TRAFFIC FORECASTING WITH MINIMUM DATA

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ABSTRACT

This paper presents a method for estimating volumes and direction of different types of traffic for short or medium term forecasting at the various aggregation levels.

The model may be applied with either rough or detailed data. It is particularly well adapted to networks undergoing a rapid development since it takes into account modifications to the number of lines by differentiating subscriber categories whose traffic evolves in a quite different manner, in volume as well as in direction.

Time series, while not necessary to apply the model, are used to determine values of its different parameters.

1. INTRODUCTION

Traffic forecasting constitutes an essential stage in the management and planning of telecommunications networks.

Good forecasting is always desired by the planner but he does not always have sufficient knowledge of past and present network conditions to accurately estimate future traffic volumes at a given horizon year. This is the case in networks undergoing rapid growth or offering new services whose characteristics are imperfectly known.

As it is expensive to observe all the lines of a subscriber network and to measure all traffic flows, forecasting must be based on mathematical models using statistical values.

Forecasts are realized here by a model which calculates traffic per line category in conjunction with a traffic growth model.

Traffic forecasting processes have to be consistent at every hierarchical and geographical level of the network.

In France we distinguish the main exchange, the local area containing one or several main exchanges and secondary and primary transit zones. Compatible forecasts must be determined for the whole network.

The model has been tested for total and long distance traffic forecasts (cf. 5.2.).

2. TRAFFIC REPRESENTATION AND MINIMUM DATA

In this paper $T$ denotes a total traffic.

Traffic measurements are made on subscriber groups or on trunk groups. Integer $j \in J_1$ will denote a type of traffic evaluated on subscriber groups, $j \in J_2$ a type of traffic evaluated on trunk groups. So, the same type of traffic will be denoted by two different integers according to the place of measurement.

Relationships between types of traffic may be represented by a directed tree. Let us consider traffic on subscriber groups. We associate $j=0$ to the total incoming and outgoing traffic, a non-zero even integer to outgoing traffics, an odd integer to incoming traffics. Figure 1 shows the corresponding directed traffic tree.

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The model has been tested for total and long distance traffic forecasts (cf. 5.2.).
There is also an associated set of intervals, each one defined by a lower bound (LB) and an upper bound (UB) on $t_{ij}$. If $t_{ij}$ is given in erlangs,

$$0 < LB_j(i) < t_{ij} < UB_j(i) < 1$$

These intervals may, or may not, overlap.

Categories may be defined, for instance, such that for $T_0$, the intervals associated with $t_{ij}$ constitute a partition of $[0,1]$. Another definition will allow lines dedicated exclusively to incoming traffic to constitute a category, the same applying to lines dedicated exclusively to outgoing traffic, and the intervals of these two categories may coincide.

The minimum input data required concerns:

- the number of categories to be considered and, for each one, the set of bounds on at least one type of traffic $j$,

- present and forecast number of lines per exchange and average percentage of lines for each category at studied area level,

- for every exchange, the estimated traffic volume $T_j$ per studied type $j$. This estimation may be deduced from the number of subscribers using values obtained for other reference exchanges.

Any other information only contributes to refine forecasts (for instance, traffic distribution per type, percentage of subscribers in each category at exchange level, ...).

3. NUMBER OF LINES VS MEAN TRAFFIC

Experience shows that mean telephone traffic per line diminishes when the number of lines increases. Figures 2 and 3 compare number of lines and mean traffic per line in France in the last decade.

At present the number of lines is increasing slowly and mean telephone traffic per line is expected to level out in a few years time at a value perhaps a little higher than today's value.

The same phenomenon applies to traffic generated by new services. As far as the recently offered videotex service (teletel and electronic directory) is concerned, the number of terminals increased in 1984 by 300%, but the mean traffic per terminal decreased by 23%.

Now, if we consider two categories, residential and business subscribers, mean traffic per subscriber may be low for both categories in rural areas, while in medium size towns and in the outskirts of big cities, it may be much higher for the second category. In big cities, the difference between mean traffics per subscriber may become smaller for both categories, as shown in figure 4.

Finer categories may be considered, according to socio-economic criteria for instance.

4. TRAFFIC DECOMPOSITION

4.1. General Principle

Let $T_j$ be the total $j$ traffic.

For every exchange and each category $i$, traffic $T_j$ will be split into a set of analytical components $t_{ij}$. These components will be determined by resolution of the following system of equations $S$, where $N$ represents the number of subscribers of the urban area where the exchange is located. Constraints on $t_{ij}$ according to the definition of categories (cf. 2.) are given by eq. (3).

$$T_j = \sum_{i=1}^{NC} n_i t_{ij}$$

$$t_{ij} / t_{i-1,j} = F_{ij}(N), i>1$$

$$LB_j(i) < t_{ij} < UB_j(i)$$

Ratios $F_{ij}(N)$ are previously determined as it is explained in the following paragraph.

4.2. Estimation of Traffic Ratios between Categories
The values of $F_{ij}$, eq. (2), are estimated first at national level, then at studied area level and exchange level.

At national level, the values of $F_{ij}$ may be deduced from traffic values defining categories or estimated from data corresponding to different zones. In all cases, national values of $F_{ij}$ become implicit values for other levels.

At studied area level, implicit values are corrected if enough values of traffic per category are provided, while at exchange level these values are ratified or modified to satisfy constraints introduced by the definition of categories.

In the absence of traffic measurements and with no precise ideas about the ratios, we take $F_{ij}$ equal to a constant value:

$$F_{ij}(N) = a_{ij} N^{b_{ij}}$$

With more precise ideas or sufficient measurements, $F_{ij}$ may be given by a monotonic growing function like

$$F_{ij}(N) = a_{ij} \exp\left(-\frac{\log N - \log M}{\lambda}\right) + E_{ij}$$

where $M$ represents the number of subscribers of a medium town of the country or the considered zone, and $E_{ij}$ the lower bound of values $t_{ij}/t_{i-1,j}$ corresponding to at least 3 urban areas of large, medium and small size, respectively.

$F_{ij}(N)$ varies according to socioeconomic population structures, the choice of categories, call charging structures and so on.

The final form is determined from observation of traffic per category in different zones. The same applies for parameters $a_{ij}$, $b_{ij}$ and $E_{ij}$. At regional level they are calculated by the least squares method and the form giving the best correlation coefficient is chosen.

4.3. Resolution of System $S$

a) Generalities

The system $S$ is solved for the first $j=k$ such that $T_k$ is known (cf. section b); values $t_{ik}$ and $F_{ik}$ are determined for every category $i$ considered.

The analytical components $t_{ij}$, the ratios $F_{ij}$ and the set of bounds $LB_{ij}(1)$ and $UB_{ij}(1)$ corresponding to each descendant $T_j$ of $T_k$ are then evaluated. This evaluation needs the proportion $P_{ijk}$ of traffic of type $k$ which is also of type $j$, per category $i$.

If $T_j$ is a descendant of $T_k$, $P_{ijk}=t_{ij}/t_{ik}$. Otherwise, $P_{ijk}=0$.

$P_{ijk}$ may be obtained from intermediate relations; we have, for instance, $P_{i,4,0}=P_{i,4,2} \times P_{i,2,0}$.

The a priori value of $P_{ijk}$ is obtained from measurements and knowledge of traffic volumes. As an example, let the total outgoing and incoming traffic, $T_0$, be given. Then,

- $P_{i,2,0} = 0$ and $P_{i,1,0} = 1$ for incoming dedicated lines; $P_{i,2,0} = 1$ and $P_{i,1,0} = 0.5$ for lines carrying outgoing traffic.
- If $i=1$ corresponds to residential subscribers and $i=2$ correspond to business subscribers, $P_{i,2,0} = 0.5$ and so on.

With sufficient measurements, $P_{i,2,0}$ is the corresponding average value.

$P_{ijk}$ may also be calculated in a more sophisticated manner, taking into account the size of the local area expressed by its number of subscribers, $N_A$, and the number of subscribers in the country, $N_N$. For instance, if $T_k$ represents the local outgoing traffic for the considered area and $T_2$ the first traffic known; we have

$$P_{i,1,2} = \frac{N_A}{N_N}, \quad 0 < P_{i,1,0} < 0.5$$

Parameter $g_i$ is estimated by the least squares method from provided values corresponding at least to two local areas of different size.

If $T_k$ is a descendant of $T_k$, $F_{ij}$ may be obtained from the previously calculated values of $F_{ik}$ and $P_{ijk}$:

$$F_{ij} = \frac{F_{ik} P_{i-1,j,k}}{F_{ik} P_{i-1,j,k} + F_{ik} P_{i-1,j,k}}$$

The set of traffic bounds per category are such that, if $T_j$ is a descendant of $T_k$

$$LB_{ij}(1) = LB_{ik}(1) \times P_{ijk}$$

$$UB_{ij}(1) = UB_{ik}(1) \times P_{ijk}$$

b) Calculations at Exchange Level

Let $T_k$ be the first known traffic.

At exchange level, the analytical components $t_{ik}$ of $T_k$ are determined by the iterative resolution of system $S$, according to the following algorithm.

Calculations are performed for the initial and preceding years.

The superscript $m$ on a variable indicates its value at iteration $m$. Iterations are limited to a previously given number, $maxm$.

Step 1. $m := 0$. For $i = 2$ to $N$, let $F_{ik}$ be equal to $F_{ik}(N_Y)$, the implicit value of $F_{ij}$, where $N_Y$ is the number of lines of the area where the exchange is located, at year $Y$. 

3.3B-1-3
Step 2. \( m := m+1. \) For \( i=1 \) to \( N \) calculate mean traffic values \( t_{ik} \) and ratios \( R_{ik} \) as follows:

\[
\begin{align*}
R_{1,k} &= 1, \quad G_{1,k} = 1, \\
for \ i \neq 1, \quad G_{i,k} &= \prod_{c=2}^{m} R_{ck}
\end{align*}
\]

\( m := m+1. \) For \( i=1 \) to \( m-1 \) calculate traffic values \( t_{ik} \) and ratios as follows:

\[
egin{align*}
G_{i,k} &= \frac{E_{T_{ik}}}{\sum_{i=1}^{N} G_{i,k}}, \\
if \ m < \text{maxm}, \quad t_{ik} &= \min(UB_{k}(i), \max(LB_{k}(i), E \times G_{ik})) \\
if \ m = \text{maxm}, \quad R_{ik} &= t_{ik} / t_{i-1,k}
\end{align*}
\]

Step 3. If \( t_{ik} \) equals one of its bounds, for at least one category \( i \), and if \( m < \text{maxm} \) go to Step 2. Should it be otherwise, set \( m1 = m \) and if, and only if, \( m1 = \text{maxm} \) redefine bounds of categories by

\[
egin{align*}
LB_{k}(i) &= \min \{ LB_{k}(i), t_{ik} \} \\
UB_{k}(i) &= \max \{ UB_{k}(i), t_{ik} \}
\end{align*}
\]

Step 4. Determine definitive values of mean traffics, ratios and bounds, for \( i=1 \) to \( N \), by

\[
egin{align*}
t_{ik} &= t_{ik}, \quad R_{ik} = R_{ik}, \\
LB_{k}(i) &= LB_{k}(i), \quad UB_{k}(i) = UB_{k}(i)
\end{align*}
\]

Values for \( t_{ij} \), \( F_{ij} \) and a possible modification of \( LB_{j}(i) \) and \( UB_{j}(i) \) may be obtained from the corresponding values for \( T_{k} \), if \( T_{k} \) is a descendant of \( T_{k} \) using equations (4), (5) and (6).

5. TRAFFIC GROWTH

5.1. A Model of Mean Traffic Growth per Category

Elementary traffic values \( t_{ij} \) are extrapolated by a growth model using historical values and the increase of subscribers within the studied area. For new exchanges, values from old ones given as references are taken into account.

A growth factor \( G_{ijp} \) of mean traffic per subscriber of category \( i \) for period \( p \) going from initial year \( Y_{0} \) to target year \( Y_{f} \) is given by

\[
G_{ijp} = E_{ijp} \cdot K_{ij} \cdot (1 + a_{ij} \cdot \theta_{p})^{p}
\]

where \( E_{ijp} \) is a coefficient taking into account call charge modifications, publicity campaigns, the introduction of new services, and so on, occurring during the whole period \( p \); \( K_{ij} \) denotes annual trends of traffic growth; \( \theta_{p} \) is the growth rate of the number of subscribers in the studied area and period \( p \); \( a_{ij} \) is such that \( 0 < a_{ij} < 0.5 \); \( p \) is expressed in years or fraction of year.

Growth factors \( G_{ijp} \) are calculated at national level from measurements of traffic on a representative sample. If measurements, or reliable samples, are not available, the growth rate of "financial" traffic (number of basic charge units) per category - if it is known - may be used as the general traffic growth. If sufficient data are available for the preceding years \( G_{ijp} \) is calculated by exponential smoothing. \( K_{ij} \) and \( a_{ij} \) may be deduced from \( G_{ijp} \).

At regional and exchange levels, \( G_{ijp} \) may be fitted to historical traffic growth if the traffic segmentations are identical for past and present years. In particular, trends \( K_{ij} \) deduced from traffic observation at exchange level have priority.

The confrontation of parameters at different levels is quite useful. Differences must be explained and implicit values may be improved.

5.2. Mean Traffic Extrapolation

Let \( t_{ij}s \) denote the mean traffic of type \( j \) per subscriber of category \( i \) calculated for exchange \( s \) at the initial year \( Y_{0} \) and \( G_{ijp} \) its associated growth factor for the period \( p \) going from the initial year to the target year \( Y_{f} \).

The corresponding mean traffic per subscriber at year \( Y_{f} \) is then

\[
t_{ijf} = t_{ij} \times G_{ijp}
\]

Forecast mean traffics per subscriber are calculated for every exchange in service at year \( Y_{0} \), even if it will be replaced or its service area modified at year \( Y_{f} \).

5.3. Experimental Results

![Figure 5. Evolution of mean traffic per line in the last decade in France (not including the region of Paris)](image-url)
This model has been tested in France to forecast total and long distance traffic. Calculations have been made for the last ten years, for every month and every year, taking into account seasonal effects. Figures 5 and 6 compare observed values and values obtained by the model.

6. TRAFFIC VOLUME FORECASTING

6.1. At Exchange Level

Let \( n_{iS} \) denote the expected number of subscribers of category \( i \), for exchange \( s \), at the future year \( Y_f \), supposing its service area to be non modified during the whole period \( p \).

Forecast volume of traffic of type \( j \) for exchange \( s \) is then

\[
T_{js} = \sum_{i=1}^{NC} [t_{ijS} \times n_{iS} + \sum_{b \in B} (t_{ijb} \Delta n_{iB})]
\]

where \( B \) is the set of exchanges \( b \) whose service area is modified by the transfer of \( \Delta n_{iB} \) of its subscribers to exchange \( s \).

Forecast traffic for new exchanges \( n \), in service at year \( Y_f \) is calculated by eq. (8) with \( t_{j1n} = 0 \) and using \( t_{ijb} \), the mean forecast traffic per subscriber for exchange \( b \), whose subscribers behaviour serves as a reference for the subscribers to be connected to exchange \( n \), represented here by \( \Delta n_{iB} \).

If \( \Delta n_{iB} \) is not known, it will be assumed that \( \Delta n_{iB} = \Delta n_{iB} \times n_{iB}/n_B \), where \( n_B \) represents the total number of subscribers of exchange \( b \), \( \Delta n_{iB} \) the total number of transferred subscribers.

Forecast traffic for exchange \( b \), from which a part, \( \Delta n_{iB} \), or all of its forecast \( n_{iB} \) subscribers will be transferred to another exchange is then

\[
T_{jb} = \sum_{i=1}^{NC} t_{ijb}(n_{iB} - \Delta n_{iB})
\]

6.2. At Higher Levels

Forecast type \( j \) traffic at area level is the sum of forecast total traffic volume of all exchanges \( s \) located within the area:

\[
T_j = \sum_s T_{js}
\]

At transit zones level, forecast volume of traffic may be calculated in a similar manner by addition of forecast volumes at the immediate lower level. However, these values have to be consistent with forecast traffic volume obtained by traffic apportionment at higher levels.

6.3. On Trunk Groups

On trunk groups, growth of some types of traffic may be slightly less than growth of the same type of traffic at the exchange level.

Let \( j \in J_1 \) and \( j' \in J_2 \) correspond to the same type of traffic measured on subscriber groups and on trunk groups respectively.

Then, \( T_{j'} = T_j^e \)

The value of depends on the representative traffic value on trunk groups calculation method.

The representative value which is generally an extreme value, tends towards the mean value as traffic grows.
7. TRAFFIC BY DIRECTION FORECASTING

7.1. Matrices Construction

The key to traffic distribution is given by an initial matrix, drawn up from measurements or using gravity or community of interests models. In the latter cases the matrix may be improved by taking into account economic or administrative relations.

The gravity model determines total traffic between an origin \( o \) and a destination \( d \) by

\[
T(o,d) = \frac{C T(o) T(d) e^{-D(o,d)/\gamma}}{e^{-\gamma} + 1}, \quad 0 < k, \alpha, \beta \leq 1
\]

\( T(o,d) \) is a function of the total traffic of ends \( T(o) \) and \( T(d) \) and the distance between them, \( D(o,d) \). Parameter \( C \) describes economic or administrative relations between the cities where exchanges are located. \( C, \alpha \) and \( \beta \) are estimated by statistical regression.

When enough measurements are provided a method based on factorial analysis \([1]\), allows us to complete missing data by an iterative process described by the following algorithm (where the number of iterations is limited to \( Z \)).

Let \( \{t(a,b)\} \) be the matrix of traffic flows generated for all exchanges \( a,b \) of the studied area, at iteration \( z \).

1. \( z := 0 \). Initialize missing traffics at the values given by the gravity model, for instance.

2. \( z := z+1 \). Perform a factorial analysis of matrices \( \{t(a,b)_{z-1}\} \) and determine the minimum number of \( x(z) \) factorial axes required to reconstitute the matrix structure.

3. Replace missing traffics by the projections of their previous values on the hyperplane determined by the \( x(z) \) axes.

Step 3. If the new values of missing traffics differ from their previous values by more than a given and if \( z < Z \), go to Step 2.

Otherwise, these last values of missing traffics will not be further modified.

The matrix so completed, as matrices obtained in another way, should be modified using Kruithof's method \([2]\) to fit previously determined global outgoing or incoming traffic values.

As for traffic by volume, traffic by direction forecasts are obtained at the various aggregation levels. Thus, elementary traffic flows matrices (between main exchanges), regional traffic matrices (between local areas or transit zones) and national traffic matrices (between primary transit zones) have to be drawn up.

Traffic by direction is, in general, well known at exchanges level within local areas, which make easier matrices construction.

Within a transit zone, traffic flows between neighboring local areas may be determined from measurements, but for distant local areas measurements are not always made. Traffic on transit

trunk groups is known in volume but is only given by direction for some particular flows (on direct transversal trunk groups). An example of this situation is given in figure 7.

![Figure 7. Traffic flows between local areas](image)

So, at regional levels, traffic by direction is not always known. Then, a model (gravity or another) have to be used to build or complete the matrix from any kind of available data.

7.2. Matrices Forecasting

To ensure consistency between matrices drawn up and projected at every level, two methods are proposed.

A. Ascending method:

- From outgoing and incoming traffics at every exchange derive initial matrices between main exchanges, between local areas, between secondary and primary transit zones.

- Determine traffic evolution at every level.

- Project initial matrices to obtain forecast matrices satisfying traffic evolution at every level.

B. Descending method:

- Forecast total traffic (long distance, international, etc) at national level.

- Split total traffic at every level in such a way that, if \( T_{j,C} \) denotes total type \( j \) traffic at a given level, for the country \( C \), primary transit zones (or main regions) \( R \), secondary transit zones (or sub-regions) \( D \), local areas \( A \) and exchanges \( s \),

\[
T_{j,C} = \sum_{R} T_{j,R}, \quad T_{j,R} = \sum_{D} T_{j,D}
\]

\[
T_{j,D} = \sum_{A} T_{j,A}, \quad T_{j,A} = \sum_{s} T_{j,s}
\]

- Determine traffic flows and matrices at every level.
To be consistent, traffic matrices are thus first projected to the horizon period and then modified to match forecast traffic volumes obtained at a higher level. In fact, each traffic volume becomes a constraint for the matrix at a lower level to be added to the constraints that might be already imposed to apply the projection method.

Introduction of these constraints avoids deviations caused by a blind use of any projection such as kruithof’s method.

8. CONCLUSION

Traffic forecasts are realized by the model in a quite satisfactory manner when good measurements are available and subscriber lines are finely segmented. Nevertheless, and this constitutes its interest, it is easily adaptable and allows us to obtain good results when measurements have not been made or are not reliable at studied area level.

Consideration of types of traffic per category allows a better understanding of the network evolution and better forecasts may be expected.

The proposed traffic matrix projection method based on statistical values provides a solution when measurements are missing.

The main characteristic of the method is its modularity, so that different traffic forecasting methods or matrix projection methods may be integrated according to the quantity and quality of available data.

REFERENCES
