ON DELAY AND LOSS IN A SWITCHING SYSTEM FOR VOICE AND DATA WITH INTERNAL OVERFLOW

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ABSTRACT
This paper deals with two types of combined delay loss systems which are suitable for overload protection in switching systems with two different types of offered traffic. Besides the protection of priority calls against overload of ordinary calls by means of cut-off priorities, an additional protection of ordinary calls against overload of priority calls is achieved by means of internal overflow. An important application of such systems is, e.g., possible in integrated switching systems for voice and data. Therefore, besides a performance analysis of systems with the same mean holding time for all calls, a method for systems with different mean holding times of ordinary and priority calls is also presented. Exact values of state probabilities and characteristic traffic values as well as waiting time distributions are calculated. Results are presented in diagrams.

1. INTRODUCTION
In many switching systems with two types of offered traffic, A1 and A2, only the traffic A2 is usually concerned by overload. It is well known that in this case so called cut-off priorities are suitable for the protection of traffic A1 against overload of the traffic A2 [1, 2, 5, 6, 7, 8, 9, 10]. If, however, an overload of the traffic A1 can also occur, the delay and loss probabilities for the traffic A2 can increase to a very high level in such systems.

Therefore, this paper deals with systems which additionally are suitable for a certain protection of the traffic A2 in situations in which A1 is overloaded. The models considered operate as combinations of a loss system and a delay-loss system. The protection of traffic A1 against overload of A2 is achieved by means of a certain kind of trunk reservation with cut-off priorities, and the protection of traffic A2 against overload of A1 with the aid of internal overflow.

After a detailed description of the modeling of the considered systems in section 2, the characteristic traffic values are calculated exactly in section 3, and the distribution of waiting times in section 4, for the case of equal holding times of the traffic parts A1 and A2. Because of the importance for the application to combined systems for voice and data, the characteristic traffic values for systems with different mean holding times are investigated in section 5. A method for the exact calculation of the corresponding waiting time distribution is presented in section 6. Results are shown in several diagrams.

2. MODEL DESCRIPTION
In both systems considered here, two independent types of traffic A1 and A2 arrive at a group of n trunks of a switching system. From the n trunks, k are reserved for calls of the traffic A2. For the traffic A1, the remaining n-k trunks act as a full access group, operating as a loss system. For the traffic A2, both models operate as a delay-loss system with s waiting places. Waiting calls are served according to the discipline "first in first out" (FIFO). Calls of the traffic A2 can, however, only be switched via one of these n-k trunks if less than m (m<n-k) of these n-k trunks are busy. The two models presented here differ with respect to the service of arriving calls of the traffic A2.

Fig. 1 Model No. 1

Fig. 2 Model No. 2
arriving calls of the traffic A2 are first offered to the group of k trunks which is hunted in a full access mode. If all k trunks are busy, arriving calls of A2 are overflowing to the group of n-k trunks, provided that less than m trunks are busy in this group.

In model No. 2, as shown schematically in fig. 2, calls of A2 are switched via one of the n-k trunks if less than m trunks are busy in this group. If m or more trunks are busy, arriving calls of A2 are overflowing to the group of k trunks which is hunted in a full access mode. Calls overflowing, when all k trunks are busy, start waiting if less than s waiting places are occupied. If all k trunks are busy and all s waiting places are occupied, overflowing calls of the traffic A2 are lost in analogy to model No. 1.

3. CHARACTERISTIC TRAFFIC VALUES FOR SYSTEMS WITH UNIFORM MEAN HOLDING TIMES

3.1. Preconditions

It is assumed that the traffic parts A1 and A2 are of Poisson type with the same mean holding time h and with the mean arrival rates A1 and A2. The total offered traffic be denoted by A and the mean terminating rate by ε, so that the following equations holds true:

\[ A1 = A1h \] (1)
\[ A2 = A2h \] (2)
\[ A = A1 + A2 \] (3)
\[ \varepsilon = \frac{A}{n} \] (4)

The state of system can be described by the values \((x,z)\), where \(x\) is the number of busy trunks in the full access group of n-k trunks, whereas \(z\) denotes the sum of the number of busy trunks of the k exclusive trunks for traffic A2 and the number \((z-k)\) of occupied waiting places in the queue. The stationary probability that the state \((x,z)\) exists be denoted by \(p(x,z)\).

3.2. State Equations of Model No. 1

The state diagram for model No. 1 is shown in fig. 3. According to this diagram the following equations are obtained:

\[ Ap(0,0) = p(1,0) + p(0,1) \] (5a)
\[ (A+x)p(x,0) = A1p(x-1,0) + (x+1)p(x+1,0) + (z+1)p(x,z+1), \quad 0 < x < n-k \] (5b)
\[ (A+z)p(0,z) = p(1,z) + (z+1)p(0,z+1) + A2p(x,z-1), \quad 0 < z < k \] (5c)
\[ (A+k)p(0,k) = p(1,k) + A2p(0,k-1) \] (5d)
\[ (A+k)p(x,z) = p(1,k)A2p(x,z-1) + (x+1)p(x+1,z) + (z+1)p(x,z+1) + A2p(x,z-1), \quad 0 < x < n-k, \quad 0 < z < k \] (5f)

\[ (A2+n-k)p(n-k,z) = A1p(n-k-1,z) \]
\[ + (z+1)p(n-k,z+1) + A2p(n-k,z-1), \quad 0 < z < k \] (5g)
\[ (A+x+k)p(x,k) = A1p(x-1,k) + (x+1)p(x+1,k) + A2p(x,z-1), \quad 0 < x < m \] (5h)
\[ (A+m+k)p(m,z) = (m+1)p(m+1,z) + (k+m)p(m,k+1) + A2p(m,k-1) \] (5i)
\[ (A+m+k)p(m,k) = A1p(m-k-1,z) + (z+1)p(m,k-1), \quad k < z < k+s \] (5j)
\[ (A2+n)p(n,z) = A1p(n-k-1,z) + (z+1)p(n-k,z+1) \] (5k)
\[ + (k+m)p(n-k,z-1) \]

including the normalizing condition

\[ \sum_{x=0}^{n-k} \sum_{z=0}^{k} p(x,z) + \sum_{x=m}^{n-k} \sum_{z=0}^{k} p(x,z) = 1. \] (5q)

Fig. 3 State diagram of model No. 1

3.3. State equations of model No. 2

The second model differs from the first model by the service discipline as described above. Therefore the following set of state equations is obtained according to the schematic state diagram in fig. 4.

\[ Ap(0,0) = p(1,0) + p(0,1) \] (6a)
\[ (A+x)p(x,0) = A1p(x-1,0) + (x+1)p(x+1,0) + p(x+1), \quad 0 < x < m \] (6b)
\[ (A+x)p(x,0) = A1p(x-1,0) + (x+1)p(x+1,0) + p(x+1), \quad m < x < n-k \] (6c)
\[ (A2+n-k)p(n-k,0) = A1p(n-k-1,0) + p(n-k,1) \] (6d)
\[(A+z)p(0,z) = p(1,z)+(z+1)p(0,z+1), \quad 0 \leq z < k \quad (6e)\]
\[(A+k)p(0,k) = p(1,k) \quad (6f)\]
\[(A+x+z)p(x,z) = Ap(x-1,z)+(x+1)p(x+1,z)+(z+1)p(x+1,z+1)+Ap(x,z+1), \quad 0 < z < k \quad (6g)\]
\[(A+m+z)p(m,z) = Ap(m-1,z)+(m+1)p(m+1,z)+(z+1)p(m+1,z+1)+Ap(m,z+1), \quad 0 < z < k \quad (6h)\]
\[(A+x+z)p(x,z) = Ap(x-1,z)+(x+1)p(x+1,z)+(z+1)p(x,z+1)+Ap(x,z+1), \quad m < x < n-k, \quad 0 < z < k \quad (6i)\]
\[(A+m+k)p(m,k) = Ap(m-1,k)+(m+1)p(m+1,k)+(k+m)p(m,k+1)+Ap(m,k-1) \quad (6j)\]
\[(A+m+k)p(m,z) = (m+1)p(m+1,z)+(k+m)p(m,z+1)+Ap(m,z-1), \quad k < z < k+s \quad (6k)\]
\[(A+1+m+k)p(m,k+s) = (m+1)p(m+1,k+s)+(k+m)p(m,k+s)+Ap(m,k+s-1) \quad (6l)\]
\[(n-k)D = \frac{(n-k)l}{(n-k)k+s} = \frac{(n-k)(k+s)}{n-k} \quad (8)\]

Fig. 4 State diagram for model No. 2

3.4. Calculation of state probabilities

Exact values of the state probabilities for both systems have been calculated iteratively by means of the so-called successive overrelaxation (SOR) method [3].

3.5. Characteristic traffic values

The loss probabilities \(B_1, B_2\) corresponding to both types of traffic \(A_1\) and \(A_2\) can be determined according to the equations

\[B_1 = \sum_{z=0}^{n-k} p(n-k,z) \quad (7)\]
\[B_2 = \sum_{x=m}^{n} p(x,k+s) \quad (8)\]

The total loss probability \(B_t\) results in

\[B_t = \frac{A_1B_1+A_2B_2}{A} \quad (9)\]

For the waiting probability \(P_w\) of calls of the traffic \(A_2\) the following formula holds true

\[P_w = \sum_{x=m}^{n-k} \sum_{z=k}^{k+s} p(x,z) \quad (10)\]

The mean queue length \(Q\) results in

\[Q = \sum_{z=k+1}^{k+s} \sum_{x=m}^{n} (z-k)p(x,z) \quad (11)\]

According to Little's theorem the following formulae are obtained for the mean waiting time \(t_w\) of all incoming calls of the traffic \(A_2\) and for the mean waiting time \(t_{w1}\) of all waiting calls

\[t_w = \frac{Q}{A_2} \quad (12)\]
\[t_{w1} = \frac{Q}{A_2} \quad (13)\]

3.6. Example No. 1

As an example, a system according to model No. 1 (with \(n=30\) trunks, restriction parameter \(m=13\) and \(s=12\) waiting places) and a system according to model No. 2 (with \(n=30, m=12\) and \(s=15\)) be considered. The basic offered traffic values (without overload) are \(A_{1n} = A_{2n} = 11\) Erlangs in both cases. The diagram in fig. 5 shows the behavior of the loss probabilities \(B_1\) and \(B_2\) and the total loss probability \(B_t\), the waiting probability \(P_w\) and the mean waiting time of \(t_{w1}\) of all waiting calls.
From the diagrams shown in fig. 5 and fig. 6 it can be seen that model No. 1 as well as model No. 2 are suitable for the realizing of the desired overload protection. In the example considered above, however, model No. 1 may be given preference as it enables a slightly larger carried total traffic (24.75 Erlangs in model No. 1 versus 24.09 Erlangs in model No. 2) in case of overload of $A_1$.

4. WAITING TIME DISTRIBUTION FOR SYSTEMS WITH UNIFORM MEAN HOLDING TIME

Calls of the traffic $A_2$ have to wait if $m$ or more of the $n-k$ trunks and all of the $k$ trunks (which are used exclusively for calls of $A_2$) are busy and less than $s$ waiting places are occupied, in both models. It is supposed that waiting calls do not give up before being served. The state spaces of the waiting processes, which are identical for the service disciplines in both models, are shown in fig. 7.

4.1. The waiting process

It is well known that the waiting process can be considered as a random walk of waiting calls. The waiting time starts with the arrival of a call of $A_2$ in a state $(x,z)$ on waiting place $(x,z+1)$ and is terminated by a transition into the taboo set, which is equivalent to the states $(x,k)$ indicated at the left border of the state space of the waiting process shown in fig. 7. During this process $x$ and $z$ may vary in the ranges $m \leq x \leq n-k$ and $k+1 \leq z \leq k+s$.

It is obvious that this process is not concerned by further waiting calls, arriving after a considered test call, because the calls are served according to "FIFO" discipline.

$$\begin{align*}
q(x,z,x+1,z) &= A_1 \frac{A_1}{A_1n} \\
q(x,z,x-1,z) &= \sum_{z'=z+s}^{k+1} q(x,z,x',z') \\
q(x,z,m+1,k) &= \sum_{z'=z+s}^{k+1} q(x,z,m+1,k) \\
q(x,z,m,k) &= \sum_{z'=z+s}^{k+1} q(x,z,m,k)
\end{align*}$$

Fig. 7 State diagram of the waiting process

4.2 The waiting time distribution

In the sequel the probability that a waiting call lasts longer than the time $t$ under the condition that the call starts waiting in a state $(x,z)$ be denoted by $w(t|x,z)$. The conditional probability for a transition from a state $(x,z)$ to a state $(x',z')$ (not belonging to the taboo set) be denoted by $q(x,z,x',z')$ and the conditional probability that a state $(x,z)$ is left by any transition to another state (which may also belong to the taboo set) be denoted by $q(x,z)$. According to the state diagram in fig. 7 the following values for these transition coefficients are obtained:

$$\begin{align*}
q(x,z,x+1,z) &= \lambda_1 \\
q(x,z,x-1,z) &= \sum_{z'=z+s}^{k+1} q(x,z,x',z') \\
q(x,z,m+1,k) &= \sum_{z'=z+s}^{k+1} q(x,z,m+1,k) \\
q(x,z,m,k) &= \sum_{z'=z+s}^{k+1} q(x,z,m,k)
\end{align*}$$
\[ q(x,z,x',z') = 0 \quad \text{for all other values} \]

For the conditional (complementary) distribution function \( w(t|x,z) \), the following set of differential equations holds true which is well known as the so-called Kolmogorov backward equation:

\[
\frac{d}{dt} w(t|x,z) = -q(x,z)w(t|x,z) + q(x,z,x+1,z)w(t|x+1,z) + q(x,z,x-1,z)w(t|x-1,z) + q(x,z,x,z-1)w(t|x,z-1) \quad (15)
\]

The fact that the waiting time of waiting calls is greater than 0 leads to the following formula for the initial conditions \( w(0|x,z) \):

\[
w(0|x,z) = 1, \quad m \leq x < n-k, k+1 \leq z \leq k+s \quad (16)
\]

With the set of differential equations (15) and these initial conditions (16) the conditional waiting times have been calculated with the aid of the Runge-Kutta method [3]. Then the absolute (complementary) distribution function \( w^*(t) \) of the waiting time of arbitrary calls can be determined by the following equation:

\[
w^*(t) = \frac{1}{w} \sum_{x=m}^{n-k+k+s} \sum_{t=k}^{s} p(x,z-1) w(t|x,z) \quad (17)
\]

5. Systems with unequal mean holding times

This section deals with systems in which the offered traffic \( A_1 \) and the offered traffic \( A_2 \) have different holding times \( h_1 \) and \( h_2 \) and, consequently, different terminating rates \( \lambda_1 = 1/h_1 \) and \( \lambda_2 = 1/h_2 \), respectively. For this case of different holding times, both models (as described in section 2) have been investigated [10]. In this paper, however, only the calculation methods for model No. 1 can be presented for lack of space.

In this case a new description of states is necessary. Now a triple of parameters \((x,u,z)\) is required to describe a state of the system, where \( x \) denotes the number of calls of \( A_1 \) being served by the group of \( n-k \) trunks, \( u \) the number of calls of \( A_2 \) being served in this group and \( z \) the total number of calls which are being served by one of the \( k \) trunks or waiting. The corresponding state diagram is shown in fig. 9.

5.1. The equations of state

Let \( p(x,u,z) \) be the probability that the system is in a state \((x,u,z)\). \( \lambda_1 \) and \( \lambda_2 \) are the arrival rates and \( \lambda_1 \) and \( \lambda_2 \) are the terminating rates of the two kinds of offered traffic \( A_1 \) \( (A_1 = \lambda_1 / \epsilon_1) \) and \( A_2 \) \( (A_2 = \lambda_2 / \epsilon_2) \) respectively.

In the sequel a boolean expression \( d_{i,j} \) is used with the meaning

\[
d_{i,j} = 0 \quad \text{if} \quad i = j \quad \text{and} \quad (19)
\]

\[
d_{i,j} = 1 \quad \text{if} \quad i \neq j \quad (20)
\]

\( d_{i,j} \) is the boolean complement of \( d_{i,j} \). For the considered system according to state diagram shown in fig. 9 the following equations of state are obtained

\[
\begin{align*}
(\lambda_1 + \lambda_2 + \epsilon_1 p(x,u,z)) p(x,u,z) &= d_{x,0} \lambda_1 p(x,u,z-1) + (z+1) \epsilon_1 p(x,u+1,z) + d_{x,0} \lambda_2 p(x-1,u,z) + (x+1) \epsilon_1 p(x+1,u,z), \\
0 &\leq x < m, 0 \leq u < m, x+u+k < k \quad (21a)
\end{align*}
\]

\[
\begin{align*}
(\lambda_1 + \lambda_2 + \epsilon_1 p(x,u,z)) p(x,u,k) &= d_{x,0} \lambda_1 p(x,u,k-1) + d_{x,0} \lambda_2 p(x,u-1,k) + (u+1) \epsilon_1 p(x+1,u,k) + (x+1) \epsilon_1 p(x+1,u,k), \\
0 &\leq x < m, 0 \leq u < m, x+u+k < k \quad (21b)
\end{align*}
\]
with the normalizing condition

\[ \sum_{x=0}^{m-k-1} \sum_{u=0}^{m} \sum_{z=0}^{k+s} p(x,u,z) = 1 \]  

These equations have been solved with the aid of the successive overrelaxation (SOR) method [3].

5.2. Characteristic traffic values

From the steady state probabilities as explained above, the following characteristic traffic values can be calculated.

The loss probabilities \( B_1 \) and \( B_2 \) can be determined as follows:

\[ B_1 = \sum_{x=0}^{m-k-1} \sum_{u=0}^{m} \sum_{z=0}^{k+s} p(n-k-u,u,z), \quad (22) \]

\[ B_2 = \sum_{x=0}^{m} \sum_{z=0}^{m-k-u} p(x,u,k+s). \quad (23) \]

The total loss probability results in

\[ B_t = \frac{A_1 B_1 + A_2 B_2}{A_1 + A_2}. \quad (24) \]

Furthermore, the following formulae are obtained for the waiting probability \( p_w \)

\[ p_w = \sum_{z=k}^{k+s} \sum_{u=0}^{m} \sum_{x=m-u}^{m} p(x,u,z), \quad (25) \]

the mean queue length \( Q \)

\[ Q = \sum_{z=k+1}^{k+s} \sum_{n=0}^{m-k-u} p(x,u,z), \quad (26) \]

the mean waiting time \( t_w \) with respect to waiting calls

\[ t_w = \frac{Q}{\lambda_2} = \frac{Q \epsilon_2}{\lambda_2 h_2} \quad (27) \]

and the mean waiting time with respect to waiting calls

\[ t_w^* = \frac{Q}{\lambda_2} = \frac{Q \epsilon_2}{\lambda_2 h_2}. \quad (28) \]

5.3 Example No. 3

For model No. 1 with the parameters of example No. 1 and the mean holding time ratio \( h_1/h_2 = 10 \) the following loss probabilities are obtained:

\( B_1 = 0.0046 \) and \( B_2 = 0.0352 \) without overload (N), \( B_1 = 0.0051 \) and \( B_2 = 0.1086 \) when \( A_1 \) is overloaded (OV1) and \( B_1 = 0.0047 \) and \( B_2 = 0.4175 \) when \( A_2 \) is overloaded (OV2).

The mean waiting time with respect to the random walk of waiting calls

\[ t_w^* = \frac{Q}{\lambda_2} = \frac{Q \epsilon_2}{\lambda_2 h_2} \quad (27) \]

and the mean waiting time with respect to waiting calls

\[ t_w = \frac{Q}{\lambda_2} = \frac{Q \epsilon_2}{\lambda_2 h_2}. \quad (28) \]

6. WAITING TIME DISTRIBUTION

6.1. Waiting process

For systems with different holding times of \( A_1 \) and \( A_2 \), a triple of parameters \((x,u,z)\) is used to describe the random walk of waiting calls. In this model a call arriving in a state \((x,u,z)\) of system in which \( x+u \leq m \) trunks of the common group and \( k+s \leq z+k+s \) of the \( k \) exclusive trunks and \( s \) waiting places are occupied, is assumed to start the waiting process in a state \((x,u,z+1)\) of the random walk. The waiting process terminates when it enters a state of the taboo set \((x,k+1)\), which is equivalent to the states in which the considered call is served.

In the sequel the probability that an arbitrary call, which starts waiting in state \((x,u,z)\), lasts longer than \( t \) be denoted by \( w(t|x,u,z) \). The conditional probability density for a transition from the state \((x,u,z)\) to a state \((x',u',z')\) not belonging to the taboo set by \( q(x,u,z) \) and the conditional probability density for a transition from the state \((x,u,z)\) to a state \((x',u',z')\) not belonging to the taboo set by \( q(x,u,z) \) and the conditional probability densities have the following values:

\[ q(x,u,z) = dx_n-k \lambda_1^x \epsilon_1^{(u+k)} \epsilon_2 \quad (29a) \]

\[ q(x,u,z,x+1,u,z) = dx_n-k \lambda_1 \quad (29b) \]

\[ q(x,u,z,z-1,u,z+1) = dx_n-k \lambda_1 \quad (29c) \]
The probability densities for all other transitions are equal to zero. (For $d_{i,j}$ see equation (19) and (20)).

For the conditional (complementary) distribution functions $w(t|x,u,z)$, the following set of differential equations holds true, according to the Kolmogorov backward equation:

$$\frac{d}{dt} w(t|x,u,z) = -q(x,u,z)w(t|x,u,z) + q(x,u,z,x+1,u,z)w(t|x+1,u,z) + q(x,u,z,x-1,u,z)w(t|x-1,u,z) + q(x,u,z,x,u-1,z)w(t|x,u-1,z) + q(x,u,z,x,u,z-1)w(t|x,u,z-1) + q(x,u,z,x-1,x+1,z-1)w(t|x-1,u+1,z-1),$$  

with the initial conditions

$$w(0|x,u,z) = 1, \quad k \leq u \leq m, \quad m-u \leq x \leq n-k-u,$$

$$k+1 \leq z \leq k+s.$$  

With this set of differential equations and the corresponding initial conditions the conditional distribution functions can be calculated, e.g., with the aid of the Runge-Kutta method [3].

Finally, the absolute (complementary) distribution function $w(t)$ of the waiting time of arbitrary calls is obtained as

$$w(t) = \sum_{u=0}^{m} \sum_{x=m-u}^{n-k-u} \sum_{z=k+s}^{k+1} p(x,u,z-1)w(t|x,u,z).$$

The absolute (complementary) distribution function $w^*(t)$ of the total waiting time with respect to waiting calls is

$$w^*(t) = \frac{1}{p_w} w(t).$$

6.2. Example No. 4

![Fig 10. Waiting time distribution](image)

For the system considered in example No. 3, with the holding time ratio $h_1/h_2 = 10$, the waiting time distributions $w^*(t)$ for waiting calls according to model No. 1 are shown for a situation without overload $N$ and for both overflow situations $Ov_1$ and $Ov_2$ in fig. 10.

7. CONCLUSION

In this paper, two different models of switching systems are presented, which are not only able to protect calls of the traffic $A_1$ against overload of traffic $A_2$, but also to prevent extremely high impairment of calls of the traffic $A_2$ if $A_1$ is overloads.

Such systems may, e.g., be applied at the integration of voice and data, therefore, besides a performance analysis of both models with equal holding times a different holding time model is investigated.

For all models, the state probabilities and the resulting characteristic traffic values as well as the waiting time distributions are calculated. Results are illustrated in examples and diagrams.

8. REFERENCES


