TRAFFIC CHARACTERISTICS OF PCR METHOD FOR CCITT SIGNALLING SYSTEM NO. 7

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Abstract - This paper studies the traffic performance of CCITT Signalling System No. 7 operating with the error correcting method of the preventive cyclic retransmission. Strict formulae for the mean and variance of the queueing delay are derived in the explicit form and the formulae of queueing delay distribution are also derived in the form of Laplace-Stieltjes transform. The results of computer simulation where the actual mechanism is fully implemented are shown indicating that the formulae are sufficiently accurate. These formulae will contribute to establishing a mathematical basis for specifying the signalling performance objectives.

1. INTRODUCTION

Signalling System No. 7 is an internationally standardized general purpose common channel signalling system optimized for use in digital telecommunications networks in conjunction with stored program controlled exchanges. Common channel signalling can be regarded as a form of specialized data communication for transferring various types of signalling and other information between processors of, for example, telephone exchanges.

In most telecommunication services, the signalling transfer time is one of the important factors for determining the service quality. Since the transfer time is a function of many variables such as traffic loading, input traffic characteristics, transmission errors and propagation delay, the mathematical formulae are required to determine the design objectives for the applications.

This paper analyzes the statistical performance of signalling transfer time of Signalling System No. 7 when used with the preventive cyclic retransmission (PCR) error correction method.

The signalling transfer time in a common channel signalling system, represented by CCITT Signalling System No. 6 has been analyzed in [1] for error-free links. The retransmission delay for error recovery has been studied in [2] and [3] for several ARQ protocols. The message transfer time including retransmission delay for HDLC balanced class of procedures has also been analyzed in [4]. However, the technique of the above paper is not directly applicable to the PCR method of Signalling System No. 7, which is subject to this paper.

The authors have already developed the mean queueing time formulae of Signalling System No. 7 taking into account the error recovery time which has been elaborated in the system performance section of the CCITT Recommendation (Yellow Book).

Beside the mean queueing delay time, since the fluctuation in queueing delay time is another important factor for many applications, further mathematical improvements are introduced to analyze its strict behavior. The present analysis focuses on the distribution of queueing and retransmission delay for the PCR method.

We first briefly review the main feature of the PCR method. Then we describe a mathematical model and its analysis. Finally, numerical results and simulation results are given to confirm analysis accuracy.

2. PCR METHOD OF SIGNALLING SYSTEM NO. 7

In Signalling System No. 7, any signalling information is transferred in a specific form called message signal units (MSUs). For the purpose of error control each MSU is supplemented by an error detecting code and sequence numbers by which the error correction is obtained basically by retransmitting an erroneous MSU. In view of the wide range of Signalling System No. 7 applications, two different error correction procedures have been specified: the basic method and the PCR method.

The PCR method is used for long distance or satellite links with a one-way propagation delay of 15 ms or more. It is a non-compelled method in which positive acknowledgements are given for correctly received signal units and errors are corrected by preventive cyclic retransmission without any explicit negative acknowledgement [5].

To illustrate how preventive cyclic retransmission is used for error recovery, we subsequently outline the operation in signalling link.

i) MSUs to be transmitted from a sending exchange to a receiving exchange may be stored in the send buffer at the sending exchange. The MSUs are transmitted on a first-come/first-served (FCFS) basis. Copies of all MSUs are stored in the retransmission buffer at send side until they are acknowledged.

ii) The transmission of new MSUs is continued without pause as long as there is any waiting MSU in the send buffer. Once the signalling link becomes idle and there are no new MSUs in the send buffer, copies of MSUs in the retransmission buffer are retransmitted in the
order of original signal sequence. When the first retransmission cycle has finished, and the retransmission buffer still contains the copies, another retransmission cycle takes place. If new MSUs need to be transmitted during the retransmission cycle, the cycle is interrupted to allow them to be sent. The cycle is resumed and the newly sent MSU is stored for possible retransmission at the end of the cycle.

iii) On the receive side, the signal sequence and the transmission error are examined and positive acknowledgements are sent back for correctly received MSUs.

iv) When all MSUs have been acknowledged, and if there are no new MSUs awaiting transmission, fill-in signal units (FISUs) are transmitted. In this manner, if an MSU is affected by error on its first transmission, that MSU will be correctly received by preventive cyclic retransmissions. Furthermore, MSUs coming after an erroneous MSU and discarded by the check on signal sequence will also be correctly received.

When the traffic volume of MSUs is not heavy and a signalling link has enough capacity for preventive cyclic retransmission, errors are effectively corrected by the PCR method, especially where long propagation time and high error rates are involved. This is because, in such a case, retransmission is done without a request from the distant receiving side.

3. MATHEMATICAL ASSUMPTIONS AND NOTATIONS

A set of assumptions which define the mathematical model of the PCR method is listed as follows:

1) MSUs to be transmitted arrive at the send buffer according to a Poisson process.
2) Transmission time of MSUs is independently distributed.
3) An FISU requires constant transmission time.
4) A constant time is required for processing a received MSU and an acknowledgement of an MSU expected to be returned at a fixed time after its transmission.
5) Errors on MSU transmissions are independent and there are no errors in the acknowledgement message.
6) Effects of double errors of MSU transmission and retransmission are ignored.

The following notations are introduced.

For MSUs:

- $\lambda_m$: arrival rate,
- $\alpha_m$: traffic volume,
- $B_m(t)$: transmission time distribution,
- $T_m$: mean transmission time,
- $C_{i-1}T_i$: $i$-th moment of transmission time,
- $B_m(s)$: Laplace-Stieltjes transform (LST) of $B_m(t)$.

For FISU:

- $T_f$: transmission time,
- $B_f(t)$: transmission time distribution,
- $B_f(s)$: LST of $B_f(t)$.

For signalling link:

- $P_u$: Signal unit error probability,
- $T_L$: Loop propagation delay.

The queueing time for MSU is denoted by the following:

\[
\text{queueing time} = \text{(total elapsed time from MSU arrival to its acknowledgement message reception)} - T_L - \text{(transmission time of that MSU)}
\]

Function $W(s)$ is denoted by the queueing time distribution. The distribution has mean $Q$, variance $Q^2$ and LST $W^*(s)$.

4. QUEUEING TIME ANALYSIS

MSUs correctly received at the receiving side are classified into the following three categories:

1) MSUs that are correctly received on the first transmission.
2) MSUs that are affected by error on their first transmission and correctly received on their retransmission.
3) MSUs that are coming after an erroneous MSU and discarded by sequence number check, then correctly received on their retransmission. We call 1) MSU-NR, 2) MSU-PR and 3) MSU-SR respectively.

In this section, the queueing time for each kind of MSU is analyzed, then their results are composed to obtain the total queueing time.

4.1 Queueing Time of MSU-NR

For queueing time analysis, we use the theory for non-preemptive M/G/1 queue with three priority levels in saturated case and consider $M_1$, $M_2$, $M_3/G_1$, $G_2$, $G_3/1$ model shown in Figure 1. Where new MSUs, MSUs for retransmission and FISUs arrive at FCFS queues $Q_1$, $Q_2$ and $Q_3$ according to a Poisson process. New MSUs are given the first priority for transmission, while MSUs for retransmission and FISUs take the second and third priorities respectively.

The main difference between this model and actual system is the arrival process of MSUs for retransmission and FISUs. However, it is well known that the waiting time of higher priority signals does not depend upon an arrival process of lower priority signals, and the only interesting feature of lower class arrival process is the arrival rate [7].

Therefore the queueing time of MSU-NR can be analyzed from this model, where the slight difference caused by the mutual dependence of transmission times is ignored.

First, equivalent arrival rates of FISUs and MSUs for retransmission are estimated. Since the
arrival process of new MSUs is considered as a Poisson process, the expected time length available for the FISUs insertion after an MSU arrival is,
\[
\tau = \int_T \left(1-T\right) A_m e^{-A_t} d\lambda
\]  
(1)
Consequently, the equivalent arrival rate of FISUs is,
\[
\lambda_f = \frac{\lambda_t \tau}{\tau} = \frac{e^{-A_t} T}{\tau}
\]  
(2)
The signalling link is operated in a saturated condition, so equivalent arrival rate of MSUs for retransmission is,
\[
\lambda_r = \frac{1-\lambda_f T_m}{T_m}
\]  
(3)
Using these results, the cumulative distribution function of new MSUs waiting time in Q1, that is the queueing time of MSU-NR, is given in the following LST representation.
\[
W_{NR}(s) = \frac{\lambda_f (1-B_m(s)) + \lambda_r (1-B_m(s))}{s - \lambda_m (1-B_m(s))}
\]  
(4)
The mean and the second moment of queueing time are
\[
W_{NR}(1) = N_R m + m t_m + \frac{\lambda_m T_m^2}{2(1-\lambda_m T_m)}
\]  
(5)
\[
W_{NR}(2) = N_R m + m t_m + \frac{\lambda_m T_m^2}{3(1-\lambda_m T_m)}
\]  
(6)
4.2 Queueing Time of MSU-PR

Queueing time of MSU-PR is decomposed into the following components.
- \(a_0\): The time interval that begins at the arrival of MSU-PR to send buffer and ends at the beginning of its first transmission (Queueing time of MSU-NR).
- \(a_i\): The sum of emission times for MSUs that arrive at the send buffer in the time interval \(a_{i-1}\).
- \(b_0\): The emission time for MSU-PR.
- \(b_i\): The sum of emission times for MSUs that arrive at the send buffer in the time interval \(b_{i-1}\).
- \(c_0\): The sum of emission times for MSUs that are retransmitted prior to MSU-PR for they have not yet been acknowledged.
- \(c_i\): The sum of emission times for MSUs that arrive at the send buffer in the time interval \(c_{i-1}\).

Let \(G(t)\) be the cumulative distribution function of the busy period of a simple M/G/1 queue with arrival rate \( \lambda_m \) and service time distribution \( B_m(t) \). Using this expression, the cumulative distribution function of the time \( \tau \) is written as,
\[
A(t) = \int_0^\infty \left( \sum_{i=0}^\infty \frac{(\lambda_m)^i}{i!} e^{-\lambda_m G(t-y)} \right) dW_{NR}(y)
\]  
(7)
where \( G^n(t) \) is the n-fold convolution of \( G(t) \).
Next let \( G^*(s) \) be the LST of \( G(t) \). Then, by taking the Laplace-Stieltjes transform of both sides of the equation (7), it is obtained that
\[
A^*(s) = W_{NR}^*(1 + \lambda_m (1-G^*(s)))
\]  
(8)
Similarly to the consideration mentioned above, the LST of cumulative distribution function of time component \( \tau \) is given by \( B_m(s+\lambda_m (1-G^*(s))) \). Furthermore the LST of cumulative distribution function of time component \( \tau \) is given by \( F^*(s+\lambda_m (1-G^*(s))) \), where \( F^*(s) \) denote the LST of cumulative distribution function of time component \( \lambda_m \) (Appendix).
Consequently, the cumulative distribution function of queueing time for MSU-PRs is give in an LST representation.
\[
W_{PR}(s) = W_{NR}^*(1 + \lambda_m (1-G^*(s)))
\]  
(9)
The mean and the second moment of queueing time are,
\[
W_{PR}(1) = N_R m + m t_m + \frac{\lambda_m T_m^2}{(1-\lambda_m T_m)} + \frac{\lambda_m T_m^2}{(1-\lambda_m T_m)^2}
\]  
(10)
\[
W_{PR}(2) = N_R m + m t_m + \frac{\lambda_m T_m^2}{(1-\lambda_m T_m)^3} + \frac{2(\lambda_m T_m^2)}{(1-\lambda_m T_m)^2}
\]  
(11)
where \( f(1) \) and \( f(2) \) denote the mean and second moment of distribution \( F(t) \).
4.3 Queueing Time of MSU-SR

Figure 2 depicts a typical sequence of transmissions and retransmissions of MSU-SRs, where MSUs are thought to be removed from the system just before its retransmission. We define time interval \( T \) as follows.
\[ T = t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 \]
where \( t_1 \) and \( t_7 \) denote the time interval of system.

4.3.1 Queueing Time without Retransmission Error

Message signal units that arrive in time interval \( T \) are considered to be MSU-SRs.

Time interval \( T \) can be decomposed by appropriate components.
\[ T = t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 \]
where \( t_1 \) and \( t_7 \) denote the time interval of system.
Fig. 2 A typical sequence of transmission and retransmissions for MSU-SRs.

Fig. 3 Queueing model for MSU-SR analysis.

Fig. 4 Model behavior.

v) The time interval for retransmission of MSU-SR \((t_e, t_f)\).

A simple queueing model is developed to analyze the queuing time of an MSU-SR. Consider the queueing model shown in Figure 3, where two kinds of signals are served by a single server. From the FCFS queue \(Q_1\), signals A are served to process 1. After the completion of process 1, they join FCFS queue \(Q_2\). Signals A continue to join queue \(Q_2\) until queue \(Q_1\) empties and the server become free. At this point signals A in queue \(Q_2\) are served process 2 and leave the system. Both queues \(Q_1\) and \(Q_2\) empty, signals B in queue \(Q_3\) are served process 3 and leave the system. By a new arrival of signal A, server switches from queue \(Q_2\) or \(Q_3\) to serve \(Q_1\) after the signal presently in service has been completed.

In this model, we assume that signal A arrives according to a Poisson process with rate \(\lambda_m\) and defines LSTs of service time distribution for processes 1 and 2 by \(B_m^*(s)\) and that for process 3 by \(W_{NR}(s)B_m^*(2s)F^*(s)\). From equation (9) it is shown that the service time for process 3 stands for the time components \(i)\), \(ii)\) and \(iii)\), while processes 1 and 2 stand for time components \(iv)\) and \(v)\) respectively.

In Figure 4, the behavior of this model is shown for the same arrival sequence as Figure 3. Signal A is thought to be removed just before the service of process 2. It is obvious from this example that queuing times of MSU-SR and signal A are in perfect agreement.

For the analysis of queuing times of MSU-SRs using the steady state behavior of this model, further we assume that the system works in a saturated condition. Therefore the arrival rate of signal B is

\[
\nu = \frac{1 - 2 \lambda_m T_m}{2 T_m + W_{NR}^1 + \nu}
\]  

Similarly to the consideration in section 4.2, the queuing time of signal A, that is the queuing time of an MSU-SR, is decomposed into the following components:

- \(\alpha\) = The remaining time of service at the arrival of signal A.
- \(\beta\) = The sum of service times of process 1 and process 2 for all signals A in queues \(Q_1\) and \(Q_2\).
- \(\sigma_1\) = The sum of service times of process 1 for signals A that arrive in queue \(Q_1\) in the time interval \(\alpha\) and \(\beta\).
- \(\sigma_1\) = The sum of service times of process 1 for signals A that arrive in queue \(Q_1\) in the time interval \(\alpha\) and \(\beta\).
- \(\delta_1\) = The sum of service times of process 1 for signals A that arrive in queue \(Q_1\) in the time interval \(\alpha\) and \(\beta\).

The sum of times \(\alpha\) and \(\beta\) can be determined by using the theory of non-preemptive priority \(M_1,M_2/G_1,G_2/1\) queue shown in Figure 5. In this model, higher and lower priority signals arrive in queues according to a Poisson process with arrival rate \(\lambda_m\) and \(\nu\) respectively. The service time for a higher priority signal is equivalent to the sum of service times of process 1 and process 2, and that for a lower priority signal is equivalent to the service time of process 3.

The waiting time for a higher priority signal in this model is equal to the sum of times \(\alpha\) and \(\beta\). Consequently, LST of the cumulative distribu-
tion function of the time \( a+S \) is

\[
W_A^*(s) = \frac{\nu [1-Y^*(s)]}{s - \lambda_m [1-z^*(s)]}
\]

where

\[
Y^*(s) = W_{NA}^*(s)B_m^*(2s)F^*(s)
\]

\[
Z^*(s) = B_m^*(2s)
\]

Using LST of busy period distribution \( G^*(s) \), the LST of cumulative distribution function of time \( a+S^{+1} \) is

\[
W_A(s) = \frac{-s}{1-\lambda_m}
\]

(13)

(14)

(15)

where \( \nu \) and \( \lambda_m \) are defined in equations (12) and (15).

Therefore the LST of the cumulative distribution function of queueing time for MSU-SR without retransmission error is,

\[
W_{SR}^*(s) = \int_0^s \left( 1-P_u \right) W_A^*(x)B_m^*(2x)H^*(x) dx
\]

\[
x = s + \lambda_m [1-G^*(s)]
\]

(16)

4.3.2 Queueing Time with Retransmission Error

Once an MSU-SR is affected by a transmission error, the error-affected MSU-SR and its subsequent MSU-SRs will be discarded temporarily at the receiving side and correctly received by another retransmission. This can be interpreted that the error-affected MSU-SR continued to occupy the signalling link until the end of its retransmission. By this interpretation, the result on the model in Figure 3 is directly applicable to the estimation of the influence of retransmission error.

The emission time of error-affected MSU-SR is thought to be prolonged. The duration of this extension is the sum of its original emission time and the time required for a retransmission cycle. Since the error probability of MSU-SR is \( P_u \), the LST of the cumulative distribution function of extended emission time \( B_m^*(s) \) is

\[
B_m^*(s) = \left( 1-P_u \right) B_m^*(s) + P_u B_m^*(2s) H^*(s)
\]

(17)

where \( H^*(s) \) denotes the LST of cumulative distribution function of time \( a+S^{+1} \) is equivalent to that of time \( \frac{T_m}{2} \) in section 4.2.

Therefore the LST of the cumulative distribution function of queueing time for MSU-SR without retransmission error is,

\[
W_{SR}^*(s) = \int_0^s \left( 1-P_u \right) W_A^*(x)B_m^*(2x)H^*(x) dx
\]

\[
x = s + \lambda_m [1-G^*(s)]
\]

(16')

The mean and second moment of queueing time are

\[
W_{SR}(s) = \left( 1-P_u \right) W_A^*(s)B_m^*(s) + P_u W_A^*(2s) H^*(s)
\]

(18)

\[
x = s + \lambda_m [1-G^*(s)]
\]

The mean and second moment of queueing time are

\[
W_{SR}(s) = \left( 1-P_u \right) W_A^*(s)B_m^*(s) + P_u W_A^*(2s) H^*(s)
\]

(18')

\[
x = s + \lambda_m [1-G^*(s)]
\]

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\]

(18'')

\[
x = s + \lambda_m [1-G^*(s)]
\]

The mean and second moment of queueing time are

\[
W_{SR}(s) = \left( 1-P_u \right) W_A^*(s)B_m^*(s) + P_u W_A^*(2s) H^*(s)
\]

(18''')

\[
x = s + \lambda_m [1-G^*(s)]
\]
analysis, the mean length of time interval T in error free retransmission can be easily determined.

\[ E(T) = \frac{\frac{W_R^{(1/2)}}{T_m} + 2 T_m + \frac{T_m T_L}{2}}{1 - 2 \frac{T_m}{T_m}} \]  \hspace{1cm} (29)

We ignore that more than one retransmission errors take place in time interval T. Then, the probability of a retransmission error in time interval T is given by \( P_u a M E[T] \). The mean extension of time interval T by a retransmission error is

\[ E(T_{SR}) = \frac{T_m + \frac{T_m T_L}{2}}{1 - 2 \frac{T_m}{T_m}} \]  \hspace{1cm} (30)

Consequently, the mean MSU-SR number for an MSU-PR is given by

\[ N = \frac{W_R^{(1/2)} + 2 T_m + \frac{T_m T_L}{2}}{1 - 2 \frac{T_m}{T_m}} (1 + P_u a M E T) \]  \hspace{1cm} (31)

From the equation (31), the proportion of MSU-SR is given by

\[ P_{SR} = \frac{P_u N}{T} \]  \hspace{1cm} (32)

Finally, the LST of the cumulative distribution function of total queueing time is,

\[ W_T(s) = (1 - P_u - P_{SR}) W_R^{(s)} + P_u W_R^{(s)} + P_{SR} W_{SR}^{(s)} \]  \hspace{1cm} (33)

The mean and variance of the queueing time can be easily obtained from equation (33).

\[ Q = (1 - P_u - P_{SR}) W_R^{(1)} + P_u W_R^{(1)} + P_{SR} W_{SR}^{(1)} \]  \hspace{1cm} (34)

\[ \sigma_q^2 = (1 - P_u - P_{SR}) W_R^{(2)} + P_u W_R^{(2)} + P_{SR} W_{SR}^{(2)} - Q^2 \]  \hspace{1cm} (35)

5. COMPARISON BETWEEN THEORETICAL DEVIATION AND SIMULATION RESULTS

In order to certify the assumptions in section 4, a simulation study on queueing delay has been performed. The actual protocol including overload procedure was implemented in full detail within this simulation study.

It is not difficult to make a theoretical calculation of the mean and variance of the queueing time. However for the theoretical calculation of queueing delay distribution, there appears no way of obtaining the explicit representation of the Laplace inversion for equation (33). The technique of numerical inversion of Laplace transform was employed [8].

We give typical calculation and simulation results. The proportion of MSU length is obtained in Table 1 [9]. Signalling link speed is 64 kbits/sec and FISU length is 48 bits.

Figures 6 and 7 show the mean and standard deviation of the queueing time and queueing delay distributions. In Figure 7 \( WC(t) \) denotes the probability that queueing time exceeds t.

From a comparison with simulation results, it is confirmed that all assumptions in this paper are acceptable and the results from our model are sufficiently accurate.

Table 1 Proportion of messages.

<table>
<thead>
<tr>
<th>Length (bits)</th>
<th>176</th>
<th>152</th>
<th>128</th>
<th>112</th>
<th>104</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent</td>
<td>7.14</td>
<td>7.94</td>
<td>7.14</td>
<td>31.75</td>
<td>46.03</td>
</tr>
</tbody>
</table>

Fig. 6 Mean and standard deviation of queueing time vs. signalling link load.

Fig. 7 Queueing delay distribution.
6. CONCLUSION

A queueing model for the PCR method of CCITT Signalling System No. 7 has been developed. The formulae for the mean and the variance of queueing time taking into account the error recovery time are given in an explicit form. Queueing time distribution is also given in terms of Laplace-Stieltjes transform.

The comparison of analytic results with results obtained from computer simulation indicates that all essential procedures determining the queueing time performance are adequately modeled and our formulae are sufficiently accurate.

APPENDIX

We define the time when the MSU-PR has transmitted and its copy has been inserted to the retransmission buffer by t. The number of MSUs in the retransmission buffer at time t is equal to the number of MSUs transmitted in the time interval [t - TL, t]. It is assumed that the probability distribution of the number of MSUs in the retransmission buffer is Poisson with mean \( \lambda_m T_L \). This assumption is not valid in a strict sense, but gives an acceptable working basis, since the loop propagation delay TL is long.

Therefore, the probability that k MSUs exists in the retransmission buffer is,

\[
q_k = \frac{(\lambda_m T_L)^k}{k!} e^{-\lambda_m T_L} \quad (A-1)
\]

The cumulative distribution function of the time for retransmitting all those MSUs is,

\[
H(t) = \sum_{k=0}^{\infty} q_k \frac{P^{(k)}(t)}{\lambda_m}
\]

LST of the function \( H(t) \) is,

\[
H^*(s) = \exp[-\lambda_m T_L (1 - B^*_m(s))]
\]

The conditioned probability that j MSUs out of k MSUs in the retransmission buffer are transmitted prior to MSU-PR is

\[
p_{jk} = \left\{ \begin{array}{cl}
\frac{1}{k+1} & 0 \leq j \leq k \\
0 & \text{otherwise}
\end{array} \right. \quad (A-4)
\]

Therefore, the total probability that j MSUs are transmitted prior to MSU-PR is

\[
p_j = \sum_{k=j}^{\infty} \frac{1}{k+1} q_k
\]

Since the distribution of the time required for j MSUs retransmission is given by \( B^{(j)}_m(t) \), the distribution of time \( C^*_0 \) and its LST is

\[
F^{(j)}(t) = \sum_{j=0}^{\infty} \frac{(\lambda_m T_L)^k}{k! (k+1)!} e^{-\lambda_m T_L} B^{(j)}_m(t)
\]

\[
F^*(s) = \frac{1}{1 - e^{-\lambda_m T_L (1 - B^*_m(s))}}
\]

The mean, second moment and third moment of distribution \( H(t) \) and \( F(t) \) are

\[
k^{(0)} = \lambda_m T_L T_m
\]

\[
k^{(1)} = \lambda_m T_L T_m^2 (C_2 + \lambda_m T_L)
\]

\[
k^{(2)} = \lambda_m T_L T_m^3 (C_3 + 3 \lambda_m T_L^2)
\]

\[
k^{(3)} = \lambda_m T_L T_m^4 / 2
\]

\[
k^{(4)} = \lambda_m T_L T_m^2 (C_2 / 2 + \lambda_m T_L / 3)
\]

\[
k^{(5)} = \lambda_m T_L T_m^3 / 4
\]

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REFERENCES


[9] CCITT Recommendation, Q.725, Sec. 5.