken for a packet generated at node S to reach node T.

Here we introduce a measure of terminal reliability which takes account of both this delay between terminal nodes, and the traffic between them.

More formally, in the presence of failure the terminal capacity > \( Y_{ST} \), and the delay between S and T ≤ MAD (maximum allowed delay). For equal link capacities this becomes the probability that there are at least \( n + 1 \) link disjoint paths between S and T and that the delay ≤ MAD.

2. Notation
- \( P_i \) = probability that link i is operative
- \( Q_i \) = probability that link i has failed \((1 - P_i)\)
- \( P_{Ni} \) = probability that node i is operative
- \( Q_{Ni} \) = probability that node i has failed \((1 - P_{Ni})\)
- \( R_{ST} \) = terminal reliability between source node S and destination node T
- \( Y_{ST} \) = average traffic between S and T
- \( e_i \) = link i, or the event that link i is good
- \( T_i \) = packet delay in link i (service delay + queuing delay)
- \( c_i \) = capacity of link i
- \( \lambda_i \) = flow in link i (average number of packets per sec. entering link i)
- \( u_k \) = average packet length (bits/packet)
- \( \pi_{ST} \) = path k between S and T
- MAD = maximum allowed delay

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From equation 2, since all links have equal flow and capacities, they will also have equal delays, and equation 1 becomes

\[ Z_{ST} = N_p + \sum_{i \in C} (T_i + P_{r_i} + N_p) \]

if all the links have equal propagation delays.

Now \( N_p < < \ell_k (T_i + P_{r_i} + N_p) \)

so \( Z_{ST} = \ell_k (T_i + P_{r_i} + N_p) \)

3.

Thus, the delay on path \( \pi(k) \) is proportional to \( \ell_k \), so for a network having links of equal capacities and uniform traffic flow, the delay can be described by the number of links \( \ell_k \) on the path.

Note that in circuit switched systems, the probability of blocking between terminal nodes S and T, using path \( \pi(k) \), and assuming independence between the links in the path, is

\[ P_b = 1 - \prod_{i=1}^{k} (1 - P_{b_i}) \]

where \( P_{b_i} \), the probability of blocking in link i, is found from the Erlang formula:

\[ P_{b_i} = \frac{A_i}{V_i} \frac{A_i^2}{2!} \cdots \]

A is the traffic offered and \( V_i \) is the number of channels in link i.

From equation 4, if \( P_{b_i} = P_b \) for all i,

\[ P_b = 1 - (1 - P_b) \]

for small \( P_b \).

Thus, in circuit switched networks, if \( P_b \) is constant and small, the probability of blocking between S and T, using a given path, is proportional to the number of links in the path.

Returning to the packet switched network, from earlier discussion of delay between terminal nodes, the terminal reliability can be restated as the probability that there are at least \( n + 1 \) link disjoint paths of length less than or equal to some given maximum path length (MPL).

4.1 Method 1. Analysis

The terminal reliability is calculated using the following steps

Step 1 Enumerate all the paths between S and T having \( \ell_k \leq \text{MPL} \).

Step 2 Calculate \( n \) from \( n = \frac{V_{ST}}{c} \), \( c = c \) for all i.

Step 3 From the paths enumerated in step 1, find all possible \( n + 1 \) link disjoint paths between the terminal nodes. Let there be \( k \) possible \( n + 1 \) link disjoint paths \( A_1, A_2, \ldots, A_k \). A group \( A_i \) is said to be good if all the elements constituting \( A_i \) (the \( i \) th \( n + 1 \) link disjoint paths) are good. The same symbol \( A_i \) is used to denote the
event that $A_i$ is good. The network success, and the terminal reliability, can be expressed as

$$S = A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k$$

7.

$$R_{ST}(MPL) = P_1[S] = P_1[A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_k].$$

Step 4. Since the events $A_1, A_2, \ldots, A_k$ are not necessarily disjoint,

$$R_{ST}(MPL) \neq P_1[A_1] + P_1[A_2] + \ldots + P_1[A_k].$$

To evaluate $R_{ST}(MPL)$ it is necessary to change the non-disjoint events $A_1, A_2, \ldots, A_k$ to disjoint events, and then the algebraic sum implied by equation 9 will be valid. The algorithm given in reference 6 is used to evaluate $R_{ST}(MPL)$ from $A_1, A_2, \ldots, A_k$. In [11] an algorithm was introduced to enumerate all the paths, in ascending order of length, having $\ell \leq MPL$.

All possible $n \times 1$ link disjoint paths (Step 3) can be found by comparing the paths enumerated in Step 1, and noting that

(a) a one-link path is disjoint from any other path
(b) all paths of length $2$ are disjoint
(c) a path of length $2$ is link disjoint from another path if the intermediate node of the length $2$ path is not the same as the second or penultimate node of the other path,
(d) if two paths, of whatever length, share the same second node, or the same penultimate node, they cannot be disjoint.

4.1.1 Example

In the network of Fig.1, all the links have equal capacities, $c = 50$k bits/sec. We wish to find the terminal reliability between nodes 1 and 4 for the conditions,

a) $Y_{14} = 90k$ bits/sec, $MPL = 2$

b) $Y_{14} = 90k$ bits/sec, $MPL = 3$

c) $Y_{14} = 135k$ bits/sec, $MPL = 3$

d) $Y_{14} = 40k$ bits/sec, $MPL = 3$

a) Step 1 (referring to Fig.1), the paths of length $\leq MPL$ are

$$P_1 = e^{10}, P_2 = e^{9}e^{13}$$

Step 2

$$n = \left[\frac{90}{50}\right] = 1$$

$$\therefore n + 1 = 2$$

Step 3

We need to find all the two-link disjoint paths; there is only one set in this example:

$$A_1 = e^{10}e^{9}e^{13}$$

Step 4

$$R_{14}(2) = P_1[A_1] = P_1 p_{10} p_{9} p_{13}$$

and if all the links have the same reliability

$$R_{14}(2) = p^2$$

b) Step 1 All the paths of length $\leq 3$ are

$$P_1 = e^{10}, P_2 = e^{9}e^{13}, P_3 = e^{1}e^{2}e^{3}, P_4 = e^{9}e^{5}e^{4}, P_5 = e^{8}e^{12}e^{4}.$$
Step 3 The three-link-disjoint path groups are:
\[ A_1 = e_{10} e_{13} e_1 e_2 e_3, \quad A_2 = e_{10} e_9 e_1 e_2 e_3 e_4, \]
\[ A_3 = e_{10} e_1 e_2 e_3 e_4 e_5 e_6, \quad A_4 = e_{10} e_1 e_2 e_3 e_4 e_5 e_6 e_7, \]
\[ A_5 = e_9 e_{13} e_1 e_2 e_3 e_8 e_{12} e_4. \]

Step 4
\[ S = e_{10} e_9 e_{13} e_1 e_2 e_3 + e_{10} e_1 e_2 e_3 e_4 e_5 + e_9 e_{10} e_1 e_2 e_3 e_8 e_{12} e_4 + e_9 e_5 e_{13} e_1 e_2 e_3 e_8 e_{12} e_4 + e_{10} e_9 e_{13} e_1 e_2 e_3 e_8 e_{12} e_4. \]

\[ R_{14} (3) = P_{10} P_9 P_1 P_2 P_3 + q_{13} P_{10} P_9 P_1 P_2 P_3 + q_9 P_{10} P_1 P_2 P_3 P_8 P_{12} P_4 + q_5 P_{13} P_9 P_{10} P_1 P_2 P_3 P_8 P_{12} P_4 + q_{10} P_{13} P_9 P_{10} P_1 P_2 P_3 P_8 P_{12} P_4 \]

If all the links have the same reliability,
\[ R_{14} (3) = p^6 + p^6(1-p^3) + 2q p^7 + q^2 p^8 + q p^8 \]

d) Step 1 - as in case (b)
Step 2
\[ n = \left\lfloor \frac{40}{50} \right\rfloor = 0 \quad n + 1 = 2 \]

Step 3
\[ A_1 = e_{10}, \quad A_2 = e_9 e_{13}, \quad A_3 = e_1 e_2 e_3, \]
\[ A_4 = e_9 e_5 e_4, \quad A_5 = e_8 e_{12} e_4. \]

Step 4
\[ S = e_{10} + e_{10} e_9 e_{13} + e_{10} e_{13} e_1 e_2 e_3 + e_{10} e_{13} e_1 e_2 e_3 e_4 + e_{10} e_{13} e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_{12} e_4 + e_{10} e_{13} e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_{12} e_4 + e_{10} e_{13} e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_{12} e_4 + e_{10} e_{13} e_1 e_2 e_3 e_4 e_5 e_6 e_7 e_8 e_{12} e_4. \]

\[ R_{14} (3) = P_{10} + q_{10} P_{10} P_9 P_{13} + q_{10} (1-P_{13}) P_1 P_2 P_3 + q_{10} P_{13} (1-P_{12}) P_9 P_{12} P_4 \]

and for equal link reliabilities, this becomes
\[ R_{14} (3) = p + q p^2 + q (1-p^2) p^3 + q^2 (1-p^3) p^3 + q^2 (1-p^3) p^4 \]

This method can be extended to include both link and node failure, assuming nodes S and T to be reliable. To do this, any intermediate nodes are enumerated included in the path enumeration of step 1. Consider case (a) above

Step 1
\[ P_1 = e_{10}, \quad P_2 = e_9 e_{13} \]

Step 2
\[ n + 1 = 2 \]

Step 3
\[ A_1 = e_{10}, \quad A_2 = e_9 e_{13} \]

Step 4
\[ R_{14} (2) = P_{10} P_9 P_{13} P_6 P_{13} \]

and for \( P = P_{13} = P \)
\[ R_{14} (2) = p^4 \]

4.2 Method 2. Simulation

The failure of elements of the network is simulated by first specifying the probability of failure for a link or node (\( q_1 \) and \( q_N \) respectively). A random number generator is used to apply a random number between 0 and 1 to each element, say \( R_i \) for link \( i \) and \( R_{N_i} \) for node \( i \). If the \( R \) value is less than the appropriate \( q \) value, the element is considered to have failed. The \( n+1 \) link disjoint groups (as defined in Step 2 of Method 1) are tested to determine whether at least one of them has survived. If so, the network is a success, if not it is a failure. The process is repeated \( N \) times, and the mean value of reliability is the (number of successes)/\( N \). The steps of the method are

Step 1 Enumerate all the paths between S and T of length \( \leq MPL \), in order of increasing number of links.

Step 2 Find \( n \) as in Method 1, and from Step 1 find all possible \( n+1 \) link disjoint paths.

Step 3 Use a random number generator to simulate element failure, as described above.

Step 4 Evaluate the groups identified in Step 2. If at least one has survived, the network is a success.

Repeat Steps 3 and 4, \( N \) times, and note the total number of successes \( NS \).

Then \[ R_{ST} (MPL) = \frac{NS}{N} \]
Results

The results shown in Figs. 2 and 3 are based on the network shown in Fig. 1, and discussed in the example. The terminal reliability between nodes 1 and 4 for fixed link capacity $c = 50k$ bits/sec, plotted against the probability of link failure (assuming reliable nodes), for different levels of traffic, and different values of delay, is shown in Fig. 2. The two sets of curves in Fig. 2 are for different ranges of link failure probabilities. The terminal unreliability is plotted against the probability of link failure, in Fig. 3.

Conclusion

We have introduced two algorithms for a measure of terminal reliability which reflects the performance of the network between the terminal nodes. Fig. 2 shows the effect of different levels of traffic, and different values of delay, on the terminal reliability. It can be seen that the terminal reliability for a certain network does not depend only on the probabilities of failure of its elements, but it also depends on the integer, traffic capacity ratio, $[\frac{c}{n}]$, and the value of the maximum allowed delay between the terminal nodes. The effect of increasing the terminal reliability of a network by increasing element reliability is not as effective as decreasing the traffic capacity ratio, especially for high probabilities of element failure. The effect on the terminal reliability of increasing the maximum allowed delay between the terminal nodes from MPL1 to MPL2 depends on the increase of the number of $n+1$ link disjoint paths due to the increase of the maximum allowed path length.

These results suggest that the analysis of reliability based on conventional reliability analysis, which takes account of all paths, regardless of length and capacity, can be misleading.

References

11. Shatwan, F., and Smith, D.G., Reliability of a computer communication network with delay, to be published.
Fig. 1 8-node network used in section 4.1.1

![8-node network diagram]

Fig. 2 Terminal Reliability as a function of link failure given various levels of traffic and delay.

\[ P = \text{prob [terminal capacity} \geq Y_{14} \text{ and delay} \leq \text{MPL}] \]

- \( a = 50 \leq Y_{14} < 100 \text{ and MPL} = 2 \)
- \( b = 50 \leq Y_{14} < 100 \text{ and MPL} = 3 \)
- \( c = 100 \leq Y_{14} < 150 \text{ and MPL} = 3 \)
- \( d = Y_{14} < 50 \text{ and MPL} = 3 \)
Fig. 3 Terminal Unreliability as a function of link failure given various levels of traffic and delay.
Q.1 (O.G. Soto)

The methodology described in your article is applied to a network in which all links have the same capacity and carry the same traffic. This allows you to apply graph theory to solve a performance problem. The applicability of the model may be limited because of this assumption. Are you investigating the possibility of using that methodology in more complex networks including different speed links, traffics and node capacities.

A.1 (F. Shatwan & D.G. Smith)

The model networks we have studied so far have the limitation you mention. We are, however, trying to extend our method to take account of some variation in the traffic parameters over the network. This is the start of our study and therefore assumes a simple model to enable us to examine the graph theory implications first.