

ERIK BROCKMEYER AND THE TELETRAFFIC THEORY

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Erik Brockmeyer (1901-1975) was employed by the Copenhagen Telephone Company (KTAS) from 1923 to 1971, and all along he worked with problems of teletraffic. (1) gives an account of his biography, publications, and contributions to the teletraffic theory.

In KTAS Brockmeyer was contemporary with such pioneers of teletraffic theory as F. Johansen, A.K. Erlang, P.V. Christensen, K. Moe, Arne Jensen, and E. Bjørn Christensen. His first reports deal with practical problems and traffic measuring problems. Later on he published more theoretical works, eg. an account of the application of probability theory to telephony, and the works mentioned below.

He was a member of the organizing committee of the First International Teletraffic Congress (ITC 1) in Copenhagen 1955. On this occasion he presented a review of traffic measuring methods in KTAS.

#### OVERFLOW THEORY

In 1954 Brockmeyer gave a solution to a key problem in teletraffic theory (2). Pure Chance Traffic Type 1 is offered to a finite trunk group and overflows to a second finite trunk group. Under the assumption of statistical equilibrium Brockmeyer gave explicit expressions for the elementary probabilities  $p(j,k)$  of  $j$  trunks of the primary group and  $k$  trunks of the secondary group being occupied simultaneously. Furthermore he determined the overflow distribution, ie. the probability  $\omega(k)$  that there will be  $k$  occupied trunks in the overflow group. Brockmeyer also presented algorithms for numerical calculations.

In 1937 L. Kosten gave the solution for an infinite overflow group. In 1956 J. Riordan derived the factorial moments of Kosten's model, and in 1976 R. Schehrer did the same for Brockmeyer's model. B. Wallström, C.E.M. Pearce and others have at previous Teletraffic Congresses generalized the Brockmeyer model in several ways.

#### ERLANG'S IDEAL GRADING

In 1948 the Copenhagen Telephone Company on the initiative of Arne Jensen published "The Life and Works of A.K. Erlang" (3). In this book Brockmeyer reviewed the mathematical works of Erlang. He also gave the theoretical basis for some of Erlang's works, i.a. Erlang's ideal grading (EIG), also referred to as Erlang's interconnection formula.

The assumptions for EIG are as follows. The offered traffic is Pure Chance Traffic Type 1. The system is in statistical equilibrium, and the

lost calls are cleared. The number of trunks is  $n$  and the availability is  $k$ . The total traffic is equally divided among inlet groups, and there are as many inlet groups as there are ways of hunting  $k$  trunks out of the total  $n$ .

Practical applications of EIG has till now been very limited. This is partly due to a limited interpretation and understanding of the model, partly due to numerical problems, which are eliminated by the advent of computers and calculators.

EIG has several advantages over other grading formulas. The blocking probability is insensitive to the holding time distribution, and it gives the lower limit of the blocking probability for gradings with random hunting. Brockmeyer noticed that small gradings with sequential hunting may give smaller blocking. By intelligent hunting methods the minimal blocking is given by Erlang's multi dimensional loss formula. EIG is easily generalized to Engset traffic and to several traffic streams with individual availability.

Palm-Jacobsen formula is based on the same assumptions as EIG as concerns the number of inlet groups, but it does not take account of the actual state probabilities. Therefore it is only applicable for small blocking probabilities. EIG is also correct at high traffic levels, and grading calculations should be based on EIG, or modifications of this.

#### OTHER WORKS AND TOPICS OF DISCUSSION

Brockmeyer published several works on traffic measurements. For the demi-automatic exchanges of that time he introduced extreme value engineering based on the recording of the last choice traffic. He also gave some approximations for Erlang's B-formula.

The above-mentioned works of Brockmeyer will be the basis for a discussion on the state of the art in grading and overflow theory. Additional material will be distributed at the special interest group meeting.

#### REFERENCES

- (1) J. Arhning and V.B. Iversen, "Erik Brockmeyer and the Teletraffic Theory", Teleteknik, English edition, 1985. To appear.
- (2) E. Brockmeyer, "The Simple Overflow Problem in the Theory of Telephone Traffic", Teleteknik, Vol. 5, pp. 361-374, 1954. In Danish. English edition published by the Copenhagen Telephone Company, April 1955. 15 pp.
- (3) E. Brockmeyer, H.L. Halstrøm and Arne Jensen, "The Life and Works of A.K. Erlang. Copenhagen 1948. 277 pp.